A Tour Of Sage

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This is a tour of Sage that closely follows the tour of Mathematica that is at the beginning of the Mathematica Book.
The Sage command line has a `sage:` prompt; you do not have to add it. If you use the Sage notebook, then put everything after the `sage:` prompt in an input cell, and press shift-enter to compute the corresponding output.

```
sage: 3 + 5
8
```

The caret symbol means “raise to a power”.

```
sage: 57.1 ^ 100
4.60904368661396e175
```

We compute the inverse of a $2 \times 2$ matrix in Sage.

```
sage: matrix([[1,2], [3,4]])^(-1)
[ -2 1]
[ 3/2 -1/2]
```

Here we integrate a simple function.

```
sage: x = var('x')  # create a symbolic variable
sage: integrate(sqrt(x)*sqrt(1+x), x)
1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) - 1) + 1/8*log(sqrt(x + 1)/sqrt(x) + 1)
```

This asks Sage to solve a quadratic equation. The symbol `==` represents equality in Sage.

```
sage: a = var('a')
sage: S = solve(x^2 + x == a, x); S
[x == -1/2*sqrt(4*a + 1) - 1/2, x == 1/2*sqrt(4*a + 1) - 1/2]
```

The result is a list of equalities.

```
sage: S[0].rhs()
-1/2*sqrt(4*a + 1) - 1/2
```

Naturally, Sage can plot various useful functions.

```
sage: show(plot(sin(x) + sin(1.6*x), 0, 40))
```
Chapter 1. Sage as a Calculator
First we create a $500 \times 500$ matrix of random numbers.

```
sage: m = random_matrix(RDF,500)
```

It takes Sage a few seconds to compute the eigenvalues of the matrix and plot them.

```
sage: e = m.eigenvalues()  #about 2 seconds
sage: w = [(i, abs(e[i])) for i in range(len(e))]
sage: show(points(w))
```
Thanks to the GNU Multiprecision Library (GMP), Sage can handle very large numbers, even numbers with millions or billions of digits.

```
sage: factorial(100)
93326215443944152681699233885626670049071596826438162146859296389521759993229915608941443976156518281280
sage: n = factorial(1000000)  #about 2.5 seconds
This computes at least 100 digits of π.
```

```
sage: N(pi, digits=100)
3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253921170679
```

This asks Sage to factor a polynomial in two variables.

```
sage: R.<x,y> = QQ[]
sage: F = factor(x^99 + y^99)
sage: F
(x + y) * (x^2 - x*y + y^2) * (x^6 - x^3*y^3 + y^6) * (x^10 - x^9*y + x^8*y^2 - x^7*y^3 + x^6*y^4 - x^5*y^5 + x^4*y^6 - x^3*y^7 + x^2*y^8 - x*y^9 + y^10) *
(x^20 + x^19*y - x^17*y^3 - x^16*y^4 + x^14*y^6 + x^13*y^7 -
```
x^{11}y^9 - x^{10}y^{10} - x^9y^{11} + x^7y^{13} + x^6y^{14} -
 x^4y^{16} - x^3y^{17} + xy^{19} + y^{20} \times (x^60 + x^57y^3 -
 x^51y^9 - x^48y^{12} + x^42y^{18} + x^39y^{21} - x^33y^{27} -
 x^30y^{30} - x^27y^{33} + x^21y^39 + x^18y^42 - x^12y^{48} -
 x^9y^51 + x^3y^57 + y^{60})
 sage: F.expand()
x^{99} + y^{99}

Sage takes just under 5 seconds to compute the numbers of ways to partition one hundred million as a sum of positive integers.

sage: z = Partitions(10^8).cardinality()  #about 4.5 seconds
sage: str(z)[:40]
'1760517045946249141360373894679135204009'
ACCESSING ALGORITHMS IN SAGE

Whenever you use Sage you are accessing one of the world’s largest collections of open source computational algorithms.