A Tour Of Sage

Release 10.3

The Sage Development Team

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CONTENTS

This is a short tour of Sage as a calculator.

The Sage command line has a prompt "`sage:`". To experiment with the following examples, you only enter the part after the prompt.

```
sage: 3 + 5
8
```

If you use Sage on the Jupyter notebook, then likewise put everything after the prompt in an input cell, and press `Shift-Enter` to get the corresponding output.

The caret symbol means "raise to a power".

```
sage: 57.1^100
4.60904368661396e175
```

We compute the inverse of a 2×2 matrix in Sage.

```
sage: matrix([[1, 2], [3, 4]])^(-1)
[  -2    1]
[ 3/2 -1/2]
```

Here we integrate a simple function.

```
sage: x = var('x')   # create a symbolic variable
sage: integrate(sqrt(x) * sqrt(1 + x), x)
1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1)/x + 1)
- 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) - 1)
```

This asks Sage to solve a quadratic equation. The symbol == represents equality in Sage.

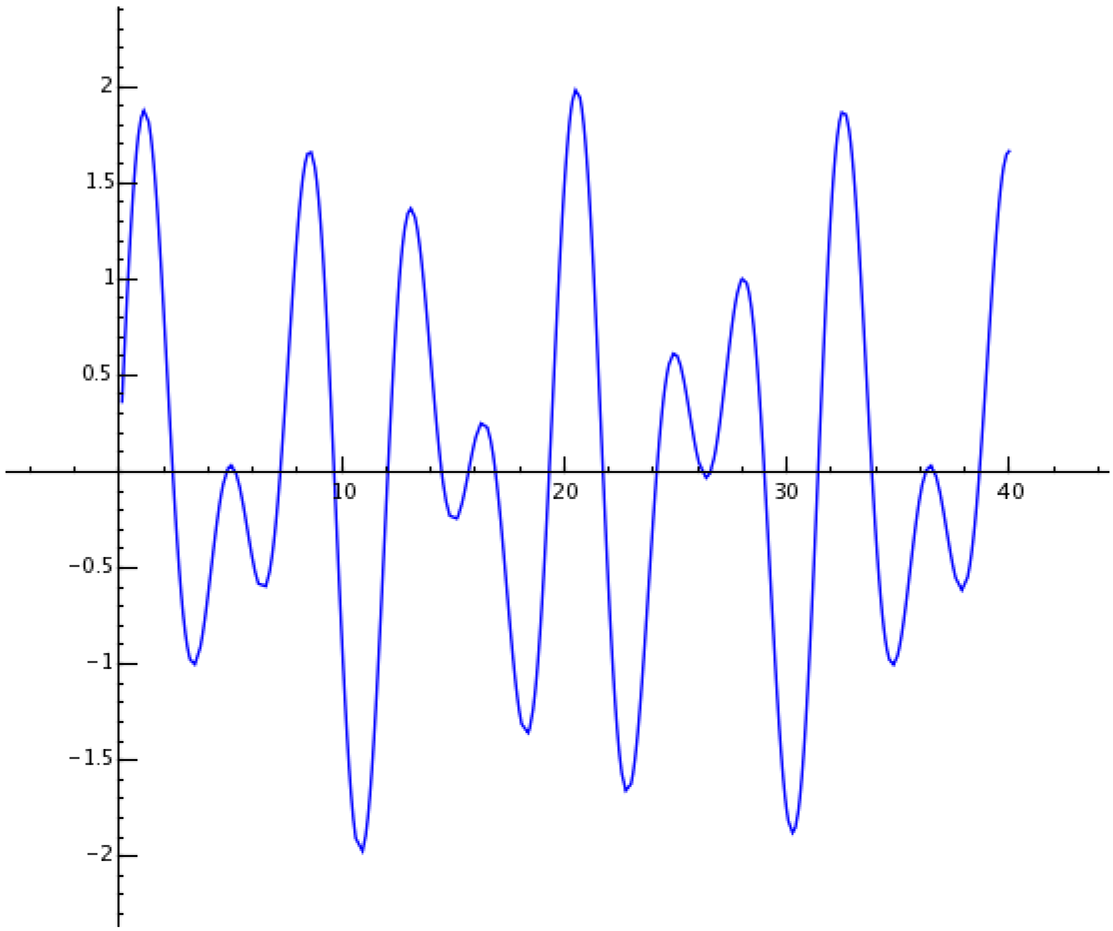
```
sage: a = var('a')
sage: S = solve(x^2 + x == a, x); S
[x == -1/2*sqrt(4*a + 1) - 1/2, x == 1/2*sqrt(4*a + 1) - 1/2]
```

The result is a list of equalities.

```
sage: S[0].rhs()  # right hand side of the equation
-1/2*sqrt(4*a + 1) - 1/2
```

Sage can plot various useful functions, of course.

```
sage: show(plot(sin(x) + sin(1.6*x), 0, 40))
```

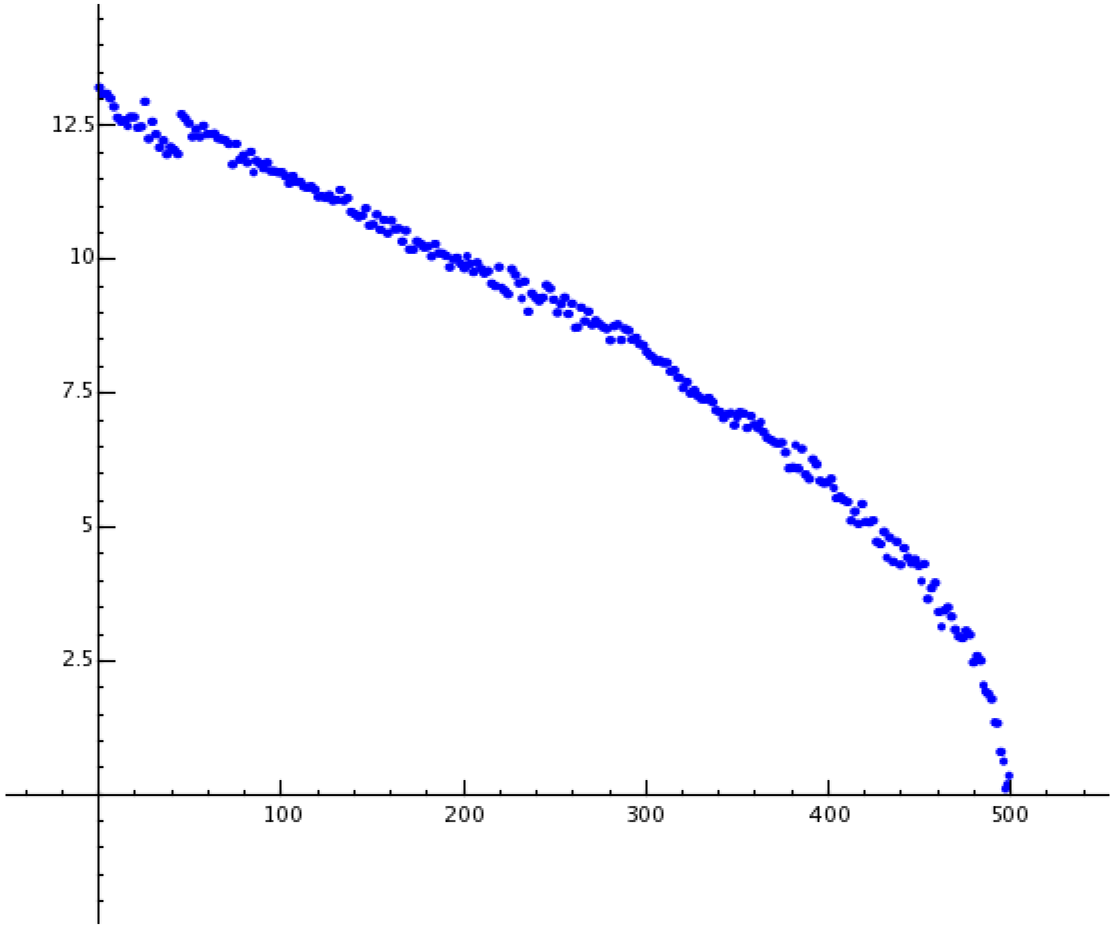


Sage is a very powerful calculator. To experience it, first we create a 500×500 matrix of random numbers.

```
sage: m = random_matrix(RDF, 500)
```

It takes Sage a second to compute the eigenvalues of the matrix and plot them.

```
sage: e = m.eigenvalues()  # about 1 second
sage: w = [(i, abs(e[i])) for i in range(len(e))]
sage: show(points(w))
```



Sage can handle very large numbers, even numbers with millions or billions of digits.

```
sage: factorial(100)
9332621544394415268169923885626670049071596826438162146859296389521759999322991560894146397615651828
```

```
sage: n = factorial(1000000)   # about 1 second
sage: len(n.digits())
5565709
```

This computes at least 100 digits of π.

```
sage: N(pi, digits=100)
3.
↪1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170
```

This asks Sage to factor a polynomial in two variables.

```
sage: R.<x,y> = QQ[]
sage: F = factor(x^99 + y^99)
sage: F
(x + y) * (x^2 - x*y + y^2) * (x^6 - x^3*y^3 + y^6) *
(x^10 - x^9*y + x^8*y^2 - x^7*y^3 + x^6*y^4 - x^5*y^5 +
 x^4*y^6 - x^3*y^7 + x^2*y^8 - x*y^9 + y^10) *
```

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```
(x^20 + x^19*y - x^17*y^3 - x^16*y^4 + x^14*y^6 + x^13*y^7 -
 x^11*y^9 - x^10*y^10 - x^9*y^11 + x^7*y^13 + x^6*y^14 -
 x^4*y^16 - x^3*y^17 + x*y^19 + y^20) * (x^60 + x^57*y^3 -
 x^51*y^9 - x^48*y^12 + x^42*y^18 + x^39*y^21 - x^33*y^27 -
 x^30*y^30 - x^27*y^33 + x^21*y^39 + x^18*y^42 - x^12*y^48 -
 x^9*y^51 + x^3*y^57 + y^60)
sage: F.expand()
x^99 + y^99
```

Sage takes less than 1 second to compute the numbers of ways to partition one hundred million as a sum of positive integers.

```
sage: z = Partitions(10^8).cardinality()  # about .1 second
sage: z
1760517045946249141360373894679135204009...
```

Sage is the world's most advanced open source math software.