Numerical Optimization

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The Sage Development Team

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This module implements a number of solutions to various knapsack problems, otherwise known as linear integer pro-
gramming problems. Solutions to the following knapsack problems are implemented:

- Solving the subset sum problem for super-increasing sequences.
- General case using Linear Programming

AUTHORS:
- Minh Van Nguyen (2009-04): initial version
- Nathann Cohen (2009-08): Linear Programming version

1.1 Definition of Knapsack problems

You have already had a knapsack problem, so you should know, but in case you do not, a knapsack problem is what happens when you have hundred of items to put into a bag which is too small, and you want to pack the most useful of them.

When you formally write it, here is your problem:

- Your bag can contain a weight of at most $W$.
- Each item $i$ has a weight $w_i$.
- Each item $i$ has a usefulness $u_i$.

You then want to maximize the total usefulness of the items you will store into your bag, while keeping sure the weight of the bag will not go over $W$.

As a linear program, this problem can be represented this way (if you define $b_i$ as the binary variable indicating whether the item $i$ is to be included in your bag):

Maximize: $\sum_i b_i u_i$

Such that: $\sum_i b_i w_i \leq W$

$\forall i, b_i$ binary variable

(For more information, see the Wikipedia article Knapsack_problem)
1.2 Examples

If your knapsack problem is composed of three items (weight, value) defined by (1,2), (1.5,1), (0.5,3), and a bag of maximum weight 2, you can easily solve it this way:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([[(1,2), (1.5,1), (0.5,3)], max=2])
[5.0, [(1, 2), (0.500000000000000, 3)]]
```

1.3 Super-increasing sequences

We can test for whether or not a sequence is super-increasing:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: seq = Superincreasing(L)
sage: seq
Super-increasing sequence of length 8
sage: seq.is_superincreasing()
True
sage: Superincreasing().is_superincreasing([1,3,5,7])
False
```

Solving the subset sum problem for a super-increasing sequence and target sum:

```
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).subset_sum(98)
[69, 21, 5, 2, 1]
```

class sage.numerical.knapsack.Superincreasing(seq=None)

Bases: SageObject

A class for super-increasing sequences.

Let \( L = (a_1, a_2, a_3, \ldots, a_n) \) be a non-empty sequence of non-negative integers. Then \( L \) is said to be super-increasing if each \( a_i \) is strictly greater than the sum of all previous values. That is, for each \( a_i \in L \) the sequence \( L \) must satisfy the property

\[
a_i > \sum_{k=1}^{i-1} a_k
\]

in order to be called a super-increasing sequence, where \(|L| \geq 2\). If \( L \) has only one element, it is also defined to be a super-increasing sequence.

If \seq\ is \None, then construct an empty sequence. By definition, this empty sequence is not super-increasing.

INPUT:

- \seq\ – (default: \None) a non-empty sequence.

EXAMPLES:
is_superincreasing(seq=None)

Determine whether or not seq is super-increasing.

If seq=None then determine whether or not self is super-increasing.

Let $L = (a_1, a_2, a_3, \ldots, a_n)$ be a non-empty sequence of non-negative integers. Then $L$ is said to be super-increasing if each $a_i$ is strictly greater than the sum of all previous values. That is, for each $a_i \in L$ the sequence $L$ must satisfy the property

$$a_i > \sum_{k=1}^{i-1} a_k$$

in order to be called a super-increasing sequence, where $|L| \geq 2$. If $L$ has exactly one element, then it is also defined to be a super-increasing sequence.

INPUT:

- seq – (default: None) a sequence to test

OUTPUT:

- If seq=None, then test self to determine whether or not it is super-increasing. In that case, return True if self is super-increasing; False otherwise.
- If seq is not None, then test seq to determine whether or not it is super-increasing. Return True if seq is super-increasing; False otherwise.

EXAMPLES:

By definition, an empty sequence is not super-increasing:

```python
sage: from sage.numerical.knapsack import Superincreasing
sage: Superincreasing().is_superincreasing([])
False
sage: Superincreasing().is_superincreasing()
False
sage: Superincreasing().is_superincreasing(tuple())
False
sage: Superincreasing().is_superincreasing(())
False
```

But here is an example of a super-increasing sequence:
A super-increasing sequence can have zero as one of its elements:

```
L = [0, 1, 2, 4]
```

```
sage: Superincreasing(L).is_superincreasing()
True
```

A super-increasing sequence can be of length 1:

```
sage: Superincreasing([randint(0, 100)]).is_superincreasing()
True
```

`largest_less_than(N)`

Return the largest integer in the sequence self that is less than or equal to N.

This function narrows down the candidate solution using a binary trim, similar to the way binary search halves the sequence at each iteration.

**INPUT:**

- N – integer; the target value to search for.

**OUTPUT:**

The largest integer in self that is less than or equal to N. If no solution exists, then return None.

**EXAMPLES:**

When a solution is found, return it:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [2, 3, 7, 25, 67, 179, 356, 819]
sage: Superincreasing(L).largest_less_than(207)
179
sage: L = (2, 3, 7, 25, 67, 179, 356, 819)
sage: Superincreasing(L).largest_less_than(2)
2
```

But if no solution exists, return None:

```
sage: L = [2, 3, 7, 25, 67, 179, 356, 819]
sage: Superincreasing(L).largest_less_than(-1) is None
True
```

`subset_sum(N)`

Solving the subset sum problem for a super-increasing sequence.

Let $S = (s_1, s_2, s_3, \ldots, s_n)$ be a non-empty sequence of non-negative integers, and let $N \in \mathbb{Z}$ be non-negative. The subset sum problem asks for a subset $A \subseteq S$ of whose elements sum to $N$. This method specializes the subset sum problem to the case of super-increasing sequences. If a solution exists, then it is also a super-increasing sequence.
Note: This method only solves the subset sum problem for super-increasing sequences. In general, solving the subset sum problem for an arbitrary sequence is known to be computationally hard.

INPUT:
- \( N \) – a non-negative integer.

OUTPUT:
- A non-empty subset of \( \text{self} \) whose elements sum to \( N \). This subset is also a super-increasing sequence. If no such subset exists, then return the empty list.

ALGORITHMS:
The algorithm used is adapted from page 355 of [HPS2008].

EXAMPLES:
Solving the subset sum problem for a super-increasing sequence and target sum:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).subset_sum(98)
[69, 21, 5, 2, 1]
```

```
sage.numerical.knapsack.knapsack(seq, binary, max=True, value_only=1, solver=False, verbose=None, integrality_tolerance=0)
```

Solves the knapsack problem

For more information on the knapsack problem, see the documentation of the \texttt{knapsack module} or the Wikipedia article \texttt{Knapsack problem}.

INPUT:
- \( \text{seq} \) – Two different possible types:
  - A sequence of tuples \((\text{weight}, \text{value}, \text{something1}, \text{something2}, \ldots)\). Note that only the first two coordinates \((\text{weight} \text{ and values})\) will be taken into account. The rest (if any) will be ignored. This can be useful if you need to attach some information to the items.
  - A sequence of reals (a value of 1 is assumed).
- \( \text{binary} \) – When set to \text{True}, an item can be taken 0 or 1 time. When set to \text{False}, an item can be taken any amount of times (while staying integer and positive).
- \( \text{max} \) – Maximum admissible weight.
- \( \text{value\_only} \) – When set to \text{True}, only the maximum useful value is returned. When set to \text{False}, both the maximum useful value and an assignment are returned.
- \( \text{solver} \) – (default: \text{None}) Specify a Mixed Integer Linear Programming (MILP) solver to be used. If set to \text{None}, the default one is used. For more information on MILP solvers and which default solver is used, see the method \text{solve} of the class \texttt{MixedIntegerLinearProgram}.
- \( \text{verbose} \) – integer (default: 0). Sets the level of verbosity. Set to 0 by default, which means quiet.
- \( \text{integrality\_tolerance} \) – parameter for use with MILP solvers over an inexact base ring; see \texttt{MixedIntegerLinearProgram.get\_values}.

OUTPUT:
If `value_only` is set to `True`, only the maximum useful value is returned. Else (the default), the function returns a pair `[value, list]`, where `list` can be of two types according to the type of `seq`:

- The list of tuples \((w_i, u_i, \ldots)\) occurring in the solution.
- A list of reals where each real is repeated the number of times it is taken into the solution.

**EXAMPLES:**

If your knapsack problem is composed of three items \((\text{weight, value})\) defined by \((1, 2), (1.5, 1), (0.5, 3)\), and a bag of maximum weight 2, you can easily solve it this way:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([(1,2), (1.5,1), (0.5,3)], max=2)  
[5.0, [(1, 2), (0.500000000000000, 3)]]
sage: knapsack([(1,2), (1.5,1), (0.5,3)], max=2, value_only=True)  
5.0
```

Besides weight and value, you may attach any data to the items:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([(1, 2, 'spam'), (0.5, 3, 'a', 'lot')])  
[3.0, [(0.500000000000000, 3, 'a', 'lot')]]
```

In the case where all the values (usefulness) of the items are equal to one, you do not need embarrass yourself with the second values, and you can just type for items \((1, 1), (1.5, 1), (0.5, 1)\) the command:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([1,1.5,0.5], max=2, value_only=True)  
2.0
```
This module implements classes and methods for the efficient solving of Linear Programs (LP) and Mixed Integer Linear Programs (MILP).

Do you want to understand how the simplex method works? See the interactive_simplex_method module (educational purposes only)

2.1 Definition

A linear program (LP) is an optimization problem (Wikipedia article Optimization_(mathematics)) in the following form

$$\max\{c^T x \mid Ax \leq b, x \geq 0\}$$

with given $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$ and unknown $x \in \mathbb{R}^n$. If some or all variables in the vector $x$ are restricted over the integers $\mathbb{Z}$, the problem is called mixed integer linear program (MILP). A wide variety of problems in optimization can be formulated in this standard form. Then, solvers are able to calculate a solution.

2.2 Example

Imagine you want to solve the following linear system of three equations:

- $w_0 + w_1 + w_2 - 14w_3 = 0$
- $w_1 + 2w_2 - 8w_3 = 0$
- $2w_2 - 3w_3 = 0$

and this additional inequality:

- $w_0 - w_1 - w_2 \geq 0$

where all $w_i \in \mathbb{Z}^+$. You know that the trivial solution is $w_i = 0$, but what is the first non-trivial one with $w_3 \geq 1$?

A mixed integer linear program can give you an answer:

1. You have to create an instance of MixedIntegerLinearProgram and – in our case – specify that it is a minimization.
2. Create a dictionary $w$ of non-negative integer variables $w$ via $w = p.new_variable(integer=True, nonnegative=True)$.
3. Add those three equations as equality constraints via add_constraint.
4. Also add the inequality constraint.
5. Add an inequality constraint $w_3 \geq 1$ to exclude the trivial solution.

6. Specify the objective function via \texttt{set_objective}. In our case that is just $w_3$. If it is a pure constraint satisfaction problem, specify it as \texttt{None}.

7. To check if everything is set up correctly, you can print the problem via \texttt{show}.

8. \texttt{Solve} it and print the solution.

The following example shows all these steps:

```python
sage: p = MixedIntegerLinearProgram(maximization=False, solver="GLPK")
sage: w = p.new_variable(integer=True, nonnegative=True)
sage: p.add_constraint(w[3] >= 1)
sage: p.set_objective(w[3])
sage: p.show()

Minimization:

$x_3$

Constraints:

0.0 <= x_0 + x_1 + x_2 - 14.0 x_3 <= 0.0
0.0 <= x_1 + 2.0 x_2 - 8.0 x_3 <= 0.0
0.0 <= 2.0 x_2 - 3.0 x_3 <= 0.0
- x_0 + x_1 + x_2 <= 0.0
- x_3 <= -1.0

Variables:

x_0 is an integer variable (min=0.0, max=+oo)
x_1 is an integer variable (min=0.0, max=+oo)
x_2 is an integer variable (min=0.0, max=+oo)
x_3 is an integer variable (min=0.0, max=+oo)

sage: print('Objective Value: {}'.format(p.solve()))
Objective Value: 2.0

sage: for i, v in sorted(p.get_values(w, convert=ZZ, tolerance=1e-3).items()):
    print(f'w_{i} = {v}')
    w_0 = 15
    w_1 = 10
    w_2 = 3
    w_3 = 2
```

Different backends compute with different base fields, for example:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.base_ring()
Real Double Field
sage: x = p.new_variable(real=True, nonnegative=True)
sage: 0.5 + 3/2*x[1]
0.5 + 1.5*x_0

sage: p = MixedIntegerLinearProgram(solver='ppl')
sage: p.base_ring()
Rational Field
sage: x = p.new_variable(nonnegative=True)
```

(continues on next page)
2.3 More about MIP variables

The underlying MILP backends always work with matrices where each column corresponds to a linear variable. The variable corresponding to the \( i \)-th column (counting from 0) is displayed as \( x_i \).

*MixedIntegerLinearProgram* maintains a dynamic mapping from the arbitrary keys indexing the components of MIPVariable objects to the backend variables (indexed by nonnegative integers). Backend variables are created when a component of a MIPVariable is accessed.

To make your code more readable, you can construct one or several MIPVariable objects that can be arbitrarily named and indexed. This can be done by calling new_variable() several times, or by the following special syntax:

```
sage: mip.<a,b> = MixedIntegerLinearProgram(solver='GLPK')
sage: a
MIPVariable a with 0 real components
sage: 5 + a[1] + 2*b[3]
5 + x_0 + 2*x_1
```

Indices can be any object, not necessarily integers. Multi-indices are also allowed:

```
sage: a[4, 'string', QQ]
x_2
x_2 - 7*x_3
sage: mip.show()
Maximization:
Constraints:
Variables:
  a[1] = x_0 is a continuous variable (min=-oo, max=+oo)
  b[3] = x_1 is a continuous variable (min=-oo, max=+oo)
  a[4, 'string', Rational Field] = x_2 is a continuous variable (min=-oo, max=+oo)
  b[2] = x_3 is a continuous variable (min=-oo, max=+oo)
```

Upper/lower bounds on a variable can be specified either as separate constraints (see add_constraint) or using the methods set_max and set_min respectively.

2.4 The default MIP variable

As a special shortcut, it is not necessary to call new_variable(). A MixedIntegerLinearProgram has a default MIPVariable, whose components are obtained by using the syntax mip[key], where key is an arbitrary key:

```
sage: mip = MixedIntegerLinearProgram(solver='GLPK')
5 + x_0 + 2*x_1
```
2.5 Index of functions and methods

Below are listed the methods of MixedIntegerLinearProgram. This module also implements the MIPSolverException exception, as well as the MIPVariable class.

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<th>Description</th>
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<td>Adds a constraint to the MixedIntegerLinearProgram</td>
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<td>base_ring()</td>
<td>Return the base ring</td>
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<td>best_known_objective_bound()</td>
<td>Return the value of the currently best known bound</td>
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<td>constraints()</td>
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<td>Returns the minimum value of a variable</td>
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<td>Tests whether the variable e is binary</td>
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<td>Tests whether the variable is an integer</td>
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<td>Tests whether the variable is real</td>
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<td>Return the parent for all linear functions</td>
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<td>new_variable()</td>
<td>Returns an instance of MIPVariable associated</td>
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<td>Returns the number of constraints assigned so far</td>
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<td>Returns the polyhedron defined by the Linear Program</td>
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<td>Removes a constraint from self</td>
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<td>remove_constraints()</td>
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<td>set_binary()</td>
<td>Sets a variable or a MIPVariable as binary</td>
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<tr>
<td>set_integer()</td>
<td>Sets a variable or a MIPVariable as integer</td>
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<tr>
<td>set_max()</td>
<td>Sets the maximum value of a variable</td>
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<td>set_objective()</td>
<td>Sets the objective of the MixedIntegerLinearProgram</td>
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<td>set_problem_name()</td>
<td>Sets the name of the MixedIntegerLinearProgram</td>
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<td>set_real()</td>
<td>Sets a variable or a MIPVariable as real</td>
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<td>show()</td>
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<td>Solves the MixedIntegerLinearProgram</td>
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<td>Write the linear program as a MPS file</td>
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AUTHORS:
- Risan (2012/02): added extension for exact computation

exception sage.numerical.mip.MIPSolverException
Bases: RuntimeError

Exception raised when the solver fails.

EXAMPLES:
class sage.numerical.mip.MIPVariable

Bases: SageObject

MIPVariable is a variable used by the class MixedIntegerLinearProgram.

Warning: You should not instantiate this class directly. Instead, use MixedIntegerLinearProgram.new_variable().

copy_for_mip(mip)

Returns a copy of self suitable for a new MixedIntegerLinearProgram instance mip.

For this to make sense, mip should have been obtained as a copy of self.mip().

EXAMPLES:

sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: pv = p.new_variable(nonnegative=True)
sage: pv[0]
x_0
sage: q = copy(p)
sage: qv = pv.copy_for_mip(q)
sage: qv[77]
x_1
sage: p.number_of_variables()
2
sage: q.number_of_variables()
1
sage: qv[33]
x_1
sage: p.number_of_variables()
2
sage: q.number_of_variables()
2
sage: qv[3]
x_0
sage: qv[5]
Traceback (most recent call last):
...
IndexError: 5 does not index a component of MIPVariable with 2 real components

items()
Return the pairs (keys,value) contained in the dictionary.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.items())
[(0, x_0), (1, x_1)]
```

`keys()`

Return the keys already defined in the dictionary.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.keys())
[0, 1]
```

`mip()`

Returns the `MixedIntegerLinearProgram` in which `self` is a variable.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p == v.mip()
True
```

`set_max(max)`

Sets an upper bound on the variable.

INPUT:

- `max` – an upper bound, or `None` to mean that the variable is unbounded.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(real=True, nonnegative=True)
sage: p.set_max(v)
sage: p.get_max(v[0])
4
sage: p.set_max(v,4)
4
sage: p.get_max(v)
4
```

`set_min(min)`

Sets a lower bound on the variable.

INPUT:

- `min` – a lower bound, or `None` to mean that the variable is unbounded.
EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(real=True, nonnegative=True)
sage: p.get_min(v)
0
sage: p.get_min(v[0])
0.0
sage: p.set_min(v,4)
sage: p.get_min(v)
4
sage: p.get_min(v[0])
4.0
```

`values()`:

Return the symbolic variables associated to the current dictionary.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.values(), key=str)
[x_0, x_1]
```

class `sage.numerical.mip.MixedIntegerLinearProgram`

Bases: `SageObject`

The `MixedIntegerLinearProgram` class is the link between Sage, linear programming (LP) and mixed integer programming (MIP) solvers.

A Mixed Integer Linear Program (MILP) consists of variables, linear constraints on these variables, and an objective function which is to be maximised or minimised under these constraints.

See the thematic tutorial on Linear Programming (Mixed Integer) or Wikipedia article Linear programming for further information on linear programming, and the MILP module for its use in Sage.

INPUT:

- `solver` – selects a solver; see Solvers (backends) for more information and installation instructions for optional solvers.
  - `solver="GLPK"`: The GNU Linear Programming Kit.
  - `solver="GLPK/exact"`: GLPK’s implementation of an exact rational simplex method.
  - `solver="Coin"`: The COIN-OR CBC (COIN Branch and Cut) solver.
  - `solver="CPLEX"`, provided by the proprietary IBM ILOG CPLEX Optimization Studio.
  - `solver="Gurobi"`: The proprietary Gurobi solver.
  - `solver="CVXOPT"`: See the CVXOPT web site.
  - `solver="PPL"`: An exact rational solver (for small scale instances) provided by the Parma Polyhedra Library (PPL).
  - `solver="InteractiveLP"`: A didactical implementation of the revised simplex method in Sage. It works over any exact ordered field, the default is QQ.
  - If `solver=None` (default), the default solver is used (see `default_mip_solver()`).
– solver can also be a callable (such as a class), see `sage.numerical.backends.generic_backend.get_solver()` for examples.

- maximization
  – When set to True (default), the MixedIntegerLinearProgram is defined as a maximization.
  – When set to False, the MixedIntegerLinearProgram is defined as a minimization.

- constraint_generation – Only used when solver=None.
  – When set to True, after solving the MixedIntegerLinearProgram, it is possible to add a constraint, and then solve it again. The effect is that solvers that do not support this feature will not be used.
  – Defaults to False.

See also:

- `default_mip_solver()` – Returns/Sets the default MIP solver.

EXAMPLES:

Computation of a maximum stable set in Petersen’s graph:

```python
sage: # needs sage.graphs
sage: g = graphs.PetersenGraph()
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: b = p.new_variable(binary=True)
sage: p.set_objective(sum([b[v] for v in g]))
```

```python
sage: for (u,v) in g.edges(sort=False, labels=None):
    ....:     p.add_constraint(b[u] + b[v], max=1)
```

```python
sage: p.solve(objective_only=True)
4.0
```

`add_constraint(linear_function, max=None, min=None, name=None, return_indices=False)`

adds a constraint to the MixedIntegerLinearProgram.

INPUT:

- `linear_function` – Four different types of arguments are admissible:
  – A linear function. In this case, one of the arguments min or max has to be specified.
  – A linear constraint of the form A <= B, A >= B, A <= B <= C, A >= B >= C or A == B.
  – A vector-valued linear function, see `linear_tensor`. In this case, one of the arguments min or max has to be specified.
  – An (in)equality of vector-valued linear functions, that is, elements of the space of linear functions tensored with a vector space. See `linear_tensor_constraints` for details.

- `max` – constant or None (default). An upper bound on the linear function. This must be a numerical value for scalar linear functions, or a vector for vector-valued linear functions. Not allowed if the linear_function argument is a symbolic (in)-equality.

- `min` – constant or None (default). A lower bound on the linear function. This must be a numerical value for scalar linear functions, or a vector for vector-valued linear functions. Not allowed if the linear_function argument is a symbolic (in)-equality.

- `name` – A name for the constraint.

- `return_indices` – boolean (optional, default False), whether to return the indices of the added constraints.
OUTPUT:

The row indices of the constraints added, if return_indices is true and the backend guarantees that removing them again yields the original MIP, None otherwise.

To set a lower and/or upper bound on the variables use the methods set_min and/or set_max of MixedIntegerLinearProgram.

EXAMPLES:

Consider the following linear program:

Maximize:
   \[ x + 5 * y \]
Constraints:
   \[ x + 0.2 \cdot y \leq 4 \]
   \[ 1.5 \cdot x + 3 \cdot y \leq 4 \]
Variables:
   \( x \) is Real (min = 0, max = None)
   \( y \) is Real (min = 0, max = None)

It can be solved as follows:

```sage
p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
x = p.new_variable(nonnegative=True)
p.set_objective(x[0] + 5*x[1])
p.add_constraint(x[0] + 0.2*x[1], max=4)
p.add_constraint(1.5*x[0] + 3*x[1], max=4)
p.solve()     # rel tol 1e-15
6.666666666666666
```

There are two different ways to add the constraint \( x[5] + 3 \cdot x[7] \leq x[6] + 3 \) to a MixedIntegerLinearProgram.

The first one consists in giving add_constraint this very expression:

```sage
```

The second (slightly more efficient) one is to use the arguments min or max, which can only be numerical values:

```sage
```

One can also define double-bounds or equality using symbols <=, >= and ==:

```sage
```

Using this notation, the previous program can be written as:

```sage
p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
x = p.new_variable(nonnegative=True)
p.set_objective(x[0] + 5*x[1])
p.add_constraint(x[0] + 0.2*x[1], <= 4)
p.add_constraint(1.5*x[0] + 3*x[1] <= 4)
p.solve()     # rel tol 1e-15
6.666666666666666
```
The two constraints can also be combined into a single vector-valued constraint:

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + 5*x[1])
sage: f_vec = vector([1, 1.5]) * x[0] + vector([0.2, 3]) * x[1]; f_vec
(1.0, 1.5)*x_0 + (0.2, 3.0)*x_1
sage: p.add_constraint(f_vec, max=vector([4, 4]))
sage: p.solve()    # rel tol 1e-15
6.666666666666666
```

Instead of specifying the maximum in the optional `max` argument, we can also use (in)equality notation for vector-valued linear functions:

```python
sage: f_vec <= 4    # constant rhs becomes vector
(1.0, 1.5)*x_0 + (0.2, 3.0)*x_1 <= (4.0, 4.0)
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + 5*x[1])
sage: p.add_constraint(f_vec <= 4)
sage: p.solve()    # rel tol 1e-15
6.666666666666666
```

Finally, one can use the matrix * `MIPVariable` notation to write vector-valued linear functions:

```python
sage: m = matrix([[1.0, 0.2], [1.5, 3.0]]); m
[ 1.00000000000000 0.200000000000000]
[ 1.50000000000000 3.000000000000000]
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + 5*x[1])
sage: p.add_constraint(m * x <= 4)
sage: p.solve()    # rel tol 1e-15
6.666666666666666
```

**base_ring()**

Return the base ring.

**OUTPUT:**

A ring. The coefficients that the chosen solver supports.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.base_ring()
Real Double Field
sage: p = MixedIntegerLinearProgram(solver='ppl')
sage: p.base_ring()
Rational Field
```

(continues on next page)
Algebraic Real Field

```
sage: # needs sage.groups sage.rings.number_field
sage: d = polytopes.dodecahedron()
sage: p = MixedIntegerLinearProgram(base_ring=d.base_ring())
sage: p.base_ring()
Number Field in sqrt5 with defining polynomial x^2 - 5 with sqrt5 = 2.
```

**best_known_objective_bound()**

Return the value of the currently best known bound.

This method returns the current best upper (resp. lower) bound on the optimal value of the objective function in a maximization (resp. minimization) problem. It is equal to the output of `get_objective_value()` if the MILP found an optimal solution, but it can differ if it was interrupted manually or after a time limit (cf `solver_parameter()`).

**Note:** Has no meaning unless `solve` has been called before.

**EXAMPLES:**

```
sage: # needs sage.groups
sage: g = graphs.CubeGraph(9)
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("mip_gap_tolerance",100)
sage: b = p.new_variable(binary=True)
sage: p.set_objective(p.sum(b[v] for v in g))
sage: for v in g:
    ....:    p.add_constraint(b[v] + p.sum(b[u] for u in g.neighbors(v)) <= 1)
sage: p.add_constraint(b[v] == 1)  # Force an easy non-0 solution
sage: p.solve()  # rel tol 100
1.0
sage: p.best_known_objective_bound()  # random
48.0
```

**constraints(indices=None)**

Returns a list of constraints, as 3-tuples.

**INPUT:**

- **indices** – select which constraint(s) to return
  - If `indices` = `None`, the method returns the list of all the constraints.
  - If `indices` is an integer `i`, the method returns constraint `i`.
  - If `indices` is a list of integers, the method returns the list of the corresponding constraints.

**OUTPUT:**

Each constraint is returned as a triple `lower_bound, (indices, coefficients), upper_bound`. For each of those entries, the corresponding linear function is the one associating to variable `indices[i]` the coefficient `coefficients[i]`, and 0 to all the others.

`lower_bound` and `upper_bound` are numerical values.
EXAMPLES:

First, let us define a small LP:

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(p[0] - p[2], min=1, max=4)
sage: p.add_constraint(p[0] - 2*p[1], min=1)
```

To obtain the list of all constraints:

```sage
sage: p.constraints()  # not tested
[(1.0, ([1, 0], [-1.0, 1.0]), 4.0), (1.0, ([2, 0], [-2.0, 1.0]), None)]
```

Or constraint 0 only:

```sage
sage: p.constraints(0)  # not tested
(1.0, ([1, 0], [-1.0, 1.0]), 4.0)
```

A list of constraints containing only 1:

```sage
sage: p.constraints([1])  # not tested
[(1.0, ([2, 0], [-2.0, 1.0]), None)]
```

default_variable()

Return the default \texttt{MIPVariable} of \texttt{self}.

EXAMPLES:

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.default_variable()
MIPVariable with 0 real components
```

get_backend()

Returns the backend instance used.

This might be useful when access to additional functions provided by the backend is needed.

EXAMPLES:

This example uses the simplex algorithm and prints information:

```sage
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max=6)
sage: p.add_constraint(3*x + 2*y, max=6)
sage: p.set_objective(x + y + 7)
sage: b = p.get_backend()
sage: b.solver_parameter("simplex_or_intopt", "simplex_only")
sage: b.solver_parameter("verbosity_simplex", "GLP_MSG_ALL")
sage: ans = p.solve()
GLPK Simplex Optimizer...
2 rows, 2 columns, 4 non-zeros
* 0: obj = 7.0000000000e+00 inf = 0.000e+00 (2)
* 2: obj = 9.4000000000e+00 inf = 0.000e+00 (0)
OPTIMAL LP SOLUTION FOUND
sage: ans  # rel tol 1e-5
9.4
```
**get_max**(v)
Returns the maximum value of a variable.

INPUT:

• v – a variable.

OUTPUT:

Maximum value of the variable, or None if the variable has no upper bound.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_max(v[1])
sage: p.set_max(v[1],6)
sage: p.get_max(v[1])
6.0
```

**get_min**(v)
Returns the minimum value of a variable.

INPUT:

• v – a variable

OUTPUT:

Minimum value of the variable, or None if the variable has no lower bound.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_min(v[1])
sage: p.set_min(v[1],6)
sage: p.get_min(v[1])
6.0
sage: p.set_min(v[1], None)
sage: p.get_min(v[1])
None
```

**get_objective_value()**
Return the value of the objective function.

**Note:** Behaviour is undefined unless solve has been called before.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max=6)
sage: p.add_constraint(3*x + 2*y, max=6)
sage: p.set_objective(x + y + 7)
```

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sage: p.solve()  # rel tol 1e-5
9.4
sage: p.get_objective_value()  # rel tol 1e-5
9.4

get_relative_objective_gap()

Return the relative objective gap of the best known solution.

For a minimization problem, this value is computed by \((\text{bestinteger} - \text{bestobjective})/(1e - 10 + |\text{bestobjective}|)\), where \text{bestinteger} is the value returned by \text{get_objective_value()} and \text{bestobjective} is the value returned by \text{best_known_objective_bound()}. For a maximization problem, the value is computed by \((\text{bestobjective} - \text{bestinteger})/(1e - 10 + |\text{bestobjective}|)\).

Note: Has no meaning unless \text{solve} has been called before.

EXAMPLES:

```python
sage: # needs sage.graphs
sage: g = graphs.CubeGraph(9)
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("mip_gap_tolerance",100)
sage: b = p.new_variable(binary=True)
sage: p.set_objective(p.sum(b[v] for v in g))
```

```python
sage: for v in g:
    ....:     p.add_constraint(b[v] + p.sum(b[u] for u in g.neighbors(v)) <= 1)
sage: p.add_constraint(b[v] == 1)  # Force an easy non-0 solution
sage: p.solve()  # rel tol 100
1.0
sage: p.get_relative_objective_gap()  # random
46.99999999999999
```

get_values(convert=None, tolerance=None, *lists)

Return values found by the previous call to \text{solve()}.  

INPUT:

- *lists – any instance of \text{MIPVariable} (or one of its elements), or lists of them.
- convert – \text{None} (default), \text{ZZ}, \text{bool}, or \text{True}.
  - if convert=\text{None} (default), return all variable values as the backend provides them, i.e., as an element of \text{base_ring()} or a float.
  - if convert=\text{ZZ}, convert all variable values from the \text{base_ring()} by rounding to the nearest integer.
  - if convert=\text{bool}, convert all variable values from the \text{base_ring()} by rounding to 0/1 and converting to bool.
  - if convert=\text{True}, use \text{ZZ} for MIP variables declared integer or binary, and convert the values of all other variables to the \text{base_ring()}.
- tolerance – \text{None}, a positive real number, or 0 (if \text{base_ring()} is an exact ring). Required if convert is not \text{None} and any integer conversion is to be done. If the variable value differs from the nearest integer by more than tolerance, raise a \text{RuntimeError}.  

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OUTPUT:

- Each instance of `MIPVariable` is replaced by a dictionary containing the numerical values found for each corresponding variable in the instance.
- Each element of an instance of a `MIPVariable` is replaced by its corresponding numerical value.

**Note:** While a variable may be declared as binary or integer, its value is an element of the `base_ring()`, or for the numerical solvers, a `float`.

For the numerical solvers, `base_ring()` is RDF, an inexact ring. Code using `get_values` should always account for possible numerical errors.

Even for variables declared as binary or integer, or known to be an integer because of the mathematical properties of the model, the returned values cannot be expected to be exact integers. This is normal behavior of the numerical solvers.

For correct operation, any user code needs to avoid exact comparisons (==, !=) and instead allow for numerical tolerances. The magnitude of the numerical tolerances depends on both the model and the solver.

The arguments `convert` and `tolerance` facilitate writing correct code. See examples below.

**EXAMPLES:**

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: y = p.new_variable(nonnegative=True)
sage: p.solve()
6.0
```

To return the value of `y[2,9]` in the optimal solution:

```sage
sage: p.get_values(y[2,9])
2.0
sage: type(_)
<class 'float'>
```

To convert the value to the `base_ring()`:

```sage
sage: p.get_values(y[2,9], convert=True)
2.0
sage: _.parent()
Real Double Field
```

To get a dictionary identical to `x` containing the values for the corresponding variables in the optimal solution:

```sage
sage: x_sol = p.get_values(x)
sage: sorted(x_sol)
[3, 5]
```

Obviously, it also works with variables of higher dimension:

```sage
sage: y_sol = p.get_values(y)
```
We could also have tried

```
sage: [x_sol, y_sol] = p.get_values(x, y)
```

Or:

```
sage: [x_sol, y_sol] = p.get_values([x, y])
```

Using convert and tolerance. First, a binary knapsack:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(binary=True)
sage: p.set_objective(3*x[1] + 4*x[2] + 5*x[3])
sage: p.solve()
8.0
sage: x_opt = p.get_values(x); x_opt
{1: 1.0, 2: 0.0, 3: 1.0}
sage: type(x_opt[1])
<class 'float'>
sage: x_opt_ZZ = p.get_values(x, convert=True, tolerance=1e-6); x_opt_ZZ
{1: 1, 2: 0, 3: 1}
sage: x_opt_ZZ[1].parent()
Integer Ring
sage: x_opt_bool = p.get_values(x, convert=bool, tolerance=1e-6); x_opt_bool
{1: True, 2: False, 3: True}
```

Thanks to total unimodularity, single-commodity network flow problems with integer capacities and integer supplies/demands have integer vertex solutions. Hence the integrality of solutions is mathematically guaranteed in an optimal solution if we use the simplex algorithm. A numerical LP solver based on the simplex method such as GLPK will return an integer solution only up to a numerical error. Hence, for correct operation, we should use tolerance:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK', maximization=False)
sage: x = p.new_variable(nonnegative=True)
sage: x.set_max(1)
sage: p.add_constraint(x['sa'] + x['sb'] == 1)
sage: p.add_constraint(x['sa'] + x['ba'] - x['ab'] - x['at'] == 0)
sage: p.add_constraint(x['sb'] + x['ab'] - x['ba'] - x['bt'] == 0)
sage: p.set_objective(10*x['sa'] + 10*x['bt'])
sage: p.solve()
0.0
sage: x_opt = p.get_values(x); x_opt
{'ab': 0.0, 'at': 1.0, 'ba': 1.0, 'bt': -0.0, 'sa': 0.0, 'sb': 1.0}
sage: x_opt_ZZ = p.get_values(x, convert=ZZ, tolerance=1e-6); x_opt_ZZ
{'ab': 0, 'at': 1, 'ba': 1, 'bt': 0, 'sa': 0, 'sb': 1}
```

```
interactive_lp_problem(form='standard')

Returns an InteractiveLPPProblem and, if available, a basis.
```

INPUT:

- **form** – (default: "standard") a string specifying return type: either None, or "std" or "standard", respectively returns an instance of InteractiveLPPProblem or of InteractiveLPPProblemStandardForm
OUTPUT:

A 2-tuple consists of an instance of class `InteractiveLPProblem` or `InteractiveLPProblemStandardForm` that is constructed based on a given `MixedIntegerLinearProgram`, and a list of basic variables (the basis) if standard form is chosen (by default), otherwise None.

All variables must have 0 as lower bound and no upper bound.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(names=['m'], solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: y = p.new_variable(nonnegative=True, name='n')
sage: v = p.new_variable(nonnegative=True)
sage: p.add_constraint(x[0] + x[1] - 7*y[0] + v[0] <= 2, name='K')
sage: p.add_constraint(x[1] + 2*y[0] - v[0] <= 3)
sage: p.add_constraint(5*x[0] + y[0] <= 21, name='L')
sage: p.set_objective(2*x[0] + 3*x[1] + 4*y[0] + 5*v[0])
sage: lp, basis = p.interactive_lp_problem()
sage: basis
['K', 'w_1', 'L']
sage: lp.constraint_coefficients()
[ 1.0  1.0 -7.0  1.0]
[ 0.0  1.0  2.0 -1.0]
[ 5.0  0.0  1.0  0.0]
sage: lp.b()
(2.0, 3.0, 21.0)
sage: lp.objective_coefficients()
(2.0, 3.0, 4.0, 5.0)
sage: lp.decision_variables()
(m_0, m_1, n_0, x_3)
sage: view(lp) #not tested
sage: d = lp.dictionary(*basis)
sage: view(d) #not tested
```

`is_binary(e)`

Tests whether the variable e is binary. Variables are real by default.

INPUT:

- e – A variable (not a `MIPVariable`, but one of its elements.)

OUTPUT:

True if the variable e is binary; False otherwise.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_binary(v[1])
False
sage: p.set_binary(v[1])
sage: p.is_binary(v[1])
True
```
**is_integer(e)**
Tests whether the variable is an integer. Variables are real by default.

**INPUT:**
- e – A variable (not a MIPVariable, but one of its elements.)

**OUTPUT:**
True if the variable e is an integer; False otherwise.

**EXAMPLES:**

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_integer(v[1])
False
sage: p.set_integer(v[1])
sage: p.is_integer(v[1])
True
```

**is_real(e)**
Tests whether the variable is real.

**INPUT:**
- e – A variable (not a MIPVariable, but one of its elements.)

**OUTPUT:**
True if the variable is real; False otherwise.

**EXAMPLES:**

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_real(v[1])
True
sage: p.set_binary(v[1])
sage: p.is_real(v[1])
False
sage: p.set_real(v[1])
sage: p.is_real(v[1])
True
```

**linear_constraints_parent()**
Return the parent for all linear constraints

See `linear_functions` for more details.

**EXAMPLES:**

```sage
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p linear_constraints_parent()
Linear constraints over Real Double Field
```
linear_functions_parent()
Return the parent for all linear functions

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.linear_functions_parent()
Linear functions over Real Double Field
```

new_variable(real=False, binary=False, integer=False, nonnegative=False, name='', indices=None)
Return a new `MIPVariable` instance.

A new variable $x$ is defined by:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
```

It behaves exactly as a usual dictionary would. It can use any key argument you may like, as $x[5]$ or $x[\text{"b"]}$, and has methods `items()` and `keys()`.

See also:

- `set_min()`, `get_min()` – set/get the lower bound of a variable.
- `set_max()`, `get_max()` – set/get the upper bound of a variable.

INPUT:

- `binary`, `integer`, `real` – boolean. Set one of these arguments to `True` to ensure that the variable gets the corresponding type.
- `nonnegative` – boolean, default `False`. Whether the variable should be assumed to be nonnegative. Rather useless for the binary type.
- `name` – string. Associates a name to the variable. This is only useful when exporting the linear program to a file using `write_mps` or `write_lp`, and has no other effect.
- `indices` – (optional) an iterable of keys; components corresponding to these keys are created in order, and access to components with other keys will raise an error; otherwise components of this variable can be indexed by arbitrary keys and are created dynamically on access

OUTPUT:

A new instance of `MIPVariable` associated to the current `MixedIntegerLinearProgram`.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(); x
MIPVariable with 0 real components
sage: x0 = x[0]; x0
x_0
```

By default, variables are unbounded:

```
sage: print(p.get_min(x0))
None
sage: print(p.get_max(x0))
None
```
To define two dictionaries of variables, the first being of real type, and the second of integer type

```python
sage: x = p.new_variable(real=True, nonnegative=True)
sage: y = p.new_variable(integer=True, nonnegative=True)
sage: p.add_constraint(x[2] + y[3,5], max=2)
sage: p.is_integer(x[2])
False
sage: p.is_integer(y[3,5])
True
```

An exception is raised when two types are supplied

```python
sage: z = p.new_variable(real=True, integer=True)
Traceback (most recent call last):
... ValueError: Exactly one of the available types has to be True
```

Unbounded variables:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(real=True)
sage: y = p.new_variable(integer=True)
sage: p.add_constraint(x[0] + x[3] <= 8)
sage: p.add_constraint(y[0] >= y[1])
sage: p.show()
Maximization:
Constraints:
  x_0 + x_1 <= 8.0
  - x_2 + x_3 <= 0.0
Variables:
  x_0 is a continuous variable (min=-oo, max=+oo)
  x_1 is a continuous variable (min=-oo, max=+oo)
  x_2 is an integer variable (min=-oo, max=+oo)
  x_3 is an integer variable (min=-oo, max=+oo)
```

On the Sage command line, generator syntax is accepted as a shorthand for generating new variables with default settings:

```python
sage: mip.<x, y, z> = MixedIntegerLinearProgram(solver='GLPK')
sage: mip.show()
Maximization:
Constraints:
Variables:
  x[0] = x_0 is a continuous variable (min=-oo, max=+oo)
  y[1] = x_1 is a continuous variable (min=-oo, max=+oo)
  z[2] = x_2 is a continuous variable (min=-oo, max=+oo)
```

`number_of_constraints()`

Return the number of constraints assigned so far.

EXAMPLES:
```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(p[0] - p[2], min=1, max=4)
sage: p.add_constraint(p[0] - 2*p[1], min=1)
sage: p.number_of_constraints()
2
```

**number_of_variables()**

Returns the number of variables used so far.

Note that this is backend-dependent, i.e. we count solver’s variables rather than user’s variables. An example of the latter can be seen below: Gurobi converts double inequalities, i.e. inequalities like \( m \leq c^T x \leq M \), with \( m < M \), into equations, by adding extra variables: \( c^T x + y = M \), \( 0 \leq y \leq M - m \).

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(p[0] - p[2], max=4)
sage: p.number_of_variables()
2
sage: p.add_constraint(p[0] - 2*p[1], min=1)
sage: p.number_of_variables()
3
sage: p = MixedIntegerLinearProgram(solver="glpk")
sage: p.add_constraint(p[0] - p[2], min=1, max=4)
sage: p.number_of_variables()
2
sage: p = MixedIntegerLinearProgram(solver="gurobi")  # optional - Gurobi
sage: p.add_constraint(p[0] - p[2], min=1, max=4)  # optional - Gurobi
sage: p.number_of_variables()  # optional - Gurobi
3
```

**polyhedron(****kwds)**

Returns the polyhedron defined by the Linear Program.

INPUT:

All arguments given to this method are forwarded to the constructor of the `Polyhedron()` class.

OUTPUT:

A `Polyhedron()` object whose \( i \)-th variable represents the \( i \)-th variable of `self`.

**Warning:** The polyhedron is built from the variables stored by the LP solver (i.e. the output of `show()`). While they usually match the ones created explicitly when defining the LP, a solver like Gurobi has been known to introduce additional variables to store constraints of the type `lower_bound <= linear_function <= upper_bound`. You should be fine if you did not install Gurobi or if you do not use it as a solver, but keep an eye on the number of variables in the polyhedron, or on the output of `show()`. Just in case.

**See also:**

- `to_linear_program()` — return the `MixedIntegerLinearProgram` object associated with a `Polyhedron()` object.

**EXAMPLES:**
A LP on two variables:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(0 <= 2*p['x'] + p['y'] <= 1)
sage: p.add_constraint(0 <= 3*p['y'] + p['x'] <= 2)

A 2-dimensional polyhedron in RDF^2 defined as the convex hull of 4 vertices
```

3-D Polyhedron:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(0 <= 2*p['x'] + p['y'] + 3*p['z'] <= 1)
sage: p.add_constraint(0 <= 2*p['y'] + p['z'] + 3*p['x'] <= 1)
sage: p.add_constraint(0 <= 2*p['z'] + p['x'] + 3*p['y'] <= 1)

A 3-dimensional polyhedron in RDF^3 defined as the convex hull of 8 vertices
```

An empty polyhedron:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.add_constraint(2*v['x'] + v['y'] + 3*v['z'] <= 1)
sage: p.add_constraint(2*v['y'] + v['z'] + 3*v['x'] <= 1)
sage: p.add_constraint(2*v['z'] + v['x'] + 3*v['y'] >= 2)

The empty polyhedron in RDF^3
```

An unbounded polyhedron:

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.add_constraint(2*p['x'] + p['y'] - p['z'] <= 1)

A 3-dimensional polyhedron in RDF^3 defined as the convex hull of 1 vertex, 1 → ray, 2 lines
```

A square (see github issue #14395)

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x,y = p['x'], p['y']
sage: p.add_constraint(x <= 1)
sage: p.add_constraint(x >= -1)
sage: p.add_constraint(y <= 1)
sage: p.add_constraint(y >= -1)

A 2-dimensional polyhedron in RDF^2 defined as the convex hull of 4 vertices
```

We can also use a backend that supports exact arithmetic:

```python
sage: p = MixedIntegerLinearProgram(solver='PPL')

(continues on next page)
**remove_constraint(i)**

Removes a constraint from self.

**INPUT:**

- i – Index of the constraint to remove.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x, y = p[0], p[1]
sage: p.add_constraint(x + y, max=10)
sage: p.add_constraint(x - y, max=0)
sage: p.add_constraint(x, max=4)
sage: p.show()
Maximization:
Constraints:
  x_0 + x_1 <= 10.0
  x_0 - x_1 <= 0.0
  x_0 <= 4.0
...
sage: p.remove_constraint(1)
sage: p.show()
Maximization:
Constraints:
  x_0 + x_1 <= 10.0
  x_0 <= 4.0
...
sage: p.number_of_constraints()
2
```

**remove_constraints(constraints)**

Remove several constraints.

**INPUT:**

- constraints – an iterable containing the indices of the rows to remove.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x, y = p[0], p[1]
sage: p.add_constraint(x + y, max=10)
sage: p.add_constraint(x - y, max=0)
sage: p.add_constraint(x, max=4)
sage: p.show()
Maximization:
Constraints:
  x_0 + x_1 <= 10.0
  x_0 - x_1 <= 0.0
  x_0 <= 4.0
...
sage: p.remove_constraints([1])
sage: p.show()
Maximization:
Constraints:
  x_0 + x_1 <= 10.0
...
```

(continues on next page)
When checking for redundant constraints, make sure you remove only the constraints that were actually added. Problems could arise if you have a function that builds lps non-interactively, but it fails to check whether adding a constraint actually increases the number of constraints. The function might later try to remove constraints that are not actually there:

```python
sage: p = MixedIntegerLinearProgram(check_redundant=True, solver='GLPK')
sage: x, y = p[0], p[1]
sage: for each in range(10):
    ....:     p.add_constraint(x + y, max=10)
sage: p.add_constraint(x, max=4)
sage: p.number_of_constraints() 3
sage: p.remove_constraints(range(1, 9))
Traceback (most recent call last):
  ... IndexError: pop index out of range
sage: p.remove_constraint(1)
sage: p.number_of_constraints() 2
```

We should now be able to add the old constraint back in:

```python
sage: for each in range(10):
    ....:     p.add_constraint(x - y, max=10)
sage: p.number_of_constraints() 3
```

**set_binary**(ee)

Sets a variable or a MIPVariable as binary.

**INPUT:**

- ee – An instance of MIPVariable or one of its elements.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
```

With the following instruction, all the variables from x will be binary:
sage: p.set_binary(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)

It is still possible, though, to set one of these variables as integer while keeping the others as they are:

sage: p.set_integer(x[3])

**set_integer**(ee)

Sets a variable or a MIPVariable as integer.

**INPUT:**

- ee – An instance of MIPVariable or one of its elements.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
```

With the following instruction, all the variables from x will be integers:

```
sage: p.set_integer(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

It is still possible, though, to set one of these variables as binary while keeping the others as they are:

sage: p.set_binary(x[3])

**set_max**(v, max)

Sets the maximum value of a variable.

**INPUT:**

- v – a variable.
- max – the maximum value the variable can take. When max=None, the variable has no upper bound.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_max(v[1])
sage: p.set_max(v[1], 6)
sage: p.get_max(v[1])
6.0
```

With a MIPVariable as an argument:

```
sage: vv = p.new_variable(real=True)
sage: p.get_max(vv)
sage: p.get_max(vv[0])
sage: p.set_max(vv, 5)
sage: p.get_max(vv[0])
5.0
```

(continues on next page)
**set_min** 

Sets the minimum value of a variable.

**INPUT:**

- v – a variable.
- min – the minimum value the variable can take. When min=None, the variable has no lower bound.

**See also:**

- `get_min()` – get the minimum value of a variable.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_min(v[1])
0.0
sage: p.set_min(v[1], 6)
sage: p.get_min(v[1])
6.0
sage: p.set_min(v[1], None)
sage: p.get_min(v[1])

With a MIPVariable as an argument:

```python
sage: vv = p.new_variable(real=True)
sage: p.get_min(vv)
sage: p.get_min(vv[0])
sage: p.set_min(vv, 5)
sage: p.get_min(vv[0])
5.0
sage: p.get_min(vv[9])
5.0
```

**set_objective**

Sets the objective of the MixedIntegerLinearProgram.

**INPUT:**

- obj – A linear function to be optimized. (can also be set to None or 0 or any number when just looking for a feasible solution )

**EXAMPLES:**

Let’s solve the following linear program:

Maximize:

\[ x + 5 \times y \]

Constraints:

\[ x + 0.2 \times y \leq 4 \]
This linear program can be solved as follows:

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 2/10*x[2], max=4)
sage: p.add_constraint(1.5*x[1] + 3*x[2], max=4)
sage: round(p.solve(),5)
6.66667
sage: p.set_objective(None)
sage: _ = p.solve()
```

**set_problem_name(name)**

Sets the name of the MixedIntegerLinearProgram.

**INPUT:**

• name – A string representing the name of the MixedIntegerLinearProgram.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.set_problem_name("Test program")
sage: p
Mixed Integer Program "Test program" (no objective, 0 variables, 0 constraints)
```

**set_real(ee)**

Sets a variable or a MIPVariable as real.

**INPUT:**

• ee – An instance of MIPVariable or one of its elements.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_real(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

With the following instruction, all the variables from x will be real:

```python
With the following instruction, all the variables from x will be real:
```

```python
sage: p.set_real(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

It is still possible, though, to set one of these variables as binary while keeping the others as they are:

```python
sage: p.set_binary(x[3])
```

**show()**

Displays the MixedIntegerLinearProgram in a human-readable way.

---

2.5. Index of functions and methods
EXAMPLES:

When constraints and variables have names

```sage
p = MixedIntegerLinearProgram(solver="GLPK")

x = p.new_variable(name="Hey")
p.set_objective(x[1] + x[2])
p.add_constraint(-3*x[1] + 2*x[2], max=2, name="Constraint_1")
p.show()
```

Maximization:

Constraints:
Constraint_1: -3.0 Hey[1] + 2.0 Hey[2] <= 2.0

Variables:
Hey[1] = x_0 is a continuous variable (min=-oo, max=+oo)
Hey[2] = x_1 is a continuous variable (min=-oo, max=+oo)

Without any names

```sage
p = MixedIntegerLinearProgram(solver="GLPK")
x = p.new_variable(nonnegative=True)
p.set_objective(x[1] + x[2])
p.add_constraint(-3*x[1] + 2*x[2], max=2)
p.show()
```

Maximization:

```
x_0 + x_1
```

Constraints:
-3.0 x_0 + 2.0 x_1 <= 2.0

Variables:

```
x_0 is a continuous variable (min=0.0, max=+oo)
x_1 is a continuous variable (min=0.0, max=+oo)
```

With \( Q \) coefficients:

```sage
p = MixedIntegerLinearProgram(solver='ppl')
x = p.new_variable(nonnegative=True)
p.set_objective(x[1] + 1/2*x[2])
p.add_constraint(-3/5*x[1] + 2/7*x[2], max=2/5)
p.show()
```

Maximization:

```
x_0 + 1/2 x_1
```

Constraints:
-constraint_0: -3/5 x_0 + 2/7 x_1 <= 2/5

Variables:

```
x_0 is a continuous variable (min=0, max=+oo)
x_1 is a continuous variable (min=0, max=+oo)
```

With a constant term in the objective:

```sage
p = MixedIntegerLinearProgram(solver='ppl')
x = p.new_variable(nonnegative=True)
p.set_objective(x[0] + 42)
p.show()
```

Maximization:

```
x_0 + 42
```

(continues on next page)
solve(log=None, objective_only=False)

Solves the MixedIntegerLinearProgram.

INPUT:

- **log** – integer (default: None) The verbosity level. Indicates whether progress should be printed during computation. The solver is initialized to report no progress.
- **objective_only** – Boolean variable.
  - When set to True, only the objective function is returned.
  - When set to False (default), the optimal numerical values are stored (takes computational time).

OUTPUT:

The optimal value taken by the objective function.

**Warning:** By default, no additional assumption is made on the domain of an LP variable. See set_min() and set_max() to change it.

EXAMPLES:

Consider the following linear program:

Maximize:
  \[ x + 5 * y \]

Constraints:
  \[ x + 0.2 * y \leq 4 \]
  \[ 1.5 * x + 3 * y \leq 4 \]

Variables:
  \( x \) is Real (min = 0, max = None)
  \( y \) is Real (min = 0, max = None)

This linear program can be solved as follows:

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 0.2*x[2], max=4)
sage: p.add_constraint(1.5*x[1] + 3*x[2], max=4)
sage: round(p.solve(),6)
6.666667
sage: x = p.get_values(x)
sage: round(x[1],6) # abs tol 1e-15
0.0
sage: round(x[2],6)
1.333333
```

Computation of a maximum stable set in Petersen's graph::

(continues on next page)
sage: # needs sage.graphs
sage: g = graphs.PetersenGraph()
sage: p = MixedIntegerLinearProgram(maximization=True, solver='GLPK')
sage: b = p.new_variable(nonnegative=True)
sage: p.set_objective(sum([b[v] for v in g]))
for (u,v) in g.edges(sort=False, labels=None):
    p.add_constraint(b[u] + b[v], max=1)
sage: p.set_binary(b)
sage: p.solve(objective_only=True)
4.0

Constraints in the objective function are respected:

sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max=6)
sage: p.add_constraint(3*x + 2*y, max=6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0

solver_parameter(name, value=None)

Return or define a solver parameter.

The solver parameters are by essence solver-specific, which means their meaning heavily depends on the
solver used.

(If you do not know which solver you are using, then you use GLPK).

Aliases:

Very common parameters have aliases making them solver-independent. For example, the following:

sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("timelimit", 60)

Sets the solver to stop its computations after 60 seconds, and works with GLPK, CPLEX, SCIP, and Gurobi.

• "timelimit" – defines the maximum time spent on a computation. Measured in seconds.

Another example is the "logfile" parameter, which is used to specify the file in which computation logs
are recorded. By default, the logs are not recorded, and we can disable this feature providing an empty
filename. This is currently working with CPLEX and Gurobi:

sage: # optional - cplex
sage: p = MixedIntegerLinearProgram(solver="CPLEX")
sage: p.solver_parameter("logfile")
''

sage: p.solver_parameter("logfile", "/dev/null")
sage: p.solver_parameter("logfile")
'/dev/null'

sage: p.solver_parameter("logfile", '')
sage: p.solver_parameter("logfile")
''
Solver-specific parameters:

- **GLPK**: We have implemented very close to comprehensive coverage of the GLPK solver parameters for the simplex and integer optimization methods. For details, see the documentation of `GLPKBackend.solver_parameter`.

- **CPLEX**’s parameters are identified by a string. Their list is available on ILOG’s website. The command

  ```python
  sage: p = MixedIntegerLinearProgram(solver="CPLEX")  # optional - CPLEX
  sage: p.solver_parameter("CPX_PARAM_TILIM", 60)       # optional - CPLEX
  ```

  works as intended.

- **Gurobi**’s parameters should all be available through this method. Their list is available on Gurobi’s website http://www.gurobi.com/documentation/5.5/reference-manual/node798.

  SCIP’s parameter can be found here: http://scip.zib.de/doc-5.0.1/html/PARAMETERS.php.

**INPUT:**

- `name` (string) – the parameter

- `value` – the parameter’s value if it is to be defined, or `None` (default) to obtain its current value.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("timelimit", 60)
sage: p.solver_parameter("timelimit")
60.0
```

**sum(L)**

Efficiently computes the sum of a sequence of `LinearFunction` elements

**INPUT:**

- `mip` – the `MixedIntegerLinearProgram` parent.

- `L` – list of `LinearFunction` instances.

**Note:** The use of the regular `sum` function is not recommended as it is much less efficient than this one

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: v = p.new_variable(nonnegative=True)

The following command:

```python
sage: s = p.sum(v[i] for i in range(90))
```

is much more efficient than:

```python
sage: s = sum(v[i] for i in range(90))
```
write_lp(filename)
Write the linear program as a LP file.
This function export the problem as a LP file.

INPUT:
• filename – The file in which you want the problem to be written.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + x[2])
sage: p.add_constraint(-3*x[1] + 2*x[2], max=2)
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix=".lp") as f:
.....:   p.write_lp(f.name)
Writing problem data to ...
9 lines were written
```

For more information about the LP file format: http://lpsolve.sourceforge.net/5.5/lp-format.htm

write_mps(filename, modern=True)
Write the linear program as a MPS file.
This function export the problem as a MPS file.

INPUT:
• filename – The file in which you want the problem to be written.
• modern – Lets you choose between Fixed MPS and Free MPS
  – True – Outputs the problem in Free MPS
  – False – Outputs the problem in Fixed MPS

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + x[2])
sage: p.add_constraint(-3*x[1] + 2*x[2], max=2, name="OneConstraint")
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix=".mps") as f:
.....:   p.write_mps(f.name)
Writing problem data to ...
17 records were written
```

For information about the MPS file format, see Wikipedia article MPS_(format)
CHAPTER THREE

SEMIDEFINITE PROGRAMMING

A semidefinite program (SDP) is an optimization problem (Wikipedia article Optimization_(mathematics)) of the following form

$$\min_{{\mathbf{X}}} \sum_{i,j=1}^{n} C_{ij} X_{ij} \quad \text{(Dual problem)}$$

Subject to:

$$\sum_{i,j=1}^{n} A_{ijk} X_{ij} = b_k, \quad k = 1 \ldots m$$

$$X \succeq 0$$

where the $X_{ij}$, $1 \leq i, j \leq n$ are $n^2$ variables satisfying the symmetry conditions $x_{ij} = x_{ji}$ for all $i, j$, the $C_{ij} = C_{ji}$, $A_{ijk} = A_{kji}$, and $b_k$ are real coefficients, and $X$ is positive semidefinite, i.e., all the eigenvalues of $X$ are nonnegative.

The closely related dual problem of this one is the following, where we denote by $A_k$ the matrix $(A_{kij})$ and by $C$ the matrix $(C_{ij})$,

$$\max_{{\mathbf{x}}} \sum_{k} b_k x_k \quad \text{(Primal problem)}$$

Subject to:

$$\sum_{k} x_k A_k \preceq C.$$ 

Here $(x_1, \ldots, x_m)$ is a vector of scalar variables. A wide variety of problems in optimization can be formulated in one of these two standard forms. Then, solvers are able to calculate an approximation to a solution. Here we refer to the latter problem as primal, and to the former problem as dual. The optimal value of the dual is always at least the optimal value of the primal, and usually (although not always) they are equal.

For instance, suppose you want to maximize $x_1 - x_0$ subject to

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} x_0 + \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} x_1 \preceq \begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x_0 + \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} x_1 \preceq \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}, \quad x_0 \geq 0, x_1 \geq 0.$$

An SDP can give you an answer to the problem above. Here is how it’s done:

1. You have to create an instance of `SemidefiniteProgram`.
2. Create a dictionary $x$ of integer variables via `new_variable()`, for example doing $x = p.new_variable()` if $p$ is the name of the SDP instance.
3. Add those two matrix inequalities as inequality constraints via `add_constraint()`.
4. Add another matrix inequality to specify nonnegativity of $x$.
5. Specify the objective function via `set_objective()`. In our case it is $x_1 - x_0$. If it is a pure constraint satisfaction problem, specify it as `None`.
6. To check if everything is set up correctly, you can print the problem via `show`. 
7. **Solve** it and print the solution.

The following example shows all these steps:

```python
sage: p = SemidefiniteProgram()
sage: x = p.new_variable()
sage: p.set_objective(x[1] - x[0])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: c1 = matrix([[1.0, 0], [0,0]], sparse=True)
sage: c2 = matrix([[0.0, 0], [0,1]], sparse=True)
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.add_constraint(c1*x[0] + c2*x[1] >= matrix.zero(2,2,sparse=True))

sage: # needs cvxopt
sage: p.solver_parameter("show_progress", True)
sage: opt = p.solve()

Objective Value: 1.0

sage: p.show()
Maximization:
  x_0 - x_1
Constraints:
  constraint_0: [ 3.0 4.0][4.0 5.0]x_0 + [ 1.0 2.0][2.0 3.0]x_1 <= [ 5.0 6.0][6.0 7.0]
  constraint_1: [ 2.0 2.0][2.0 2.0]x_0 + [ 1.0 1.0][1.0 1.0]x_1 <= [ 3.0 3.0][3.0 3.0]
  constraint_2: [ 0.0 0.0][ 0.0 -1.0]x_0 + [-1.0 0.0][ 0.0 0.0]x_1 <= [ 0 0][0 0]
Variables:
  x_0, x_1
```

Most solvers, e.g. the default Sage SDP solver CVXOPT, solve simultaneously the pair of primal and dual problems. Thus we can get the optimizer $X$ of the dual problem as follows, as diagonal blocks, one per each constraint, via `dual_variable()`. E.g.:

```python
sage: p.dual_variable(0) # rel tol 2e-03
[ 85555.0  -85555.0]
[-85555.0  85555.0]
```

We can see that the optimal value of the dual is equal (up to numerical noise) to $opt$.

```python
sage: opt - ((p.dual_variable(0)*a3).trace()) # tol 8e-08
0.0
```

---

Chapter 3. Semidefinite Programming
Dual variable blocks at optimality are orthogonal to “slack variables”, that is, matrices $C - \sum x_k A_k$, cf. (Primal problem) above, available via `slack()`. E.g.:

```python
sage: (p.slack(0)*p.dual_variable(0)).trace()  # tol 2e-07
0.0
```

More interesting example, the Lövasz theta of the 7-gon:

```python
sage: # needs sage.graphs
sage: c = graphs.CycleGraph(7)
sage: c2 = c.distance_graph(2).adjacency_matrix()
sage: c3 = c.distance_graph(3).adjacency_matrix()
sage: p.<y> = SemidefiniteProgram()
sage: p.add_constraint((1/7)*matrix.identity(7)>=-y[0]*c2-y[1]*c3)
sage: p.set_objective(y[0]*(c2**2).trace()+y[1]*(c3**2).trace())
sage: x = p.solve(); x + 1  #_
3.31766...
```

Unlike in the previous example, the slack variable is very far from 0:

```python
sage: p.slack(0).trace()  # tol 1e-14  #_
1.0
```

The default CVXOPT backend computes with the Real Double Field, for example:

```python
sage: # needs cvxopt
sage: p = SemidefiniteProgram(solver='cvxopt')
sage: p.base_ring()
Real Double Field
sage: x = p.new_variable()
sage: 0.5 + 3/2*x[1]
0.5 + 1.5*x_0
```

For representing an SDP with exact data, there is another backend:

```python
sage: from sage.numerical.backends.matrix_sdp_backend import MatrixSDPBackend
sage: p = SemidefiniteProgram(solver=MatrixSDPBackend, base_ring=QQ)
sage: p.base_ring()
Rational Field
sage: x = p.new_variable()
sage: 1/2 + 3/2 * x[1]
1/2 + 3/2*x_0
```
3.1 Linear Variables and Expressions

To make your code more readable, you can construct *SDPVariable* objects that can be arbitrarily named and indexed. Internally, this is then translated back to the $x_i$ variables. For example:

```python
sage: sdp.<a,b> = SemidefiniteProgram()
sage: a
SDPVariable
sage: 5 + a[1] + 2*b[3]
5 + x_0 + 2*x_1
```

Indices can be any object, not necessarily integers. Multi-indices are also allowed:

```python
sage: a[4, 'string', QQ]
x_2
x_2 - 7*x_3
sage: sdp.show()
Maximization:

Constraints:  
Variables:

     a[1], b[3], a[(4, 'string', Rational Field)], b[2]
```

3.2 Index of functions and methods

Below are listed the methods of *SemidefiniteProgram*. This module also implements the *SDPSolverException* exception, as well as the *SDPVariable* class.

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**AUTHORS:**

- Ingolfur Edvardsson (2014/08): added extension for exact computation
• Dima Pasechnik (2014-) : supervision, minor fixes, duality

**exception** `sage.numerical.sdp.SDPSolverException`

Bases: `RuntimeError`

Exception raised when the solver fails.

SDPSolverException is the exception raised when the solver fails.

**EXAMPLES:**

```python
sage: from sage.numerical.sdp import SDPSolverException
sage: SDPSolverException("Error")
SDPSolverException('Error'...)
```

**class** `sage.numerical.sdp.SDPVariable`

Bases: `Element`

SDPVariable is a variable used by the class SemidefiniteProgram.

**Warning:** You should not instantiate this class directly. Instead, use `SemidefiniteProgram.new_variable()`.

**items()**

Return the pairs (keys,value) contained in the dictionary.

**EXAMPLES:**

```python
sage: p = SemidefiniteProgram()
sage: v = p.new_variable()
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.items())
[(0, x_0), (1, x_1)]
```

**keys()**

Return the keys already defined in the dictionary.

**EXAMPLES:**

```python
sage: p = SemidefiniteProgram()
sage: v = p.new_variable()
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.keys())
[0, 1]
```

**values()**

Return the symbolic variables associated to the current dictionary.

**EXAMPLES:**

```python
sage: p = SemidefiniteProgram()
sage: v = p.new_variable()
sage: p.set_objective(v[0] + v[1])
sage: sorted(v.values(), key=str)
[x_0, x_1]
```
class sage.numerical.sdp.SDPVariableParent

Bases: Parent

Parent for SDPVariable.

Warning: This class is for internal use. You should not instantiate it yourself. Use SemidefiniteProgram.new_variable() to generate sdp variables.

Element

alias of SDPVariable

class sage.numerical.sdp.SemidefiniteProgram

Bases: SageObject

The SemidefiniteProgram class is the link between Sage, semidefinite programming (SDP) and semidefinite programming solvers.

A Semidefinite Programming (SDP) consists of variables, linear constraints on these variables, and an objective function which is to be maximised or minimised under these constraints.

See the Wikipedia article Semidefinite_programming for further information on semidefinite programming, and the SDP module for its use in Sage.

INPUT:

• solver – selects a solver:
  
  – CVXOPT (solver="CVXOPT"). See the CVXOPT website.
  
  – If solver=None (default), the default solver is used (see default_sdp_solver())

• maximization
  
  – When set to True (default), the SemidefiniteProgram is defined as a maximization.
  
  – When set to False, the SemidefiniteProgram is defined as a minimization.

See also:

• default_sdp_solver() – Returns/Sets the default SDP solver.

EXAMPLES:

Computation of a basic Semidefinite Program:

```sage
p = SemidefiniteProgram(maximization=False)
p = p.new_variable()
p.set_objective(x[0] - x[1])
a1 = matrix([[1, 2.], [2., 3.]])
a2 = matrix([[3, 4.], [4., 5.]])
a3 = matrix([[5, 6.], [6., 7.]])
b1 = matrix([[1, 1.], [1., 1.]])
b2 = matrix([[2, 2.], [2., 2.]])
b3 = matrix([[3, 3.], [3., 3.]])
p.add_constraint(a1*x[0] + a2*x[1] <= a3)
p.add_constraint(b1*x[0] + b2*x[1] <= b3)
N(p.solve(), 2) # needs cvxopt
-3.0
```
add_constraint(linear_function, name=None)

Adds a constraint to the SemidefiniteProgram.

INPUT:

• linear_function – Two different types of arguments are possible:
  - A linear function. In this case, arguments min or max have to be specified.
  - A linear constraint of the form A <= B, A >= B <= C, A >= B >= C or A == B. In this case, arguments min and max will be ignored.

• name – A name for the constraint.

EXAMPLES:

Let’s solve the following semidefinite program:

\[ \text{maximize } x + 5y \]
\[ \text{subject to } \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} y \preceq \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \]

This SDP can be solved as follows:

```
sage: p = SemidefiniteProgram(maximization=True)
sage: x = p.new_variable()
sage: p.set_objective(x[1] + 5*x[2])
sage: a1 = matrix([[1,2],[2,3]])
sage: a2 = matrix([[1,1],[1,1]])
sage: a3 = matrix([[1,-1],[-1,1]])
sage: N(p.solve(), digits=3) # needs cvxopt
16.2
```

One can also define double-bounds or equality using the symbol >= or ==:

```
sage: p = SemidefiniteProgram(maximization=True)
sage: x = p.new_variable()
sage: p.set_objective(x[1] + 5*x[2])
sage: a1 = matrix([[1,2],[2,3]])
sage: a2 = matrix([[1,1],[1,1]])
sage: a3 = matrix([[1,-1],[-1,1]])
sage: p.add_constraint(a3 >= a1*x[1] + a2*x[2])
sage: N(p.solve(), digits=3) # needs cvxopt
16.2
```

base_ring()

Return the base ring.

OUTPUT:

A ring. The coefficients that the chosen solver supports.

EXAMPLES:

```
sage: p = SemidefiniteProgram(solver='cvxopt')
sage: p.base_ring()
Real Double Field
```

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**dual_variable**(*i*, **sparse**=**False**)

The *i*-th dual variable.

Available after self.solve() is called, otherwise the result is undefined.

**INPUT:**
- **index** (integer) – the constraint’s id

**OUTPUT:**

The matrix of the *i*-th dual variable.

**EXAMPLES:**

Dual objective value is the same as the primal one:

```python
sage: p = SemidefiniteProgram(maximization=False)
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)

sage: # needs cvxopt
sage: p.solve()  # doctest: +TOL 1e-08
-3.0
sage: x = p.get_values(x).values()
sage: -(a3*p.dual_variable(0)).trace() - (b3*p.dual_variable(1)).trace()  # doctest: +TOL 1e-07
-3.0
```

Dual variable is orthogonal to the slack

```python
sage: # needs cvxopt
sage: g = sum((p.slack(j)*p.dual_variable(j)).trace() for j in range(2)); g  # doctest: +TOL 1.2e-08
0.0
```

**gen**(*i*)

Return the linear variable *x*<sub>i</sub>.

**EXAMPLES:**

```python
sage: sdp = SemidefiniteProgram()
sage: sdp.gen(0)
\[x_0\]
```

**get_backend**

Return the backend instance used.
This might be useful when access to additional functions provided by the backend is needed.

EXAMPLES:

This example prints a matrix coefficient:

```python
sage: p = SemidefiniteProgram(solver="cvxopt")
sage: x = p.new_variable()
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a1)
sage: b = p.get_backend()
sage: b.get_matrix()[0][0]
([-1.0 -2.0]
-1, [-2.0 -3.0])
```

`get_values(*lists)`

Return values found by the previous call to `solve()`.

INPUT:

- Any instance of `SDPVariable` (or one of its elements), or lists of them.

OUTPUT:

- Each instance of `SDPVariable` is replaced by a dictionary containing the numerical values found for each corresponding variable in the instance.
- Each element of an instance of a `SDPVariable` is replaced by its corresponding numerical value.

EXAMPLES:

```python
sage: p = SemidefiniteProgram(solver = "cvxopt", maximization = False)
sage: x = p.new_variable()
sage: p.set_objective(x[3] - x[5])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: N(p.solve(), 3)  # needs cvxopt
-3.0
```

To return the optimal value of `x[3]`:

```python
sage: N(p.get_values(x[3]),3)  # needs cvxopt
-1.0
```

To get a dictionary identical to `x` containing optimal values for the corresponding variables
linear_constraints_parent()

Return the parent for all linear constraints.

See linear_functions for more details.

EXAMPLES:

```
sage: p = SemidefiniteProgram()
sage: p.linear_constraints_parent()
Linear constraints over Real Double Field
```

linear_function(x)

Construct a new linear function.

EXAMPLES:

```
sage: p = SemidefiniteProgram()
sage: p.linear_function({0:1})
x_0
```

linear_functions_parent()

Return the parent for all linear functions.

EXAMPLES:

```
sage: p = SemidefiniteProgram()
sage: p.linear_functions_parent()
Linear functions over Real Double Field
```

new_variable(name="")

Returns an instance of SDPVariable associated to the current instance of SemidefiniteProgram.

A new variable x is defined by:

```
sage: p = SemidefiniteProgram()
sage: x = p.new_variable()
```

It behaves exactly as an usual dictionary would. It can use any key argument you may like, as x[5] or x["b"], and has methods items() and keys().

INPUT:

- dim – integer. Defines the dimension of the dictionary. If x has dimension 2, its fields will be of the form x[key1][key2]. Deprecated.
- name – string. Associates a name to the variable.

EXAMPLES:

```
sage: p = SemidefiniteProgram()
sage: x = p.new_variable()
```

(continues on next page)
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: p.add_constraint(a1*x[0] + a1*x[3] <= 0)
sage: p.show()
Maximization:
Constraints:
  constraint_0: [1.0 2.0][2.0 3.0]x_0 + [1.0 2.0][2.0 3.0]x_1 <= [0 0][0 0]
Variables:
  x_0, x_1

number_of_constraints()
Return the number of constraints assigned so far.

EXAMPLES:

sage: p = SemidefiniteProgram(solver = "cvxopt")
sage: x = p.new_variable()
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.add_constraint(b1*x[0] + a2*x[1] <= b3)
sage: p.number_of_constraints()
3

number_of_variables()
Return the number of variables used so far.

EXAMPLES:

sage: p = SemidefiniteProgram()
sage: a = matrix([[1, 2.], [2., 3.]])
sage: p.number_of_variables()
3

set_objective(obj)
Sets the objective of the SemidefiniteProgram.

INPUT:

• obj – A semidefinite function to be optimized. (can also be set to None or 0 when just looking for a feasible solution)

EXAMPLES:

Let’s solve the following semidefinite program:

 maximize \( x + 5y \)
 subject to \( \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} y \preceq \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \)
This SDP can be solved as follows:

```python
sage: p = SemidefiniteProgram(maximization=True)
sage: x = p.new_variable()
sage: p.set_objective(x[1] + 5*x[2])
sage: a1 = matrix([[1,2],[2,3]])
sage: a2 = matrix([[1,1],[1,1]])
sage: a3 = matrix([[1,-1],[-1,1]])
```

```python
sage: N(p.solve(), digits=3) # needs cvxopt
16.2
```

```python
sage: p.set_objective(None)
sage: _ = p.solve() # needs cvxopt
```

**set_problem_name(name)**

Sets the name of the SemidefiniteProgram.

**INPUT:**

- `name` – A string representing the name of the SemidefiniteProgram.

**EXAMPLES:**

```python
sage: p = SemidefiniteProgram()
sage: p.set_problem_name("Test program")
sage: p
Semidefinite Program "Test program" ( maximization, 0 variables, 0 constraints )
```

**show()**

Display the `SemidefiniteProgram` in a human-readable way.

**EXAMPLES:**

When constraints and variables have names

```python
sage: p = SemidefiniteProgram()
sage: x = p.new_variable(name="hihi")
sage: a1 = matrix([[1,2],[2,3]])
sage: a2 = matrix([[2,3],[3,4]])
sage: a3 = matrix([[3,4],[4,5]])
sage: p.set_objective(x[0] - x[1])
sage: p.add_constraint(a1*x[0] + a2*x[1]<= a3)
sage: p.show()
Maximization:
  hihi[0] - hihi[1]
Constraints:
  constraint_0: [1.0 2.0][2.0 3.0]hihi[0] + [2.0 3.0][3.0 4.0]hihi[1] <= [3.0␣
 ˓→4.0][4.0 5.0]
Variables:
  hihi[0], hihi[1]
```

**slack(i, sparse=False)**

Slack of the `i`-th constraint

Available after `self.solve()` is called, otherwise the result is undefined.
INPUT:
• index (integer) – the constraint’s id.

OUTPUT:
The matrix of the slack of the \(i\)-th constraint

EXAMPLES:

```python
sage: p = SemidefiniteProgram(maximization = False)
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: # needs cvxopt
sage: p.solve()  # tol 1e-08
-3.0
sage: B1 = p.slack(1); B1  # tol 1e-08
[0.0 0.0]
[0.0 0.0]
sage: B1.is_positive_definite()
True
sage: x = sorted(p.get_values(x).values())
sage: x[0]*b1 + x[1]*b2 - b3 + B1  # tol 1e-09
[0.0 0.0]
[0.0 0.0]
```

**solve**(objective_only=False)

Solve the `SemidefiniteProgram`.

INPUT:
• objective_only – Boolean variable.
  – When set to True, only the objective function is returned.
  – When set to False (default), the optimal numerical values are stored (takes computational time).

OUTPUT:
The optimal value taken by the objective function.

**solver_parameter**(name, value=None)

Return or define a solver parameter.

The solver parameters are by essence solver-specific, which means their meaning heavily depends on the solver used.

(If you do not know which solver you are using, then you are using CVXOPT).

INPUT:
• name (string) – the parameter
• value – the parameter's value if it is to be defined, or None (default) to obtain its current value.

EXAMPLES:

```
sage: # needs cvxopt
sage: p.<x> = SemidefiniteProgram(solver="cvxopt",
.....:     maximization=False)

sage: p.solver_parameter("show_progress", True)

sage: p.solver_parameter("show_progress")
    True

sage: p.set_objective(x[0] - x[1])

sage: a1 = matrix([[1, 2.], [2., 3.]]),

sage: a2 = matrix([[3, 4.], [4., 2.]])

sage: a3 = matrix([[5, 6.], [6., 7.]]),

sage: b1 = matrix([[1, 1.], [1., 1.]]),

sage: b2 = matrix([[2, 2.], [2., 1.]]),

sage: b3 = matrix([[3, 3.], [3., 3.]]),

sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)

sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)

sage: N(p.solve(), 4)

    pcost  dcost  gap  pres  dres  k/t
0: 1...
...
Optimal solution found.

-11.
```

\texttt{sum(L)}

Efficiently computes the sum of a sequence of \texttt{LinearFunction} elements.

INPUT:

• \texttt{L} – list of \texttt{LinearFunction} instances.

\textbf{Note:} The use of the regular \texttt{sum} function is not recommended as it is much less efficient than this one.

EXAMPLES:

```
sage: p = SemidefiniteProgram()

sage: v = p.new_variable()

The following command:

```
sage: s = p.sum(v[i] for i in range(90))
```

is much more efficient than:

```
sage: s = sum(v[i] for i in range(90))
```
CHAPTER FOUR

LINEAR FUNCTIONS AND CONSTRAINTS

This module implements linear functions (see `LinearFunction`) in formal variables and chained (in)equalities between them (see `LinearConstraint`). By convention, these are always written as either equalities or less-or-equal. For example:

```python
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: f = 1 + x[1] + 2*x[2]; f         # a linear function
    1 + x_0 + 2*x_1
sage: type(f)
<class 'sage.numerical.linear_functions.LinearFunction'>

sage: c = (0 <= f); c   # a constraint
    0 <= 1 + x_0 + 2*x_1
sage: type(c)
<class 'sage.numerical.linear_functions.LinearConstraint'>
```

Note that you can use this module without any reference to linear programming, it only implements linear functions over a base ring and constraints. However, for ease of demonstration we will always construct them out of linear programs (see `mip`).

Constraints can be equations or (non-strict) inequalities. They can be chained:

```python
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
x_0 == x_1 == x_2 == x_3

sage: type(ieq_01234)
<class 'sage.numerical.linear_functions.LinearConstraint'>
```

If necessary, the direction of inequality is flipped to always write inequalities as less or equal:

```python
sage: x[5] >= ieq_01234
    x_0 <= x_1 <= x_2 <= x_3 <= x_4 <= x_5

sage: (x[5] <= x[6]) >= ieq_01234
    x_0 <= x_1 <= x_2 <= x_3 <= x_4 <= x_5 <= x_6

sage: (x[5] <= x[6]) <= ieq_01234
    x_5 <= x_6 <= x_0 <= x_1 <= x_2 <= x_3 <= x_4
```
Warning: The implementation of chained inequalities uses a Python hack to make it work, so it is not completely robust. In particular, while constants are allowed, no two constants can appear next to each other. The following does not work for example:

```
sage: x[0] <= 3 <= 4
True
```

If you really need this for some reason, you can explicitly convert the constants to a `LinearFunction`:

```
sage: from sage.numerical.linear_functions import LinearFunctionsParent
sage: LF = LinearFunctionsParent(QQ)
sage: x[1] <= LF(3) <= LF(4)
x_1 <= 3 <= 4
```

**class** `sage.numerical.linear_functions.LinearConstraint`

Bases: `LinearFunctionOrConstraint`

A class to represent formal Linear Constraints.

A Linear Constraint being an inequality between two linear functions, this class lets the user write `LinearFunction1 <= LinearFunction2` to define the corresponding constraint, which can potentially involve several layers of such inequalities (`A <= B <= C`), or even equalities like `A == B == C`.

Trivial constraints (meaning that they have only one term and no relation) are also allowed. They are required for the coercion system to work.

**Warning:** This class has no reason to be instantiated by the user, and is meant to be used by instances of `MixedIntegerLinearProgram`.

**INPUT:**

- `parent` – the parent, a `LinearConstraintsParent_class`
- `terms` – a list/tuple/iterable of two or more linear functions (or things that can be converted into linear functions).
- `equality` – boolean (default: `False`). Whether the terms are the entries of a chained less-or-equal (`<=`) inequality or a chained equality.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
x_0 + 2*x_1 <= -5 + x_2
```

**equals** (*left*, *right*)

Compare *left* and *right*.

**OUTPUT:**

Boolean. Whether all terms of *left* and *right* are equal. Note that this is stronger than mathematical equivalence of the relations.

**EXAMPLES:**
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
True
sage: (x[1] + 1 >= 2).equals(x[1] + 1-1 >= 1-1)
False

**equations()**

Iterate over the unchained(!) equations

**OUTPUT:**

An iterator over pairs (lhs, rhs) such that the individual equations are lhs == rhs.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: eqns = 1 == b[0] == b[2] == 3 == b[3]; eqns
1 == x_0 == x_1 == 3 == x_2
sage: for lhs, rhs in eqns.equations():
    ....:     print(str(lhs) + ' == ' + str(rhs))
1 == x_0
x_0 == x_1
x_1 == 3
3 == x_2
```

**inequalities()**

Iterate over the unchained(!) inequalities

**OUTPUT:**

An iterator over pairs (lhs, rhs) such that the individual equations are lhs <= rhs.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: ieq = 1 <= b[0] <= b[2] <= 3 <= b[3]; ieq
1 <= x_0 <= x_1 <= 3 <= x_2
sage: for lhs, rhs in ieq.inequalities():
    ....:     print(str(lhs) + ' <= ' + str(rhs))
1 <= x_0
x_0 <= x_1
x_1 <= 3
3 <= x_2
```

**is_equation()**

Whether the constraint is a chained equation

**OUTPUT:**

Boolean.

**EXAMPLES:**
```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: (b[0] == b[1]).is_equation()
   True
sage: (b[0] <= b[1]).is_equation()
   False
```

**is_less_or_equal()**

Whether the constraint is a chained less-or-equal inequality

**OUTPUT:**

Boolean.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: (b[0] == b[1]).is_less_or_equal()
   False
sage: (b[0] <= b[1]).is_less_or_equal()
   True
```

**is_trivial()**

Test whether the constraint is trivial.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: LC = p.linear_constraints_parent()
sage: ieq = LC(1,2); ieq
   1 <= 2
sage: ieq.is_trivial()
   False
sage: ieq = LC(1); ieq
   trivial constraint starting with 1
sage: ieq.is_trivial()
   True
```

sage.numerical.linear_functions.LinearConstraintsParent()

Return the parent for linear functions over base_ring.

The output is cached, so only a single parent is ever constructed for a given base ring.

**INPUT:**

- linear_functions_parent – a LinearFunctionsParent_class. The type of linear functions that the constraints are made out of.

**OUTPUT:**

The parent of the linear constraints with the given linear functions.

**EXAMPLES:**
```python
sage: from sage.numerical.linear_functions import (LinearFunctionsParent, LinearConstraintsParent)

sage: LF = LinearFunctionsParent(QQ)

sage: LinearConstraintsParent(LF)
Linear constraints over Rational Field
```

**class** `sage.numerical.linear_functions.LinearConstraintsParent_class`  
Bases: `Parent`  
Parent for `LinearConstraint`

**Warning:** This class has no reason to be instantiated by the user, and is meant to be used by instances of `MixedIntegerLinearProgram`. Also, use the `LinearConstraintsParent()` factory function.

**INPUT/OUTPUT:**  
See `LinearFunctionsParent()`

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram()

sage: LC = p.linear_constraints_parent(); LC
Linear constraints over Real Double Field

sage: from sage.numerical.linear_functions import LinearConstraintsParent

sage: LinearConstraintsParent(p.linear_functions_parent()) is LC
True
```

**linear_functions_parent()**  
Return the parent for the linear functions

**EXAMPLES:**

```python
sage: LC = MixedIntegerLinearProgram().linear_constraints_parent()

sage: LC.linear_functions_parent()
Linear functions over Real Double Field
```

**class** `sage.numerical.linear_functions.LinearFunction`  
Bases: `LinearFunctionOrConstraint`  
An elementary algebra to represent symbolic linear functions.

**Warning:** You should never instantiate `LinearFunction` manually. Use the element constructor in the parent instead.

**EXAMPLES:**

For example, do this:

```python
sage: p = MixedIntegerLinearProgram()

sage: parent = p.linear_functions_parent()

sage: parent({0 : 1, 3 : -8})
x_0 - 8*x_3
```

instead of this:
coefficient\( (x) \)

Return one of the coefficients.

**INPUT:**

- \( x \) – a linear variable or an integer. If an integer \( i \) is passed, then \( x_i \) is used as linear variable.

**OUTPUT:**

A base ring element. The coefficient of \( x \) in the linear function. Pass \(-1\) for the constant term.

**EXAMPLES:**

```python
sage: mip.<b> = MixedIntegerLinearProgram()

sage: lf = -8 * b[3] + b[0] - 5; lf
-5 - 8*x_0 + x_1

sage: lf.coefficient(b[3])
-8.0

sage: lf.coefficient(0)  # x_0 is b[3]
-8.0

sage: lf.coefficient(4)
0.0

sage: lf.coefficient(-1)
-5.0
```

dict()

Return the dictionary corresponding to the Linear Function.

**OUTPUT:**

The linear function is represented as a dictionary. The value are the coefficient of the variable represented by the keys (which are integers). The key \(-1\) corresponds to the constant term.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram()

sage: LF = p.linear_functions_parent()

sage: lf = LF({0 : 1, 3 : -8})

sage: lf.dict()
{0: 1.0, 3: -8.0}
```

equals(\( \text{left}, \text{right} \))

Logically compare \( \text{left} \) and \( \text{right} \).

**OUTPUT:**

Boolean.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram()

sage: x = p.new_variable()

sage: (x[1] + 1).equals(3/3 + 1*x[1] + 0*x[2])
True
```
**is_zero()**

Test whether *self* is zero.

**OUTPUT:**

Boolean.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: (x[1] - x[1] + 0*x[2]).is_zero()
True
```

**iteritems()**

Iterate over the index, coefficient pairs.

**OUTPUT:**

An iterator over the (key, coefficient) pairs. The keys are integers indexing the variables. The key -1 corresponds to the constant term.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram(solver = 'ppl')
sage: x = p.new_variable()
sage: f = 0.5 + 3/2*x[1] + 0.6*x[3]
sage: for id, coeff in sorted(f.iteritems()):
    ....:     print('id = {}  coeff = {}'.format(id, coeff))
    id = -1  coeff = 1/2
    id = 0   coeff = 3/2
    id = 1   coeff = 3/5
```

class sage.numerical.linear_functions.LinearFunctionOrConstraint

Based on ModuleElement

Base class for LinearFunction and LinearConstraint.

This class exists solely to implement chaining of inequalities in constraints.

**sage.numerical.linear_functions.LinearFunctionsParent()**

Return the parent for linear functions over base_ring.

The output is cached, so only a single parent is ever constructed for a given base ring.

**INPUT:**

- base_ring – a ring. The coefficient ring for the linear functions.

**OUTPUT:**

The parent of the linear functions over base_ring.

**EXAMPLES:**

```
sage: from sage.numerical.linear_functions import LinearFunctionsParent
sage: LinearFunctionsParent(QQ)
linear functions over Rational Field
```
class sage.numerical.linear_functions.LinearFunctionsParent_class

Bases: Parent

The parent for all linear functions over a fixed base ring.

Warning: You should use LinearFunctionsParent() to construct instances of this class.

INPUT/OUTPUT:
See LinearFunctionsParent()

EXAMPLES:

```python
sage: from sage.numerical.linear_functions import LinearFunctionsParent_class
sage: LinearFunctionsParent_class
<class 'sage.numerical.linear_functions.LinearFunctionsParent_class'>
```

**gen(i)**

Return the linear variable $x_i$.

**INPUT:**

- $i$ – non-negative integer.

**OUTPUT:**

The linear function $x_i$.

**EXAMPLES:**

```python
sage: LF = MixedIntegerLinearProgram().linear_functions_parent()
sage: LF.gen(23)
x_23
```

**set_multiplication_symbol**(symbol='*')

Set the multiplication symbol when pretty-printing linear functions.

**INPUT:**

- `symbol` – string, default: '*'. The multiplication symbol to be used.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: f = -1-2*x[0]-3*x[1]
sage: LF = f.parent()
sage: LF._get_multiplication_symbol()
'*'
sage: f
-1 - 2*x_0 - 3*x_1
sage: LF.set_multiplication_symbol(' ')  # symbol=' '
sage: f
-1 - 2 x_0 - 3 x_1
sage: LF.set_multiplication_symbol()  # symbol=''
sage: f
-1 - 2*x_0 - 3*x_1
```
\textbf{tensor}(\textit{free\_module})

Return the tensor product with \textit{free\_module}.

INPUT:

- \textit{free\_module} – vector space or matrix space over the same base ring.

OUTPUT:

Instance of \texttt{sage.numerical.linear\_tensor.LinearTensorParent\_class}.

EXAMPLES:

```
sage: LF = MixedIntegerLinearProgram().linear_functions_parent()
sage: LF.tensor(RDF^3)
Tensor product of Vector space of dimension 3 over Real Double Field
and Linear functions over Real Double Field
sage: LF.tensor(QQ^2)
Traceback (most recent call last):
... 
ValueError: base rings must match
```

\texttt{sage.numerical.linear\_functions.is\_LinearConstraint}(\textit{x})

Test whether \textit{x} is a linear constraint

INPUT:

- \textit{x} – anything.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: ieq = (x[0] <= x[1])
sage: from sage.numerical.linear\_functions import is\_LinearConstraint
sage: is\_LinearConstraint(ieq)
True
sage: is\_LinearConstraint('a string')
False
```

\texttt{sage.numerical.linear\_functions.is\_LinearFunction}(\textit{x})

Test whether \textit{x} is a linear function

INPUT:

- \textit{x} – anything.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: from sage.numerical.linear\_functions import is\_LinearFunction
```
sage: is_LinearFunction(x[0] - 2*x[2])
True
sage: is_LinearFunction('a string')
False
In Sage, matrices assume that the base is a ring. Hence, we cannot construct matrices whose entries are linear functions in Sage. Really, they should be thought of as the tensor product of the $R$-module of linear functions and the $R$-vector/matrix space, with the latter viewed as an $R$-module ($R$ is usually $\mathbb{Q}$ or $\mathbb{R}$ for our purposes).

You should not construct any tensor products by calling the parent directly. This is also why none of the classes are imported in the global namespace. The come into play whenever you have vector or matrix MIP linear expressions/constraints. The intended way to construct them is implicitly by acting with vectors or matrices on linear functions. For example:

```
sage: mip.<x> = MixedIntegerLinearProgram('ppl')  # base ring is QQ
sage: 3 + x[0] + 2*x[1]                      # a linear function
3 + x_0 + 2x_1
sage: x[0] * vector([3,4]) + 1           # vector linear function
(1, 1) + (3, 4)x_0
sage: x[0] * matrix([[3,1],[4,0]]) + 1   # matrix linear function
[1 + 3x_0 x_0]
[4x_0 1 ]
```

Internally, all linear functions are stored as a dictionary whose

- keys are the index of the linear variable (and -1 for the constant term)
- values are the coefficient of that variable. That is, a number for linear functions, a vector for vector-valued functions, etc.

The entire dictionary can be accessed with the `dict()` method. For convenience, you can also retrieve a single coefficient with `coefficient()`. For example:

```
sage: mip.<b> = MixedIntegerLinearProgram()
sage: f_scalar = (3 + b[7] + 2*b[9]); f_scalar
3 + x_0 + 2x_1
sage: f_scalar.dict()
{-1: 3.0, 0: 1.0, 1: 2.0}
sage: f_scalar.dict()[1]
2.0
sage: f_scalar.coefficient(b[9])
2.0
sage: f_scalar.coefficient(1)
2.0

sage: f_vector = b[7] * vector([3,4]) + 1; f_vector
(1.0, 1.0) + (3.0, 4.0)x_0
sage: f_vector.coefficient(-1)
(continues on next page)
```
sage: f_vector.coefficient(b[7])
(3.0, 4.0)
sage: f_vector.coefficient(0)
(3.0, 4.0)
sage: f_vector.coefficient(1)
(0.0, 0.0)
sage: f_matrix = b[7] * matrix([[0,1], [2,0]]) + b[9] - 3; f_matrix
[-3 + x_1 x_0]
[2*x_0 -3 + x_1]
sage: f_matrix.coefficient(-1)
[-3.0 0.0]
[ 0.0 -3.0]
sage: f_matrix.coefficient(0)
[0.0 1.0]
[2.0 0.0]
sage: f_matrix.coefficient(1)
[1.0 0.0]
[0.0 1.0]

Just like sage.numerical.linear_functions, (in)equalities become symbolic inequalities. See linear_tensor_constraints for details.

Note: For brevity, we just use LinearTensor in class names. It is understood that this refers to the above tensor product construction.

sage.numerical.linear_tensor.LinearTensorParent(linear_functions_parent)

Return the parent for the tensor product over the common base_ring.

The output is cached, so only a single parent is ever constructed for a given base ring.

INPUT:

• free_module_parent – module. A free module, like vector or matrix space.
• linear_functions_parent – linear functions. The linear functions parent.

OUTPUT:

The parent of the tensor product of a free module and linear functions over a common base ring.

EXAMPLES:

sage: from sage.numerical.linear_functions import LinearFunctionsParent
sage: from sage.numerical.linear_tensor import LinearTensorParent
sage: LinearTensorParent(QQ^3, LinearFunctionsParent(QQ))
Tensor product of Vector space of dimension 3 over Rational Field and Linear...
functions over Rational Field

sage: LinearTensorParent(ZZ^3, LinearFunctionsParent(QQ))
Traceback (most recent call last):
...
ValueError: base rings must match
class sage.numerical.linear_tensor.LinearTensorParent_class\( (\text{free\_module}, \text{linear\_functions}) \)
Bases: Parent

The parent for all linear functions over a fixed base ring.

**Warning:** You should use \text{LinearTensorParent()} to construct instances of this class.

INPUT/OUTPUT:
See \text{LinearTensorParent()}

EXAMPLES:

```python
sage: from sage.numerical.linear_tensor import LinearTensorParent_class
sage: LinearTensorParent_class
<class 'sage.numerical.linear_tensor.LinearTensorParent_class'>
```

**Element**
alias of \text{LinearTensor}

**free\_module()**
Return the linear functions.
See also \text{free\_module()}.

OUTPUT:
Parent of the linear functions, one of the factors in the tensor product construction.

EXAMPLES:

```python
sage: mip.<x> = MixedIntegerLinearProgram()
sage: lt = x[0] * vector(RDF, [1,2])
sage: lt.parent().free_module()
Vector space of dimension 2 over Real Double Field
sage: lt.parent().free_module() is vector(RDF, [1,2]).parent()
True
```

**is\_matrix\_space()**
Return whether the free module is a matrix space.

OUTPUT:
Boolean. Whether the \text{free\_module()} factor in the tensor product is a matrix space.

EXAMPLES:

```python
sage: mip = MixedIntegerLinearProgram()
sage: LF = mip.linear_functions_parent()
sage: LF.tensor(RDF^2).is_matrix_space()
False
sage: LF.tensor(RDF^(2,2)).is_matrix_space()
True
```

**is\_vector\_space()**
Return whether the free module is a vector space.

OUTPUT:
Boolean. Whether the `free_module()` factor in the tensor product is a vector space.

EXAMPLES:

```python
sage: mip = MixedIntegerLinearProgram()
sage: LF = mip.linear_functions_parent()
sage: LF.tensor(RDF^2).is_vector_space()
True
sage: LF.tensor(RDF^(2,2)).is_vector_space()
False
```

`linear_functions()`

Return the linear functions.

See also `free_module()`.

OUTPUT:

Parent of the linear functions, one of the factors in the tensor product construction.

EXAMPLES:

```python
sage: mip.<x> = MixedIntegerLinearProgram()
sage: lt = x[0] * vector([1,2])
sage: lt.parent().linear_functions()
Linear functions over Real Double Field
sage: lt.parent().linear_functions() is mip.linear_functions_parent()
True
```

`sage.numerical.linear_tensor.is_LinearTensor(x)`

Test whether `x` is a tensor product of linear functions with a free module.

INPUT:

* `x` — anything.

OUTPUT:

Boolean.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=False)
sage: from sage.numerical.linear_tensor import is_LinearTensor
sage: is_LinearTensor(x[0] - 2*x[2])
False
sage: is_LinearTensor('a string')
False
```
Here is an example of a linear function tensored with a vector space:

```sage
mip.<x> = MixedIntegerLinearProgram('ppl')  # base ring is QQ
lt = x[0] * vector([3,4]) + 1; lt
(1, 1) + (3, 4)*x_0
sage: type(lt)
<class 'sage.numerical.linear_tensor_element.LinearTensor'>
```

**class sage.numerical.linear_tensor_element.LinearTensor**

Bases: ModuleElement

A linear function tensored with a free module

**Warning:** You should never instantiate `LinearTensor` manually. Use the element constructor in the parent instead.

**EXAMPLES:**

```sage
parent = MixedIntegerLinearProgram().linear_functions_parent().tensor(RDF^2)
sage: parent({0: [1,2], 3: [-7,-8]})
(1.0, 2.0)*x_0 + (-7.0, -8.0)*x_3
```

**coefficient(x)**

Return one of the coefficients.

**INPUT:**

- x – a linear variable or an integer. If an integer $i$ is passed, then $x_i$ is used as linear variable. Pass $-1$ for the constant term.

**OUTPUT:**

A constant, that is, an element of the free module factor. The coefficient of $x$ in the linear function.

**EXAMPLES:**

```sage
mip.<b> = MixedIntegerLinearProgram()
sage: lt = vector([1,2]) * b[3] + vector([4,5]) * b[0] - 5; lt
(-5.0, -5.0) + (1.0, 2.0)*x_0 + (4.0, 5.0)*x_1
sage: lt.coefficient(b[3])
(1.0, 2.0)
sage: lt.coefficient(0)  # x_0 is b[3]
```
**dict()**

Return the dictionary corresponding to the tensor product.

**OUTPUT:**

The linear function tensor product is represented as a dictionary. The value are the coefficient (free module elements) of the variable represented by the keys (which are integers). The key $-1$ corresponds to the constant term.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram().linear_functions_parent().tensor(RDF^2)
sage: lt = p({0:[1,2], 3:[4,5]})
sage: lt.dict()
{0: (1.0, 2.0), 3: (4.0, 5.0)}
```
Here is an example of a vector-valued linear function:

```
sage: mip.<x> = MixedIntegerLinearProgram('ppl')  # base ring is QQ
sage: x[0] * vector([3,4]) + 1  # vector linear function
(1, 1) + (3, 4)*x_0
```

Just like `linear_functions`, (in)equalities become symbolic inequalities:

```
sage: 3 + x[0] + 2*x[1] <= 10
3 + x_0 + 2*x_1 <= 10
sage: x[0] * vector([3,4]) + 1 <= 10
(1, 1) + (3, 4)*x_0 <= (10, 10)
sage: x[0] * matrix([[0,0,1],[0,1,0],[1,0,0]]) + x[1] * identity_matrix(3) >= 0
[0 0 0] [x_1 0 x_0]
[0 0 0] <= [0 x_0 + x_1 0 ]
[0 0 0] [x_0 0 x_1]
```

class `sage.numerical.linear_tensor_constraints.LinearTensorConstraint`(`parent, lhs, rhs, equality`)

Bases: `Element`

Formal constraint involving two module-valued linear functions.

**Note**: In the code, we use “linear tensor” as abbreviation for the tensor product (over the common base ring) of a `linear function` and a free module like a vector/matrix space.

**Warning**: This class has no reason to be instantiated by the user, and is meant to be used by instances of `MixedIntegerLinearProgram`.

**INPUT:**

- `parent` – the parent, a `LinearTensorConstraintsParent_class`
- `lhs, rhs` – two `sage.numerical.linear_tensor_element.LinearTensor`. The left and right hand side of the constraint (in)equality.
- `equality` – boolean (default: `False`). Whether the constraint is an equality. If `False`, it is a `<=` inequality.

**EXAMPLES:**
```python
sage: mip.<b> = MixedIntegerLinearProgram()
sage: (b[2]+2*b[3]) * vector([1,2]) <= b[8] * vector([2,3]) - 5
(1.0, 2.0)*x_0 + (2.0, 4.0)*x_1 <= (-5.0, -5.0) + (2.0, 3.0)*x_2
```

**is_equation()**

Whether the constraint is a chained equation

**OUTPUT:**

Boolean.

**EXAMPLES:**

```python
sage: mip.<b> = MixedIntegerLinearProgram()
sage: (b[0] * vector([1,2]) == 0).is_equation()  
True
sage: (b[0] * vector([1,2]) >= 0).is_equation()  
False
```

**is_less_or_equal()**

Whether the constraint is a chained less-or_equal inequality

**OUTPUT:**

Boolean.

**EXAMPLES:**

```python
sage: mip.<b> = MixedIntegerLinearProgram()
sage: (b[0] * vector([1,2]) == 0).is_less_or_equal()  
False
sage: (b[0] * vector([1,2]) >= 0).is_less_or_equal()  
True
```

**lhs()**

Return the left side of the (in)equality.

**OUTPUT:**

Instance of `sage.numerical.linear_tensor_element.LinearTensor`. A linear function valued in a free module.

**EXAMPLES:**

```python
sage: mip.<x> = MixedIntegerLinearProgram()
sage: (x[0] * vector([1,2]) == 0).lhs()  
(1.0, 2.0)*x_0
```

**rhs()**

Return the right side of the (in)equality.

**OUTPUT:**

Instance of `sage.numerical.linear_tensor_element.LinearTensor`. A linear function valued in a free module.

**EXAMPLES:**
sage: mip.<x> = MixedIntegerLinearProgram()
sage: (x[0] * vector([1,2]) == 0).rhs()
(0.0, 0.0)

sage.numerical.linear_tensor_constraints.LinearTensorConstraintsParent()

Return the parent for linear functions over base_ring.

The output is cached, so only a single parent is ever constructed for a given base ring.

INPUT:

- linear_functions_parent – a LinearFunctionsParent_class. The type of linear functions that the constraints are made out of.

OUTPUT:

The parent of the linear constraints with the given linear functions.

EXAMPLES:

sage: from sage.numerical.linear_functions import LinearFunctionsParent
sage: from sage.numerical.linear_tensor import LinearTensorParent
sage: from sage.numerical.linear_tensor_constraints import LinearTensorConstraintsParent, LinearTensorConstraintsParent
sage: LF = LinearFunctionsParent(QQ)
sage: LT = LinearTensorParent(QQ^2, LF)
sage: LinearTensorConstraintsParent(LT)
Linear constraints in the tensor product of Vector space of dimension 2 over Rational Field and Linear functions over Rational Field

class sage.numerical.linear_tensor_constraints.LinearTensorConstraintsParent_class(linear_tensor_parent)

Bases: Parent

Parent for LinearTensorConstraint

Warning: This class has no reason to be instantiated by the user, and is meant to be used by instances of MixedIntegerLinearProgram. Also, use the LinearTensorConstraintsParent() factory function.

INPUT/OUTPUT:

See LinearTensorConstraintsParent()

EXAMPLES:

sage: p = MixedIntegerLinearProgram()
sage: LT = p.linear_functions_parent().tensor(RDF^2); LT
Tensor product of Vector space of dimension 2 over Real Double Field and Linear functions over Real Double Field
sage: from sage.numerical.linear_tensor_constraints import LinearTensorConstraintsParent
sage: LTC = LinearTensorConstraintsParent(LT); LTC
Linear constraints in the tensor product of Vector space of dimension 2 over Real Double Field and Linear functions over Real Double Field
sage: type(LTC)
Element

alias of \texttt{LinearTensorConstraint}

\textbf{linear\_functions()}  
Return the parent for the linear functions 

\textbf{OUTPUT:}

Instance of \texttt{sage.numerical.linear\_functions.LinearFunctionsParent\_class}.

\textbf{EXAMPLES:}

\begin{verbatim}
sage: mip.<x> = MixedIntegerLinearProgram()
sage: ieq = (x[0] * vector([1,2]) >= 0)
sage: ieq.parent().linear\_functions()
Linear functions over Real Double Field
\end{verbatim}

\textbf{linear\_tensors()}  
Return the parent for the linear functions 

\textbf{OUTPUT:}

Instance of \texttt{sage.numerical.linear\_tensor.LinearTensorParent\_class}.

\textbf{EXAMPLES:}

\begin{verbatim}
sage: mip.<x> = MixedIntegerLinearProgram()
sage: ieq = (x[0] * vector([1,2]) >= 0)
sage: ieq.parent().linear\_tensors()
Tensor product of Vector space of dimension 2 over Real Double Field and Linear functions over Real Double Field
\end{verbatim}

\texttt{sage.numerical.linear\_tensor\_constraints.\texttt{is\_LinearTensorConstraint}(x)}  
Test whether \(x\) is a constraint on module-valued linear functions.

\textbf{INPUT:}

\begin{itemize}
  \item \(x\) – anything.
\end{itemize}

\textbf{OUTPUT:}

Boolean.

\textbf{EXAMPLES:}

\begin{verbatim}
sage: mip.<x> = MixedIntegerLinearProgram()
sage: vector_ieq = (x[0] * vector([1,2]) <= x[1] * vector([2,3]))
sage: from sage.numerical.linear\_tensor\_constraints import is\_LinearTensorConstraint
sage: is\_LinearTensorConstraint(vector\_ieq)
True
sage: is\_LinearTensorConstraint('a string')
False
\end{verbatim}
8.1 Functions and Methods

\texttt{sage.numerical.optimize.binpacking(items, maximum, k=1, solver=None, verbose=None, integrality_tolerance=0)}

Solve the bin packing problem.

The Bin Packing problem is the following:

Given a list of items of weights \( p_i \) and a real value \( k \), what is the least number of bins such that all the items can be packed in the bins, while ensuring that the sum of the weights of the items packed in each bin is at most \( k \)?

For more informations, see Wikipedia article Bin_packing_problem.

Two versions of this problem are solved by this algorithm:

- Is it possible to put the given items in \( k \) bins?
- What is the assignment of items using the least number of bins with the given list of items?

**INPUT:**

- \texttt{items} – list or dict; either a list of real values (the items’ weight), or a dictionary associating to each item its weight.
- \texttt{maximum} – (default: 1); the maximal size of a bin
- \texttt{k} – integer (default: None); Number of bins
  - When set to an integer value, the function returns a partition of the items into \( k \) bins if possible, and raises an exception otherwise.
  - When set to None, the function returns a partition of the items using the least possible number of bins.
- \texttt{solver} – (default: None) Specify a Mixed Integer Linear Programming (MILP) solver to be used. If set to None, the default one is used. For more information on MILP solvers and which default solver is used, see the method \texttt{solve} of the class \texttt{MixedIntegerLinearProgram}.
- \texttt{verbose} – integer (default: 0); sets the level of verbosity. Set to 0 by default, which means quiet.
- \texttt{integrality_tolerance} – parameter for use with MILP solvers over an inexact base ring; see \texttt{MixedIntegerLinearProgram.get_values()}. 
OUTPUT:

A list of lists, each member corresponding to a bin and containing either the list of the weights inside it when `items` is a list of items’ weight, or the list of items inside it when `items` is a dictionary. If there is no solution, an exception is raised (this can only happen when `k` is specified or if `maximum` is less than the weight of one item).

EXAMPLES:

Trying to find the minimum amount of boxes for 5 items of weights $1/5, 1/4, 2/3, 3/4, 5/7$:

```python
sage: from sage.numerical.optimize import binpacking
sage: values = [1/5, 1/3, 2/3, 3/4, 5/7]
sage: bins = binpacking(values)  # needs sage.numerical.mip
sage: len(bins)  # needs sage.numerical.mip
3
```

Checking the bins are of correct size

```python
sage: all(sum(b) <= 1 for b in bins)  # needs sage.numerical.mip
True
```

Checking every item is in a bin

```python
sage: b1, b2, b3 = bins  # needs sage.numerical.mip
sage: all((v in b1 or v in b2 or v in b3) for v in values)  # needs sage.numerical.mip
True
```

And only in one bin

```python
sage: sum(len(b) for b in bins) == len(values)  # needs sage.numerical.mip
True
```

One way to use only three boxes (which is best possible) is to put $1/5 + 3/4$ together in a box, $1/3 + 2/3$ in another, and $5/7$ by itself in the third one.

Of course, we can also check that there is no solution using only two boxes

```python
sage: from sage.numerical.optimize import binpacking
sage: binpacking([0.2,0.3,0.8,0.9], k=2)  # needs sage.numerical.mip
Traceback (most recent call last):
... ValueError: this problem has no solution
```

We can also provide a dictionary keyed by items and associating to each item its weight. Then, the bins contain the name of the items inside it

```python
sage: values = {'a':1/5, 'b':1/3, 'c':2/3, 'd':3/4, 'e':5/7}
sage: bins = binpacking(values)  # needs sage.numerical.mip
```

(continues on next page)
sage.numerical.optimize.find_fit(data, model, initial_guess=None, parameters=None, variables=None, solution_dict=False)

Finds numerical estimates for the parameters of the function model to give a best fit to data.

INPUT:

• data – A two dimensional table of floating point numbers of the form \([[[x_{1,1}, x_{1,2}, \ldots, x_{1,k}, f_1], [x_{2,1}, x_{2,2}, \ldots, x_{2,k}, f_2], \ldots, [x_{n,1}, x_{n,2}, \ldots, x_{n,k}, f_n]]\) given as either a list of lists, matrix, or numpy array.

• model – Either a symbolic expression, symbolic function, or a Python function. model has to be a function of the variables \((x_1, x_2, \ldots, x_k)\) and free parameters \((a_1, a_2, \ldots, a_l)\).

• initial_guess – (default: None) Initial estimate for the parameters \((a_1, a_2, \ldots, a_l)\), given as either a list, tuple, vector or numpy array. If None, the default estimate for each parameter is 1.

• parameters – (default: None) A list of the parameters \((a_1, a_2, \ldots, a_l)\). If model is a symbolic function it is ignored, and the free parameters of the symbolic function are used.

• variables – (default: None) A list of the variables \((x_1, x_2, \ldots, x_k)\). If model is a symbolic function it is ignored, and the variables of the symbolic function are used.

• solution_dict – (default: False) if True, return the solution as a dictionary rather than an equation.

EXAMPLES:

First we create some data points of a sine function with some “random” perturbations:

```
sage: set_random_seed(0)
sage: data = [(i, 1.2 * sin(0.5*i-0.2) + 0.1 * normalvariate(0, 1))
          for i in xsrange(0, 4*pi, 0.2)]
sage: var('a, b, c, x')
```

We define a function with free parameters \(a, b\) and \(c\):

```
sage: model(x) = a * sin(b * x - c)
```

We search for the parameters that give the best fit to the data:

```
sage: find_fit(data, model)
```

We can also use a Python function for the model:

```
sage: def f(x, a, b, c): return a * sin(b * x - c)
sage: fit = find_fit(data, f, parameters=[a, b, c], variables=[x], solution_dict=True)
```
We search for a formula for the $n$-th prime number:

```python
sage: # needs sage.libs.pari
sage: dataprime = [(i, nth_prime(i)) for i in range(1, 5000, 100)]

sage: find_fit(dataprime, a * x * log(b * x),
        parameters=[a, b], variables=[x])
[\begin{verbatim}
a == 1.11..., b == 1.24...
\end{verbatim}]
```

ALGORITHM:
Uses scipy.optimize.leastsq which in turn uses MINPACK’s lmdif and lmder algorithms.

```python
sage.numerical.optimize.find_local_maximum(f, a, b, tol=1.48e-08, maxfun=500)

Numerically find a local maximum of the expression $f$ on the interval $[a, b]$ (or $[b, a]$) along with the point at which the maximum is attained.

Note that this function only finds a local maximum, and not the global maximum on that interval – see the examples with find_local_maximum().

See the documentation for find_local_maximum() for more details and possible workarounds for finding the global minimum on an interval.

EXAMPLES:

```python
sage: f = lambda x: x*cos(x)
sage: find_local_maximum(f, 0, 5)
(0.561096338191..., 0.8603335890...)

sage: find_local_maximum(f, 0, 5, tol=0.1, maxfun=10)
(0.561090323458..., 0.857926501456...)

sage: find_local_maximum(8*e^(-x)*sin(x) - 1, 0, 7)
(1.579175535558..., 0.7853981...)
```

```python
sage.numerical.optimize.find_local_minimum(f, a, b, tol=1.48e-08, maxfun=500)

Numerically find a local minimum of the expression $f$ on the interval $[a, b]$ (or $[b, a]$) and the point at which it attains that minimum. Note that $f$ must be a function of (at most) one variable.

Note that this function only finds a local minimum, and not the global minimum on that interval – see the examples below.

INPUT:

- $f$ – a function of at most one variable.
- $a$, $b$ – endpoints of interval on which to minimize $f$.
- $tol$ – the convergence tolerance
- $maxfun$ – maximum function evaluations

OUTPUT:

- $\text{minval}$ – (float) the minimum value that $f$ takes on in the interval $[a, b]$
- $x$ – (float) the point at which $f$ takes on the minimum value
EXAMPLES:

```python
sage: f = lambda x: x^2*cos(x)
sage: find_local_minimum(f, 1, 5)
(-3.28837139559..., 3.4256184695...)  # Needs sage.plot
sage: find_local_minimum(f, 1, 5, tol=1e-3)
(-3.28837136189098..., 3.42575079030572...)  # Needs sage.symbolic
sage: find_local_minimum(f, 1, 5, tol=1e-2, maxfun=10)
(-3.28837084598..., 3.4250840220...)  # Needs sage.symbolic
```

Only local minima are found; if you enlarge the interval, the returned minimum may be larger! See [github issue #2607](https://github.com/sage/sage/issues/2607).

```python
sage: # Needs sage.symbolic
sage: f(x) = -x*sin(x^2)
sage: find_local_minimum(f, -2.5, -1)
(-2.182769784677722, -2.1945027498534686)
```

Enlarging the interval returns a larger minimum:

```python
sage: # Needs sage.plot sage.symbolic
sage: plot(f, (x, -2.5, -1)).ymin()  # Needs sage.plot
-2.182...
sage: plot(f, (x, -2.5, 2)).ymin()  # Needs sage.plot
-2.182...
```

ALGORITHM:
Uses `scipy.optimize.fminbound` which uses Brent’s method.

AUTHOR:
- William Stein (2007-12-07)

```python
sage.numerical.optimize.find_root(f, a, b, xtol=1e-12, rtol=8.88178419700125e-16, maxiter=100, full_output=False)
```

Numerically find a root of $f$ on the closed interval $[a, b]$ (or $[b, a]$) if possible, where $f$ is a function in the one variable. Note: this function only works in fixed (machine) precision, it is not possible to get arbitrary precision approximations with it.

INPUT:
- $f$ – a function of one variable or symbolic equality
- $a$, $b$ – endpoints of the interval
- $xtol$, $rtol$ – the routine converges when a root is known to lie within $xtol$ of the value return. Should be $\geq 0$. The routine modifies this to take into account the relative precision of doubles. By default, $rtol$ is

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is $4*numpy.finfo(float).eps$, the minimum allowed value for `scipy.optimize.brentq`, which is what this method uses underneath. This value is equal to $2.0^{-50}$ for IEEE-754 double precision floats as used by Python.

- **maxiter** – integer; if convergence is not achieved in `maxiter` iterations, an error is raised. Must be $\geq 0$.
- **full_output** – bool (default: `False`), if `True`, also return object that contains information about convergence.

**EXAMPLES:**

An example involving an algebraic polynomial function:

```python
sage: R.<x> = QQ[]
sage: f = (x+17)*(x-3)*(x-1/8)^3
sage: find_root(f, 0,4)
2.999999999999995
sage: find_root(f, 0,1)  # abs tol 1e-6 (note -- precision of answer isn't very good, on some machines)
0.124999
sage: find_root(f, -20,-10)
-17.0
```

In Pomerance’s book on primes he asserts that the famous Riemann Hypothesis is equivalent to the statement that the function $f(x)$ defined below is positive for all $x \geq 2.01$:

```python
sage: def f(x):
....:     return sqrt(x) * log(x) - abs(Li(x) - prime_pi(x))
```

We find where $f$ equals, i.e., what value that is slightly smaller than 2.01 that could have been used in the formulation of the Riemann Hypothesis:

```python
sage: find_root(f, 2, 4, rtol=0.0001)
2.0082...
```

This agrees with the plot:

```python
sage: plot(f,2,2.01)
```

The following example was added due to github issue #4942 and demonstrates that the function need not be defined at the endpoints:

```python
sage: find_root(x^2*log(x,2)-1,0, 2)
1.41421356237
```

The following is an example, again from github issue #4942 where Brent’s method fails. Currently no other method is implemented, but at least we acknowledge the fact that the algorithm fails:

```python
sage: find_root(1/(x-1)+1,0, 2)
0.0
sage: find_root(1/(x-1)+1,0.00001, 2)
Traceback (most recent call last):
  ... NotImplementedError: Brent's method failed to find a zero for f on the interval
```

An example of a function which evaluates to NaN on the entire interval:
sage: f(x) = 0.0 / max(0, x)
sage: find_root(f, -1, 0)
Traceback (most recent call last):
  ... 
RuntimeError: f appears to have no zero on the interval

sage.numerical.optimize.minimize(func, x0, gradient=None, hessian=None, algorithm='default', verbose=False, **args)

This function is an interface to a variety of algorithms for computing the minimum of a function of several variables.

INPUT:

• func – Either a symbolic function or a Python function whose argument is a tuple with \( n \) components
• x0 – Initial point for finding minimum.
• gradient – Optional gradient function. This will be computed automatically for symbolic functions. For Python functions, it allows the use of algorithms requiring derivatives. It should accept a tuple of arguments and return a NumPy array containing the partial derivatives at that point.
• hessian – Optional hessian function. This will be computed automatically for symbolic functions. For Python functions, it allows the use of algorithms requiring derivatives. It should accept a tuple of arguments and return a NumPy array containing the second partial derivatives of the function.
• algorithm – String specifying algorithm to use. Options are 'default' (for Python functions, the simplex method is the default) (for symbolic functions bfgs is the default):
  – 'simplex' – using the downhill simplex algorithm
  – 'powell' – use the modified Powell algorithm
  – 'bfgs' – (Broyden-Fletcher-Goldfarb-Shanno) requires gradient
  – 'cg' – (conjugate-gradient) requires gradient
  – 'ncg' – (newton-conjugate gradient) requires gradient and hessian
• verbose – (optional, default: False) print convergence message

Note: For additional information on the algorithms implemented in this function, consult SciPy’s documentation on optimization and root finding

EXAMPLES:

Minimize a fourth order polynomial in three variables (see the Wikipedia article Rosenbrock_function):

sage: vars = var('x y z')
# needs sage.symbolic
sage: f = 100*(y-x^2)^2 + (1-x)^2 + 100*(z-y^2)^2 + (1-y)^2
# needs sage.symbolic
sage: minimize(f, [.1,.3,.4]) # abs tol 1e-6
# needs sage.symbolic
(1.0, 1.0, 1.0)

Try the newton-conjugate gradient method; the gradient and hessian are computed automatically:
We get additional convergence information with the `verbose` option:

```
sage: minimize(f, [.1, .3, .4], algorithm="ncg", verbose=True)  # Needs sage.symbolic
Optimization terminated successfully.
... (0.999999..., 0.999999..., 0.999999...)
```

Same example with just Python functions:

```
sage: def rosen(x):  # The Rosenbrock function
.....:     return sum(100.0*(x[1]-x[0]**2)**2 + (1-x[0])**2)
sage: minimize(rosen, [.1,.3,.4])  # abs tol 3e-5
(1.0, 1.0, 1.0)
```

Same example with a pure Python function and a Python function to compute the gradient:

```
sage: def rosen(x):  # The Rosenbrock function
.....:     return sum(100.0*(x[1]-x[0]**2)**2 + (1-x[0])**2)
sage: import numpy
sage: from numpy import zeros
sage: def rosen_der(x):
.....:     xm = x[1:-1]
.....:     xm_m1 = x[:-2]
.....:     xm_p1 = x[2:]
.....:     der = zeros(x.shape, dtype=float)
.....:     der[1:-1] = 200*(xm-xm_m1**2) - 400*(xm_p1 - xm**2)*xm - 2*(1-xm)
.....:     der[0] = -400*x[0]*(x[1]-x[0]**2) - 2*(1-x[0])
.....:     der[-1] = 200*(x[-1]-x[-2]**2)
.....:     return der
sage: minimize(rosen, [.1,.3,.4], gradient=rosen_der,  # abs tol 1e-6
.....:     algorithm="bfgs")
(1.0, 1.0, 1.0)
```

`sage.numerical.optimize.minimize_constrained(func, cons, x0, gradient=None, algorithm='default', **args)`

Minimize a function with constraints.

**INPUT:**

- **func** – Either a symbolic function, or a Python function whose argument is a tuple with \( n \) components
- **cons** – constraints. This should be either a function or list of functions that must be positive. Alternatively, the constraints can be specified as a list of intervals that define the region we are minimizing in. If the constraints are specified as functions, the functions should be functions of a tuple with \( n \) components (assuming \( n \) variables). If the constraints are specified as a list of intervals and there are no constraints for a given variable, that component can be \((\text{None}, \text{None})\).
- **x0** – Initial point for finding minimum
- **algorithm** – Optional, specify the algorithm to use:
- 'default' – default choices
- 'l-bfgs-b' – only effective if you specify bound constraints. See [ZBN1997].

• gradient – Optional gradient function. This will be computed automatically for symbolic functions. This is only used when the constraints are specified as a list of intervals.

EXAMPLES:

Let us maximize $x + y - 50$ subject to the following constraints: $50x + 24y \leq 2400$, $30x + 33y \leq 2100$, $x \geq 45$, and $y \geq 5$:

```python
sage: f = lambda p: -p[0]-p[1]+50
sage: c_1 = lambda p: p[0]-45
sage: c_2 = lambda p: p[1]-5
sage: c_3 = lambda p: -50*p[0]-24*p[1]+2400
sage: c_4 = lambda p: -30*p[0]-33*p[1]+2100
sage: a = minimize_constrained(f, [c_1, c_2, c_3, c_4], [2, 3])
```

(45.0, 6.25...)

Let’s find a minimum of $\sin(xy)$:

```python
sage: x, y = var('x y')
# needs sage.symbolic
sage: f(x, y) = sin(x*y)
# needs sage.symbolic
sage: minimize_constrained(f, [(None, None), (4, 10)], [5, 5])
```

(4.8..., 4.8...)

Check if L-BFGS-B finds the same minimum:

```python
sage: minimize_constrained(f, [(None, None), (4, 10)], [5, 5],
                        # needs sage.symbolic
                        ....: algorithm='l-bfgs-b')
(4.7..., 4.9...)
```

Rosenbrock function (see the Wikipedia article Rosenbrock_function):

```python
sage: from scipy.optimize import rosen, rosen_der
sage: minimize_constrained(rosen, [(-50, -10), (5, 10)], [1, 1],
                        ....: gradient=rosen_der, algorithm='l-bfgs-b')
(-10.0, 10.0)
```

```python
sage: minimize_constrained(rosen, [(-50, -10), (5, 10)], [1, 1],
                        ....: algorithm='l-bfgs-b')
(-10.0, 10.0)
```
CHAPTER
NINE

INTERACTIVE SIMPLEX METHOD

This module, meant for educational purposes only, supports learning and exploring of the simplex method.

Do you want to solve Linear Programs efficiently? use MixedIntegerLinearProgram instead.

The methods implemented here allow solving Linear Programming Problems (LPPs) in a number of ways, may require explicit (and correct!) description of steps and are likely to be much slower than “regular” LP solvers. If, however, you want to learn how the simplex method works and see what happens in different situations using different strategies, but don’t want to deal with tedious arithmetic, this module is for you!

Historically it was created to complement the Math 373 course on Mathematical Programming and Optimization at the University of Alberta, Edmonton, Canada.

AUTHORS:


EXAMPLES:

Most of the module functionality is demonstrated on the following problem.

**Corn & Barley**

A farmer has 1000 acres available to grow corn and barley. Corn has a net profit of 10 dollars per acre while barley has a net profit of 5 dollars per acre. The farmer has 1500 kg of fertilizer available with 3 kg per acre needed for corn and 1 kg per acre needed for barley. The farmer wants to maximize profit. (Sometimes we also add one more constraint to make the initial dictionary infeasible: the farmer has to use at least 40% of the available land.)

Using variables $C$ and $B$ for land used to grow corn and barley respectively, in acres, we can construct the following LP problem:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P
```

LP problem (use 'view(...)' or '%display typeset' for details)

It is recommended to copy-paste such examples into your own worksheet, so that you can run these commands with
typeset mode on (%display typeset) and get

\[
\begin{align*}
\max & \quad 10C + 5B \\
& \quad C + B \leq 1000 \\
& \quad 3C + B \leq 1500 \\
& \quad C, B \geq 0
\end{align*}
\]

Since it has only two variables, we can solve it graphically:

```sage
sage: P.plot() #
needs sage.plot
```

The simplex method can be applied only to problems in standard form, which can be created either directly

```sage
sage: InteractiveLPProblemStandardForm(A, b, c, ['C', 'B'])
LP problem (use ...)
```
or from an already constructed problem of “general type”:

```sage
sage: P = P.standard_form()
```

In this case the problem does not require any modifications to be written in standard form, but this step is still necessary to enable methods related to the simplex method.

The simplest way to use the simplex method is:

```sage
sage: P.run_simplex_method()
\begin{equation*}
\begin{align*}
\text{The optimal value: } & 6250. \\
\text{An optimal solution: } & (250, 750).
\end{align*}
\end{equation*}
```

(This method produces quite long formulas which have been omitted here.) But, of course, it is much more fun to do most of the steps by hand. Let’s start by creating the initial dictionary:

```sage
sage: D = P.initial_dictionary()
sage: D
LP problem dictionary (use ...)
```

Using typeset mode as recommended, you’ll see

\[
\begin{align*}
x_3 & = 1000 - C - B \\
x_4 & = 1500 - 3C - B \\
z & = 0 + 10C + 5B
\end{align*}
\]

With the initial or any other dictionary you can perform a number of checks:

```sage
sage: D.is_feasible()
True
sage: D.is_optimal()
False
```

You can look at many of its pieces and associated data:
Most importantly, you can perform steps of the simplex method by picking an entering variable, a leaving variable, and updating the dictionary:

\begin{verbatim}
sage: D.enter("C")
sage: D.leave(4)
sage: D.update()
\end{verbatim}

If everything was done correctly, the new dictionary is still feasible and the objective value did not decrease:

\begin{verbatim}
sage: D.is_feasible()
True
sage: D.objective_value()
5000
\end{verbatim}

If you are unsure about picking entering and leaving variables, you can use helper methods that will try their best to tell you what are your next options:

\begin{verbatim}
sage: D.possible_entering()
[B]
sage: D.possible_leaving()
Traceback (most recent call last):
  ...:
ValueError: leaving variables can be determined
  for feasible dictionaries with a set entering variable
  or for dual feasible dictionaries
\end{verbatim}

It is also possible to obtain feasible sets and final dictionaries of problems, work with revised dictionaries, and use the dual simplex method!

Note: Currently this does not have a display format for the terminal.

### 9.1 Classes and functions

**class** `sage.numerical.interactive_simplex_method.InteractiveLPProblem(A, b, c, x='x', constraint_type='<=', variable_type='''', problem_type='max', base_ring=None, is_primal=True, objective_constant_term=0)`

Bases: `SageObject`

Construct an LP (Linear Programming) problem.
This class supports LP problems with “variables on the left” constraints.

INPUT:

- A – a matrix of constraint coefficients
- b – a vector of constraint constant terms
- c – a vector of objective coefficients
- x – (default: "x") a vector of decision variables or a string giving the base name
- constraint_type – (default: "\leq") a string specifying constraint type(s): either "\leq", "\geq", "\equiv", or a list of them
- variable_type – (default: "") a string specifying variable type(s): either "\geq", "\leq", "" (the empty string), or a list of them, corresponding, respectively, to non-negative, non-positive, and free variables
- problem_type – (default: "max") a string specifying the problem type: "max", "min", "-max", or "-min"
- base_ring – (default: the fraction field of a common ring for all input coefficients) a field to which all input coefficients will be converted
- is_primal – (default: True) whether this problem is primal or dual: each problem is of course dual to its own dual, this flag is mostly for internal use and affects default variable names only
- objective_constant_term – (default: 0) a constant term of the objective

EXAMPLES:

We will construct the following problem:

\[
\begin{align*}
\text{max} & \quad 10C + 5B \\
& \quad C + B \leq 1000 \\
& \quad 3C + B \leq 1500 \\
& \quad C, B \geq 0
\end{align*}
\]

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="\geq")
```

Same problem, but more explicitly:

```
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"],
.....:            constraint_type="\leq", variable_type="\geq")
```

Even more explicitly:

```
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], problem_type="max",
.....:            constraint_type=["\leq", "\leq"], variable_type=["\geq", "\geq"])
```

Using the last form you should be able to represent any LP problem, as long as all like terms are collected and in constraints variables and constants are on different sides.
A()  
Return coefficients of constraints of self, i.e. $A$.

OUTPUT:
• a matrix

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constraint_coefficients()
[1 1]
[3 1]
sage: P.A()
[1 1]
[3 1]
```

Abcx()  
Return $A$, $b$, $c$, and $x$ of self as a tuple.

OUTPUT:
• a tuple

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.Abcx()
([1 1]
 [3 1],
(1000, 1500),
(10, 5),
(C, B))
```

add_constraint(coefficients, constant_term, constraint_type='<=')  
Return a new LP problem by adding a constraint to `self`.

INPUT:
• coefficients – coefficients of the new constraint
• constant_term – a constant term of the new constraint
• constraint_type – (default: "<=") a string indicating the constraint type of the new constraint

OUTPUT:
• an LP problem

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
```
sage: P = InteractiveLPProblem(A, b, c)
sage: P1 = P.add_constraint(([2, 4]), 2000, "\leq")
sage: P1.Abcx()
(1 1
[3 1]
[2 4], (1000, 1500, 2000), (10, 5), (x1, x2))
sage: P1.constraint_types()
('\leq', '\leq', '\leq')
sage: P.Abcx()
(1 1
[3 1], (1000, 1500), (10, 5), (x1, x2))
sage: P.constraint_types()
('\leq', '\leq')
sage: P2 = P.add_constraint(([2, 4, 6]), 2000, "\leq")
Traceback (most recent call last):
  ...TypeError: number of columns must be the same, not 2 and 3
sage: P3 = P.add_constraint(([2, 4]), 2000, "<")
Traceback (most recent call last):
  ...ValueError: unknown constraint type

b()

Return constant terms of constraints of self, i.e. b.

OUTPUT:
  • a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="\geq")
sage: P.constant_terms()
(1000, 1500)
sage: P.b()
(1000, 1500)

base_ring()

Return the base ring of self.

Note: The base ring of LP problems is always a field.

OUTPUT:
  • a ring

EXAMPLES:
c()

Return coefficients of the objective of self, i.e. $c$.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_coefficients()  
(10, 5)
sage: P.c()  
(10, 5)
```

```
c()  
Return coefficients of the objective of self, i.e. $c$.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_coefficients()  
(10, 5)
sage: P.c()  
(10, 5)
```

```
constant_terms()

Return constant terms of constraints of self, i.e. $b$.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constant_terms()  
(1000, 1500)
sage: P.b()  
(1000, 1500)
```

```
constant_terms()

Return constant terms of constraints of self, i.e. $b$.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constant_terms()  
(1000, 1500)
sage: P.b()  
(1000, 1500)
```

```
constraint_coefficients()

Return coefficients of constraints of self, i.e. $A$.

OUTPUT:

• a matrix

EXAMPLES:

```
```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constraint_coefficients()
[1 1]
[3 1]
sage: P.A()
[1 1]
[3 1]

constraint_types()
Return a tuple listing the constraint types of all rows.

OUTPUT:
• a tuple of strings

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=", constraint_type=["<=""]
sage: P.constraint_types()
('<=', '==')

decision_variables()
Return decision variables of self, i.e. x.

OUTPUT:
• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.decision_variables()
(C, B)
sage: P.x()
(C, B)

dual(y=None)
Construct the dual LP problem for self.

INPUT:
• y – (default: depends on style()) a vector of dual decision variables or a string giving the base name

OUTPUT:
• an InteractiveLPProblem

EXAMPLES:
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="\ge")
sage: DP = P.dual()
sage: DP.b() == P.c()
True
sage: DP.dual(["C", "B"])) == P
True

feasible_set()

Return the feasible set of self.

OUTPUT:

• a Polyhedron

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="\ge")
sage: P.feasible_set()
A 2-dimensional polyhedron in QQ^2
defined as the convex hull of 4 vertices

is_bounded()

Check if self is bounded.

OUTPUT:

• True is self is bounded, False otherwise

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="\ge")
sage: P.is_bounded()
True

Note that infeasible problems are always bounded:

sage: b = (-1000, 1500)
sage: P = InteractiveLPProblem(A, b, c, variable_type="\ge")
sage: P.is_feasible()
False
sage: P.is_bounded()
True

is_feasible(*x)

Check if self or given solution is feasible.

INPUT:
• (optional) anything that can be interpreted as a valid solution for this problem, i.e. a sequence of values for all decision variables

OUTPUT:
• True if this problem or given solution is feasible, False otherwise

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, variable_type=">=")
sage: P.is_feasible()
True
sage: P.is_feasible(100, 200)
True
sage: P.is_feasible(1000, 200)
False
sage: P.is_feasible([1000, 200])
False
sage: P.is_feasible(1000)
Traceback (most recent call last):
...
TypeError: given input is not a solution for this problem
```

**is_negative()**

Return True when the problem is of type "-max" or "-min".

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.is_negative()
False
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=", problem_type="-min")
sage: P.is_negative()
True
```

**is_optimal(**x**)**

Check if given solution is feasible.

INPUT:
• anything that can be interpreted as a valid solution for this problem, i.e. a sequence of values for all decision variables

OUTPUT:
• True if the given solution is optimal, False otherwise

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
```
sage: c = (15, 5)
sage: P = InteractiveLPProblem(A, b, c, variable_type=">=")
sage: P.is_optimal(100, 200)
False
sage: P.is_optimal(500, 0)
True
sage: P.is_optimal(499, 3)
True
sage: P.is_optimal(501, -3)
False

is_primal()
Check if we consider this problem to be primal or dual.
This distinction affects only some automatically chosen variable names.

OUTPUT:
• boolean

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.is_primal()
True
sage: P.dual().is_primal()
False

m()
Return the number of constraints of self, i.e. \( m \).

OUTPUT:
• an integer

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_constraints()
2
sage: P.m()
2

n()
Return the number of decision variables of self, i.e. \( n \).

OUTPUT:
• an integer

EXAMPLES:
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_variables()
2
sage: P.n()
2

n_constraints()

Return the number of constraints of self, i.e. \( m \).

OUTPUT:

• an integer

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_constraints()
2
sage: P.m()
2

n_variables()

Return the number of decision variables of self, i.e. \( n \).

OUTPUT:

• an integer

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_variables()
2
sage: P.n()
2

objective_coefficients()

Return coefficients of the objective of self, i.e. \( c \).

OUTPUT:

• a vector

EXAMPLES:
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_coefficients()
(10, 5)
sage: P.c()
(10, 5)

**objective_constant_term()**

Return the constant term of the objective.

**OUTPUT:**

• a number

**EXAMPLES:**

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_constant_term()
0
sage: P.optimal_value()
6250
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"],
.....: variable_type=">=", objective_constant_term=-1250)
sage: P.objective_constant_term()
-1250
sage: P.optimal_value()
5000

**objective_value(*x)*

Return the value of the objective on the given solution.

**INPUT:**

• anything that can be interpreted as a valid solution for this problem, i.e. a sequence of values for all decision variables

**OUTPUT:**

• the value of the objective on the given solution taking into account **objective_constant_term()** and **is_negative()**

**EXAMPLES:**

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, variable_type=">=")
sage: P.objective_value(100, 200)
2000

**optimal_solution()**

Return an optimal solution of self.

**OUTPUT:**
Numerical Optimization, Release 10.2

• a vector or None if there are no optimal solutions

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ['C', 'B'], variable_type='>=')
sage: P.optimal_solution()
(250, 750)
```

**optimal_value()**

Return the optimal value for self.

**OUTPUT:**

• a number if the problem is bounded, \( \pm \infty \) if it is unbounded, or None if it is infeasible

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ['C', 'B'], variable_type='>=')
sage: P.optimal_value()
6250
```

**plot(*args, **kwds)**

Return a plot for solving self graphically.

**INPUT:**

• xmin, xmax, ymin, ymax – bounds for the axes, if not given, an attempt will be made to pick reasonable values

• alpha – (default: 0.2) determines how opaque are shadows

**OUTPUT:**

• a plot

This only works for problems with two decision variables. On the plot the black arrow indicates the direction of growth of the objective. The lines perpendicular to it are level curves of the objective. If there are optimal solutions, the arrow originates in one of them and the corresponding level curve is solid; all points of the feasible set on it are optimal solutions. Otherwise the arrow is placed in the center. If the problem is infeasible or the objective is zero, a plot of the feasible set only is returned.

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ['C', 'B'], variable_type='>='

sage: p = P.plot() # needs sage.plot
sage: p.show() # needs sage.plot
```

In this case the plot works better with the following axes ranges:
plot_feasible_set(xmin=None, xmax=None, ymin=None, ymax=None, alpha=0.2)

Return a plot of the feasible set of self.

INPUT:

- xmin, xmax, ymin, ymax – bounds for the axes, if not given, an attempt will be made to pick reasonable values
- alpha – (default: 0.2) determines how opaque are shadows

OUTPUT:

- a plot

This only works for a problem with two decision variables. The plot shows boundaries of constraints with a shadow on one side for inequalities. If the feasible_set() is not empty and at least part of it is in the given boundaries, it will be shaded gray and \( F \) will be placed in its middle.

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: p = P.plot_feasible_set()  # needs sage.plot
sage: p.show()  # needs sage.plot
```

In this case the plot works better with the following axes ranges:

```python
sage: p = P.plot_feasible_set(0, 1000, 0, 1500)  # needs sage.plot
sage: p.show()  # needs sage.plot
```

problem_type()

Return the problem type.

Needs to be used together with is_negative.

OUTPUT:

- a string, one of "max", "min".

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: Pproblem_type() 'max'
```
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type="=>", problem_type="-min")

standard_form(transformation=False, **kwds)

Construct the LP problem in standard form equivalent to self.

INPUT:

- transformation – (default: False) if True, a map converting solutions of the problem in standard form to the original one will be returned as well
- you can pass (as keywords only) slack_variables, auxiliary_variable,``objective_name`` to the constructor of InteractiveLPProblemStandardForm

OUTPUT:

- an InteractiveLPProblemStandardForm by itself or a tuple with variable transformation as the second component

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, variable_type="=>")

A more complicated transformation map:

sage: P = InteractiveLPProblem(A, b, c, variable_type=["<=", ""],
....: objective_constant_term=42)

sage: PSF, f = P.standard_form(True)

Vector space morphism represented by the matrix:
[1 0]
[0 1]
[0 -1]
Domain: Vector space of dimension 3 over Rational Field
Codomain: Vector space of dimension 2 over Rational Field
Codomain: Vector space of dimension 2 over Rational Field

**sage:** PSF.optimal_solution()
(0, 1000, 0)

**sage:** P.optimal_solution()
(0, 1000)

**sage:** P.is_optimal(PSF.optimal_solution())
Traceback (most recent call last):
...
TypeError: given input is not a solution for this problem

**sage:** P.is_optimal(f(PSF.optimal_solution()))
True

**sage:** PSF.optimal_value()
5042
**sage:** P.optimal_value()
5042

---

**variable_types()**
Return a tuple listing the variable types of all decision variables.

**OUTPUT:**

• a tuple of strings

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=[">=", ""])
sage: P.variable_types()
(‘>=’, ‘’)
```

---

**x()**
Return decision variables of self, i.e. $x$.

**OUTPUT:**

• a vector

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.decision_variables()
(C, B)
sage: P.x()
(C, B)
```
class sage.numerical.interactive_simplex_method.InteractiveLPProblemStandardForm(A, b, c, x='x', problem_type='max', slack_variables=None, auxiliary_variable=None, base_ring=None, is_primal=True, objective_name=None, objective_constant_term=0)

Bases: InteractiveLPProblem

Construct an LP (Linear Programming) problem in standard form.

**Note:** This class is for educational purposes only: if you want to solve Linear Programs efficiently, use MixedIntegerLinearProgram instead.

The used standard form is:

\[ \pm \max cx \]
\[ Ax \leq b \]
\[ x \geq 0 \]

**INPUT:**

- A – a matrix of constraint coefficients
- b – a vector of constraint constant terms
- c – a vector of objective coefficients
- x – (default: "x") a vector of decision variables or a string the base name giving
- problem_type – (default: "max") a string specifying the problem type: either "max" or "-max"
- slack_variables – (default: depends on style()) a vector of slack variables or a string giving the base name
- auxiliary_variable – (default: same as x parameter with adjoined "0" if it was given as a string, otherwise "x0") the auxiliary name, expected to be the same as the first decision variable for auxiliary problems
- base_ring – (default: the fraction field of a common ring for all input coefficients) a field to which all input coefficients will be converted
- is_primal – (default: True) whether this problem is primal or dual: each problem is of course dual to its own dual, this flag is mostly for internal use and affects default variable names only
- objective_name – a string or a symbolic expression for the objective used in dictionaries, default depends on style()
- objective_constant_term – (default: 0) a constant term of the objective

**EXAMPLES:**
Unlike `InteractiveLPProblem`, this class does not allow you to adjust types of constraints (they are always "\(<\leq\)"") and variables (they are always "\(\geq\)"). And the problem type may only be "max" or "-max". You may give custom names to slack and auxiliary variables, but in most cases defaults should work:

```python
sage: P.decision_variables()
(x1, x2)
sage: P.slack_variables()
(x3, x4)
```

`add_constraint(coefficients, constant_term, slack_variable=None)`

Return a new LP problem by adding a constraint to `self`.

```
INPUT:
• coefficients – coefficients of the new constraint
• constant_term – a constant term of the new constraint
• slack_variable – (default: depends on `style()`) a string giving the name of the slack variable of the new constraint

OUTPUT:
• an LP problem in standard form
```

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.Abcx()
([1 1]
 [3 1], (1000, 1500), (10, 5), (x1, x2))
sage: P.slack_variables()
(x3, x4)
sage: P1 = P.add_constraint(([2, 4]), 2000)
sage: P1.Abcx()
([1 1]
 [3 1], [2 4], (1000, 1500, 2000), (10, 5), (x1, x2))
sage: P1.slack_variables()
(x3, x4, x5)
sage: P2 = P.add_constraint(([2, 4]), 2000, slack_variable='c')
sage: P2.slack_variables()
(x3, x4, c)
sage: P3 = P.add_constraint(([2, 4, 6]), 2000)
```

(continues on next page)
auxiliary_problem\( (\text{objective\_name=\text{None}}) \)

Construct the auxiliary problem for \( \text{self} \).

INPUT:

\( \cdot \) \text{objective\_name} – a string or a symbolic expression for the objective used in dictionaries, default depends on \text{style()}\)

OUTPUT:

\( \cdot \) an LP problem in standard form

The auxiliary problem with the auxiliary variable \( x_0 \) is

\[
\begin{align*}
\max & -x_0 \\
-x_0 + A_i x & \leq b_i \text{ for all } i \\
x & \geq 0
\end{align*}
\]

Such problems are used when the \text{initial\_dictionary()} is infeasible.

EXAMPLES:

\begin{verbatim}
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: AP = P.auxiliary_problem()
\end{verbatim}

auxiliary_variable()

Return the auxiliary variable of \( \text{self} \).

Note that the auxiliary variable may or may not be among \text{decision\_variables()}.

OUTPUT:

\( \cdot \) a variable of the \text{coordinate\_ring()} of \( \text{self} \)

EXAMPLES:

\begin{verbatim}
\begin{verbatim}
\end{verbatim}
\end{verbatim}
**coordinate_ring()**

Return the coordinate ring of `self`.

**OUTPUT:**

- a polynomial ring over the `base_ring()` of `self` in the `auxiliary_variable()`, `decision_variables()`, and `slack_variables()` with “neglex” order

**EXAMPLES:**

```python
sage: A = ([[1, 1], [3, 1], [-1, -1]])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4, x5 over Rational Field
sage: P.base_ring()
Rational Field
sage: P.auxiliary_variable()
x0
sage: P.decision_variables()
(x1, x2)
sage: P.slack_variables()
(x3, x4, x5)
```

**dictionary(*x_B)*

Construct a dictionary for `self` with given basic variables.

**INPUT:**

- basic variables for the dictionary to be constructed

**OUTPUT:**

- a `dictionary`

**Note:** This is a synonym for `self.revised_dictionary(x_B).dictionary()`, but basic variables are mandatory.

**EXAMPLES:**

```python
sage: A = ([[1, 1], [3, 1]])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary("x1", "x2")
sage: D.basic_variables()
(x1, x2)
```

**feasible_dictionary(auxiliary_dictionary)**

Construct a feasible dictionary for `self`.

**INPUT:**

- `auxiliary_dictionary` – an optimal dictionary for the `auxiliary_problem()` of `self` with the optimal value 0 and a non-basic auxiliary variable
OUTPUT:

• a feasible dictionary for self

EXAMPLES:

```sage
A = ([1, 1], [3, 1], [-1, -1])
b = (1000, 1500, -400)
c = (10, 5)
P = InteractiveLPProblemStandardForm(A, b, c)
AP = P.auxiliary_problem()
D = AP.initial_dictionary()
D.enter(0)
D.leave(5)
D.update()
D.enter(1)
D.leave(0)
D.update()
D.is_optimal()
True
D.objective_value()
0
D.basic_solution()
(0, 400, 0)
D = P.feasible_dictionary(D)
D.is_optimal()
False
D.is_feasible()
True
D.objective_value()
4000
D.basic_solution()
(400, 0)
```

`final_dictionary()`

Return the final dictionary of the simplex method applied to self.

See `run_simplex_method()` for the description of possibilities.

OUTPUT:

• a dictionary

EXAMPLES:

```sage
A = ([1, 1], [3, 1])
b = (1000, 1500)
c = (10, 5)
P = InteractiveLPProblemStandardForm(A, b, c)
D = P.final_dictionary()
D.is_optimal()
True
D.objective_value()
4000
D.basic_solution()
(400, 0)
```

`final_revised_dictionary()`

Return the final dictionary of the revised simplex method applied to self.

See `run_revised_simplex_method()` for the description of possibilities.
OUTPUT:

• a revised dictionary

EXAMPLES:

```
sage: A = ([[1, 1], [3, 1]])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.final_revised_dictionary()
sage: D.is_optimal()
True
```

`initial_dictionary()`

Construct the initial dictionary of self.

The initial dictionary “defines” `slack_variables()` in terms of the `decision_variables()`, i.e. it has slack variables as basic ones.

OUTPUT:

• a dictionary

EXAMPLES:

```
sage: A = ([[1, 1], [3, 1]])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
```

`inject_variables(scope=None, verbose=True)`

Inject variables of self into scope.

INPUT:

• `scope` – namespace (default: global)
• `verbose` – if True (default), names of injected variables will be printed

OUTPUT:

• none

EXAMPLES:

```
sage: A = ([[1, 1], [3, 1]])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: 3*x1 + x2
x2 + 3*x1
```

`objective_name()`

Return the objective name used in dictionaries for this problem.

OUTPUT:
- a symbolic expression

EXAMPLES:

```sage
A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.objective_name()
z
sage: sage.numerical.interactive_simplex_method.style("Vanderbei")
'Vanderbei'
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.objective_name()
zeta
sage: sage.numerical.interactive_simplex_method.style("UAlberta")
'UAlberta'
sage: P = InteractiveLPProblemStandardForm(A, b, c, objective_name="custom")
sage: P.objective_name()
custom
```

`static random_element(m, n, bound=5, special_probability=0.2, **kwds)`

Construct a random `InteractiveLPProblemStandardForm`.

**INPUT:**

- `m` – the number of constraints/basic variables
- `n` – the number of decision/non-basic variables
- `bound` – (default: 5) a bound on coefficients
- `special_probability` – (default: 0.2) probability of constructing a problem whose initial dictionary is allowed to be primal infeasible or dual feasible

All other keyword arguments are passed to the constructor.

**EXAMPLES:**

```sage
InteractiveLPProblemStandardForm.random_element(3, 4)
LP problem (use 'view(...)' or '%display typeset' for details)
```

`revised_dictionary(*x_B)`

Construct a revised dictionary for `self`.

**INPUT:**

- basic variables for the dictionary to be constructed; if not given, `slack_variables()` will be used, perhaps with the `auxiliary_variable()` to give a feasible dictionary

**OUTPUT:**

- a revised dictionary

**EXAMPLES:**

```sage
A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
```
sage: D = P.revised_dictionary("x1", "x2")
sage: D.basic_variables()
(x1, x2)

If basic variables are not given the initial dictionary is constructed:

sage: P.revised_dictionary().basic_variables()
(x3, x4)
sage: P.initial_dictionary().basic_variables()
(x3, x4)

Unless it is infeasible, in which case a feasible dictionary for the auxiliary problem is constructed:

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.initial_dictionary().is_feasible()
False
sage: P.revised_dictionary().basic_variables()
(x3, x4, x0)

run_revised_simplex_method()

Apply the revised simplex method and return all steps.

OUTPUT:

- `HtmlFragment` with HTML/\LaTeX\ code of all encountered dictionaries

**Note:** You can access the `final_revised_dictionary()`, which can be one of the following:

- an optimal dictionary with the `auxiliary_variable()` among `basic_variables()` and a non-zero optimal value indicating that `self` is infeasible;
- a non-optimal dictionary that has marked entering variable for which there is no choice of the leaving variable, indicating that `self` is unbounded;
- an optimal dictionary.

**EXAMPLES:**

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.run_revised_simplex_method()
\begin{equation*}
\end{equation*}
Entering: $x_{1}$. Leaving: $x_{0}$.
\begin{equation*}
\end{equation*}
Entering: $x_{5}$. Leaving: $x_{4}$.
\begin{equation*}
\end{equation*}
... 
\end{equation*}
Entering: $x_{2}$. Leaving: $x_{3}$.
\begin{equation*}
... 
\end{equation*}
The optimal value: $6250$. An optimal solution: $\left(250,\,750\right)$.

\texttt{run\_simplex\_method()}

Apply the simplex method and return all steps and intermediate states.

OUTPUT:

• \texttt{HtmlFragment} with HTML/\LaTeX{} code of all encountered dictionaries

Note: You can access the \texttt{final\_dictionary()}, which can be one of the following:

• an optimal dictionary for the \texttt{auxiliary\_problem()} with a non-zero optimal value indicating that \texttt{self} is infeasible;

• a non-optimal dictionary for \texttt{self} that has marked entering variable for which there is no choice of the leaving variable, indicating that \texttt{self} is unbounded;

• an optimal dictionary for \texttt{self}.

\texttt{slack\_variables()}

Return slack variables of \texttt{self}.

Slack variables are differences between the constant terms and left hand sides of the constraints.

If you want to give custom names to slack variables, you have to do so during construction of the problem.
OUTPUT:

• a tuple

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: P.slack_variables()
(x3, x4)
sage: P = InteractiveLPProblemStandardForm(A, b, c, ['C', 'B'],
....:    slack_variables=['L', 'F'])
sage: P.slack_variables()
(L, F)
```

```python
class sage.numerical.interactive_simplex_method.LPAbstractDictionary
Bases: sageObject

Abstract base class for dictionaries for LP problems.

Instantiating this class directly is meaningless, see LPDictionary and LPRevisedDictionary for useful extensions.

**add_row**(nonbasic_coefficients, constant, basic_variable=None)

Return a dictionary with an additional row based on a given dictionary.

**INPUT:**

• nonbasic_coefficients– a list of the coefficients for the new row (with which nonbasic variables are subtracted in the relation for the new basic variable)

• constant– the constant term for the new row

• basic_variable– (default: depends on style()) a string giving the name of the basic variable of the new row

**OUTPUT:**

• a new dictionary of the same class

**EXAMPLES:**

```python
sage: A = ([−1, 1, 7], [8, 2, 13], [34, 17, 12])
sage: b = (2, 17, 6)
sage: c = (55/10, 21/10, 14/30)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary("x1", "x2", "x4")
sage: D1 = D.add_row([7, 11, 19], 42, basic_variable='c')
sage: D1.row_coefficients("c")
(7, 11, 19)
```
```
```
```
```
```
```
```
```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.base_ring()
Rational Field
sage: D = P.revised_dictionary()
sage: D.base_ring()
Rational Field

basic_solution(include_slack_variables=False)

Return the basic solution of self.

The basic solution associated to a dictionary is obtained by setting to zero all nonbasic_variables(), in which case basic_variables() have to be equal to constant_terms() in equations. It may refer to values of decision_variables() only or include slack_variables() as well.

INPUT:

• include_slack_variables – (default: False) if True, values of slack variables will be appended at the end

OUTPUT:

• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)

sage: D = P.initial_dictionary()
sage: D.basic_solution()
(0, 0)
sage: D.basic_solution(True)
(0, 0, 1000, 1500)
sage: D = P.revised_dictionary()
sage: D.basic_solution()
(0, 0)
sage: D.basic_solution(True)
(0, 0, 1000, 1500)

basic_variables()

Return the basic variables of self.

OUTPUT:

• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
column_coefficients(v)

Return the coefficients of a nonbasic variable.

INPUT:
• v – a nonbasic variable of self, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

OUTPUT:
• a vector of coefficients of a nonbasic variable

EXAMPLES:

```sage
sage: A = ([[1, 1], [3, 1]])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.column_coefficients(1)
(1, 3)
```

constant_terms()

Return the constant terms of relations of self.

OUTPUT:
• a vector.

EXAMPLES:

```sage
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.constant_terms()
(1000, 1500)
```

coordinate_ring()

Return the coordinate ring of self.

OUTPUT:
• a polynomial ring in auxiliary_variable(), decision_variables(), and slack_variables() of self over the base_ring()

EXAMPLES:

```sage
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
```
sage: D = P.initial_dictionary()
sage: D.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4
over Rational Field
sage: D = P.revised_dictionary()
sage: D.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4
over Rational Field

dual_ratios()

Return ratios used to determine the entering variable based on leaving.

OUTPUT:

• A list of pairs \((r_j, x_j)\) where \(x_j\) is a non-basic variable and \(r_j = c_j/a_{ij}\) is the ratio of the objective
coefficient \(c_j\) to the coefficient \(a_{ij}\) of \(x_j\) in the relation for the leaving variable \(x_i\):

\[
x_i = b_i - \cdots - a_{ij} x_j - \cdots.
\]

The order of pairs matches the order of \texttt{nonbasic_variables()}, but only \(x_j\) with negative \(a_{ij}\) are considered.

EXAMPLES:

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary(2, 3, 5)
sage: D.leave(3)
sage: D.dual_ratios()
[(5/2, x1), (5, x4)]
sage: D = P.revised_dictionary(2, 3, 5)
sage: D.leave(3)
sage: D.dual_ratios()
[(5/2, x1), (5, x4)]

enter(v)

Set \(v\) as the entering variable of \texttt{self}.

INPUT:

• \(v\) – a non-basic variable of \texttt{self}, can be given as a string, an actual variable, or an integer interpreted
as the index of a variable. It is also possible to enter \texttt{None} to reset choice.

OUTPUT:

• none, but the selected variable will be used as entering by methods that require an entering variable
and the corresponding column will be typeset in green

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
We can also use indices of variables:

```python
sage: D.enter(1)
```

Or variable names without quotes after injecting them:

```python
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.enter(x1)
```

The same works for revised dictionaries as well:

```python
sage: D = P.revised_dictionary()
sage: D.enter(x1)
```

**entering()**

Return the currently chosen entering variable.

**OUTPUT:**

- a variable if the entering one was chosen, otherwise None

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.entering() is None
True
sage: D.enter(1)
sage: D.entering()
x1
```

**entering_coefficients()**

Return coefficients of the entering variable.

**OUTPUT:**

- a vector

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.entering_coefficients()
(1, 3)
```
**is_dual_feasible()**
Check if self is dual feasible.

OUTPUT:
- True if all `objective_coefficients()` are non-positive, False otherwise

EXAMPLES:
```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_dual_feasible()
False
sage: D = P.revised_dictionary()
sage: D.is_dual_feasible()
False
```

**is_feasible()**
Check if self is feasible.

OUTPUT:
- True if all `constant_terms()` are non-negative, False otherwise

EXAMPLES:
```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_feasible()
True
sage: D = P.revised_dictionary()
sage: D.is_feasible()
True
```

**is_optimal()**
Check if self is optimal.

OUTPUT:
- True if self `is_feasible()` and `is_dual_feasible()` (i.e. all `constant_terms()` are non-negative and all `objective_coefficients()` are non-positive), False otherwise.

EXAMPLES:
```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_optimal()
False
sage: D = P.revised_dictionary()
```
(continues on next page)
sage: D.is_optimal()
False
sage: D = P.revised_dictionary(1, 2)
sage: D.is_optimal()
True

**leave(v)**

Set v as the leaving variable of self.

**INPUT:**

- v – a basic variable of self, can be given as a string, an actual variable, or an integer interpreted as the index of a variable. It is also possible to leave None to reset choice.

**OUTPUT:**

- none, but the selected variable will be used as leaving by methods that require a leaving variable and the corresponding row will be typeset in red

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.leave("x4")
```

We can also use indices of variables:

```python
sage: D.leave(4)
```

Or variable names without quotes after injecting them:

```python
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.leave(x4)
```

The same works for revised dictionaries as well:

```python
sage: D = P.revised_dictionary()
sage: D.leave(x4)
```

**leaving()**

Return the currently chosen leaving variable.

**OUTPUT:**

- a variable if the leaving one was chosen, otherwise None

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
```
leaving_coefficients()

Return coefficients of the leaving variable.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary(2, 3)
sage: D.leave(3)
sage: D.leaving_coefficients()
(-2, -1)
```

The same works for revised dictionaries as well:

```
sage: D = P.revised_dictionary(2, 3)
sage: D.leave(3)
sage: D.leaving_coefficients()
(-2, -1)
```

nonbasic_variables()

Return non-basic variables of self.

OUTPUT:

• a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```

objective_coefficients()

Return coefficients of the objective of self.

OUTPUT:

• a vector

EXAMPLES:
```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_coefficients()
(10, 5)
```

**objective_name()**

Return the objective name of `self`.

**OUTPUT:**

- a symbolic expression

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_name()
z
```

**objective_value()**

Return the value of the objective at the `basic_solution()` of `self`.

**OUTPUT:**

- a number

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_value()
0
```

**possible_dual_simplex_method_steps()**

Return possible dual simplex method steps for `self`.

**OUTPUT:**

- A list of pairs (leaving, entering), where leaving is a basic variable that may `leave()` and entering is a list of non-basic variables that may `enter()` when leaving leaves. Note that entering may be empty, indicating that the problem is infeasible (since the dual one is unbounded).

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
```

(continues on next page)
possible_entering()

Return possible entering variables for self.

OUTPUT:

• a list of non-basic variables of self that can enter() on the next step of the (dual) simplex method

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.possible_entering()
[x1, x2]
sage: D = P.revised_dictionary()
sage: D.possible_entering()
[x1, x2]
```

possible_leaving()

Return possible leaving variables for self.

OUTPUT:

• a list of basic variables of self that can leave() on the next step of the (dual) simplex method

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.possible_leaving()
[x4]
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.possible_leaving()
[x4]
```

possible_simplex_method_steps()

Return possible simplex method steps for self.

OUTPUT:

• A list of pairs (entering, leaving), where entering is a non-basic variable that may enter() and leaving is a list of basic variables that may leave() when entering enters. Note that leaving may be empty, indicating that the problem is unbounded.
EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.possible_simplex_method_steps()
[(x1, [x4]), (x2, [x3])]
sage: D = P.revised_dictionary()
sage: D.possible_simplex_method_steps()
[(x1, [x4]), (x2, [x3])]
```

**ratios()**

Return ratios used to determine the leaving variable based on entering.

**OUTPUT:**

- A list of pairs \((r_i, x_i)\) where \(x_i\) is a basic variable and \(r_i = b_i / a_{ik}\) is the ratio of the constant term \(b_i\) to the coefficient \(a_{ik}\) of the entering variable \(x_k\) in the relation for \(x_i\):

  \[
  x_i = b_i - \cdots - a_{ik} x_k - \cdots.
  \]

  The order of pairs matches the order of `basic_variables()` but only \(x_i\) with positive \(a_{ik}\) are considered.

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.ratios()
[(1000, x3), (500, x4)]
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.ratios()
[(1000, x3), (500, x4)]
```

**row_coefficients(v)**

Return the coefficients of the basic variable \(v\).

These are the coefficients with which nonbasic variables are subtracted in the relation for \(v\).

**INPUT:**

- \(v\) – a basic variable of `self`, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

**OUTPUT:**

- a vector of coefficients of a basic variable

**EXAMPLES:**
sage: A = ([−1, 1], [8, 2])
sage: b = (2, 17)
sage: c = (55/10, 21/10)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.final_dictionary()
sage: D.row_coefficients("x1")
(1/10, -1/5)

We can also use indices of variables:

sage: D.row_coefficients(1)
(1/10, -1/5)

Or use variable names without quotes after injecting them:

sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.row_coefficients(x1)
(1/10, -1/5)

run_dual_simplex_method()

Apply the dual simplex method and return all steps/intermediate states.

If either entering or leaving variables were already set, they will be used.

OUTPUT:

• HtmlFragment with HTML/LaTeX code of all encountered dictionaries

EXAMPLES:

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.run_dual_simplex_method()
Traceback (most recent call last):
...
ValueError: leaving variables can be determined for feasible
dictionaries with a set entering variable or for dual feasible
dictionaries

Let’s start with a dual feasible dictionary then:

sage: D = P.dictionary(2, 3, 5)
sage: D.is_dual_feasible()
True
sage: D.is_optimal()
False
sage: D.run_dual_simplex_method()
\begin{equation*}
...
\end{equation*}
Leaving: $x_{3}$. Entering: $x_{1}$.
This method detects infeasible problems:

```python
sage: A = ([1, 0],)
sage: b = (-1,)
sage: c = (0, -1)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.run_dual_simplex_method()
```

The problem is infeasible because of $x_{3}$ constraint.

**run_simplex_method()**

Apply the simplex method and return all steps and intermediate states.

If either entering or leaving variables were already set, they will be used.

**OUTPUT:**

- **HtmlFragment** with HTML/LaTeX code of all encountered dictionaries

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.run_simplex_method()
```

Traceback (most recent call last):

```python
... ValueError: entering variables can be determined for feasible
dictionaries or for dual feasible dictionaries with a set leaving
variable
```

Let's start with a feasible dictionary then:

```python
sage: D = P.dictionary(1, 3, 4)
sage: D.is_feasible()
True
sage: D.is_optimal()
False
sage: D.run_simplex_method()
```

Entering: $x_{5}$. Leaving: $x_{4}$.
Entering: $x_2$. Leaving: $x_3$.

This method detects unbounded problems:

\begin{itemize}
\item \texttt{sage: A = ([1, 0],)}
\item \texttt{sage: b = (1,)}
\item \texttt{sage: c = (0, 1)}
\item \texttt{sage: P = InteractiveLPProblemStandardForm(A, b, c)}
\item \texttt{sage: D = P.initial_dictionary()}
\item \texttt{sage: D.run_simplex_method()}
\end{itemize}

The problem is unbounded in $x_2$ direction.

**update()**

Update self using previously set entering and leaving variables.

**EXAMPLES:**

\begin{itemize}
\item \texttt{sage: A = ([1, 1], [3, 1])}
\item \texttt{sage: b = (1000, 1500)}
\item \texttt{sage: c = (10, 5)}
\item \texttt{sage: P = InteractiveLPProblemStandardForm(A, b, c)}
\item \texttt{sage: D = P.initial_dictionary()}
\item \texttt{sage: D.objective_value()}
\item \texttt{0}
\item \texttt{sage: D.enter("x1")}
\item \texttt{sage: D.leave("x4")}
\item \texttt{sage: D.update()}
\item \texttt{sage: D.objective_value()}
\item \texttt{5000}
\end{itemize}

**class** \texttt{sage.numerical.interactive_simplex_method.LPDictionary}(A, b, c, objective_value, basic_variables, nonbasic_variables, objective_name)

**Bases:** \texttt{LPAbstractDictionary}

Construct a dictionary for an LP problem.

A dictionary consists of the following data:

\[
\begin{align*}
    x_B &= b - Ax_N \\
    z &= z^* + cx_N
\end{align*}
\]

**INPUT:**
• $A$ – a matrix of relation coefficients
• $b$ – a vector of relation constant terms
• $c$ – a vector of objective coefficients
• `objective_value` – current value of the objective $z^*$
• `basic_variables` – a list of basic variables $x_B$
• `nonbasic_variables` – a list of non-basic variables $x_N$
• `objective_name` – a “name” for the objective $z$

**OUTPUT:**
• a dictionary for an LP problem

**Note:** This constructor does not check correctness of input, as it is intended to be used internally by `InteractiveLPProblemStandardForm`.

**EXAMPLES:**

The intended way to use this class is indirect:

```sage
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D
LP problem dictionary (use ...)
```

But if you want you can create a dictionary without starting with an LP problem, here is construction of the same dictionary as above:

```sage
sage: A = matrix(QQ, ([1, 1], [3, 1]))
sage: b = vector(QQ, (1000, 1500))
sage: c = vector(QQ, (10, 5))
sage: R = PolynomialRing(QQ, "x1, x2, x3, x4", order="neglex")
sage: from sage.numerical.interactive_simplex_method import LPDictionary
sage: D2 = LPDictionary(A, b, c, 0, R.gens()[2:], R.gens()[:2], "z")
sage: D2 == D
True
```

**add_row** *(nonbasic_coefficients, constant, basic_variable=None)*

Return a dictionary with an additional row based on a given dictionary.

**INPUT:**

• `nonbasic_coefficients` – a list of the coefficients for the new row (with which nonbasic variables are subtracted in the relation for the new basic variable)
• `constant` – the constant term for the new row
• `basic_variable` – (default: depends on `style()`) a string giving the name of the basic variable of the new row

**OUTPUT:**
• a dictionary

EXAMPLES:

```
sage: A = ([−1, 1, 7], [8, 2, 13], [34, 17, 12])
sage: b = (2, 17, 6)
sage: c = (55/10, 21/10, 14/30)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary("x1", "x2", "x4")
sage: D1 = D.add_row([7, 11, 19], 42, basic_variable='c')
sage: D1.row_coefficients("c")
(7, 11, 19)
sage: D1.constant_terms()[-1]
42
sage: D1.basic_variables()[-1]
c
```

**basic_variables()**

Return the basic variables of self.

**OUTPUT:**

• a vector

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.basic_variables()
(x3, x4)
```

**column_coefficients(v)**

Return coefficients of a nonbasic variable.

**INPUT:**

• v – a nonbasic variable of self, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

**OUTPUT:**

• a vector

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.column_coefficients(1)
(1, 3)
```

**constant_terms()**

Return the constant terms of relations of self.
OUTPUT:
  • a vector.

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.constant_terms()
(1000, 1500)
```

**nonbasic_variables()**

Return non-basic variables of *self*.

OUTPUT:
  • a vector

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```

**objective_coefficients()**

Return coefficients of the objective of *self*.

OUTPUT:
  • a vector

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_coefficients()
(10, 5)
```

**objective_name()**

Return the objective name of *self*.

OUTPUT:
  • a symbolic expression

EXAMPLES:
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_name()
z

**objective_value()**

Return the value of the objective at the `basic_solution()` of `self`.

**OUTPUT:**

• a number

**EXAMPLES:**

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_value()
0

**static random_element**(m, n, `bound=5`, `special_probability=0.2`)

Construct a random dictionary.

**INPUT:**

• m – the number of constraints/basic variables
• n – the number of decision/non-basic variables
• bound – (default: 5) a bound on dictionary entries
• special_probability – (default: 0.2) probability of constructing a potentially infeasible or potentially optimal dictionary

**OUTPUT:**

• an LP problem dictionary

**EXAMPLES:**

```python
sage: from sage.numerical.interactive_simplex_method \.....: import random_dictionary
sage: random_dictionary(3, 4) # indirect doctest
LP problem dictionary (use 'view(...)' or '%display typeset' for details)
```

**row_coefficients**(v)

Return the coefficients of the basic variable v.

These are the coefficients with which nonbasic variables are subtracted in the relation for v.

**INPUT:**

• v – a basic variable of `self`, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

**OUTPUT:**
• a vector of coefficients of a basic variable

EXAMPLES:

```
sage: A = ([−1, 1], [8, 2])
sage: b = (2, 17)
sage: c = (55/10, 21/10)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.final_dictionary()
sage: D.row_coefficients("x1")
(1/10, -1/5)
```

We can also use indices of variables:

```
sage: D.row_coefficients(1)
(1/10, -1/5)
```

Or use variable names without quotes after injecting them:

```
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.row_coefficients(x1)
(1/10, -1/5)
```

**update()**

Update self using previously set entering and leaving variables.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_value()
0
sage: D.enter("x1")
sage: D.leave("x4")
sage: D.update()
sage: D.objective_value()
5000
```

```python
class sage.numerical.interactive_simplex_method.LPRevisedDictionary(problem, basic_variables)
Bases: LPAbstractDictionary

Construct a revised dictionary for an LP problem.

INPUT:

• problem – an LP problem in standard form
• basic_variables – a list of basic variables or their indices

OUTPUT:

• a revised dictionary for an LP problem

A revised dictionary encodes the same relations as a regular dictionary, but stores only what is “necessary to efficiently compute data for the simplex method”.
```
Let the original problem be
\[ \pm \max cx, \quad Ax \leq b, \quad x \geq 0 \]

Let \( \bar{x} \) be the vector of decision_variables() \( x \) followed by the slack_variables(). Let \( \bar{c} \) be the vector of objective_coefficients() \( c \) followed by zeroes for all slack variables. Let \( \bar{A} = (A|I) \) be the matrix of constraint_coefficients() \( A \) augmented by the identity matrix as columns corresponding to the slack variables. Then the problem above can be written as
\[ \pm \max \bar{c} \bar{x}, \quad \bar{A} \bar{x} = \bar{b}, \quad \bar{x} \geq 0 \]

and any dictionary is a system of equations equivalent to \( \bar{A} \bar{x} = \bar{b} \), but resolved for basic_variables() \( x_B \) in terms of nonbasic_variables() \( x_N \) together with the expression for the objective in terms of \( x_N \). Let \( c_B() \) and \( c_N() \) be vectors “splitting \( \bar{c} \) into basic and non-basic parts”. Let \( B() \) and \( A_N() \) be the splitting of \( \bar{A} \). Then the corresponding dictionary is
\[
\begin{align*}
    x_B &= B^{-1}b - B^{-1}A_Nx_N, \\
    z &= yb + (c_N - y^T A_N)x_N
\end{align*}
\]

where \( y = c_B^T B^{-1} \). To proceed with the simplex method, it is not necessary to compute all entries of this dictionary. On the other hand, any entry is easy to compute, if you know \( B^{-1} \), so we keep track of it through the update steps.

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: from sage.numerical.interactive_simplex_method 
....: import LPRevisedDictionary
sage: D = LPRevisedDictionary(P, [1, 2])
sage: D.basic_variables()
(x1, x2)
sage: D
LP problem dictionary (use ...)
```

The same dictionary can be constructed through the problem:

```python
sage: P.revised_dictionary(1, 2) == D
True
```
When this dictionary is typeset, you will see two tables like these ones:

<table>
<thead>
<tr>
<th>𝑥_𝐵</th>
<th>𝑐_𝐵</th>
<th>𝐵^−1</th>
<th>𝑦</th>
<th>𝐵^−1𝑏</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝑥₁</td>
<td>10</td>
<td>−1/2</td>
<td>3/2</td>
<td>5/2</td>
</tr>
<tr>
<td>𝑥₂</td>
<td>5</td>
<td>3/2</td>
<td>−1/2</td>
<td>5/2</td>
</tr>
</tbody>
</table>

More details will be shown if entering and leaving variables are set, but in any case the top table shows 𝐵^−1 and a few extra columns, while the bottom one shows several rows: these are related to columns and rows of dictionary entries.

**A(𝑣)**

Return the column of constraint coefficients corresponding to 𝑣.

**INPUT:**
• 𝑣 – a variable, its name, or its index

**OUTPUT:**
• a vector

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.A(1)
(1, 3)
sage: D.A(0)
(-1, -1)
sage: D.A("x3")
(1, 0)
```

**A_N()**

Return the 𝐴_𝑁 matrix, constraint coefficients of non-basic variables.

**OUTPUT:**
• a matrix

**EXAMPLES:**

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
```

(continues on next page)
B()
Return the $B$ matrix, i.e. constraint coefficients of basic variables.

OUTPUT:
• a matrix

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.B()
[1 1]
[3 1]
```

B_inverse()
Return the inverse of the $B()$ matrix.

This inverse matrix is stored and computed during dictionary update in a more efficient way than generic inversion.

OUTPUT:
• a matrix

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.B_inverse()
[-1/2 1/2]
[ 3/2 -1/2]
```

E()
Return the eta matrix between `self` and the next dictionary.

OUTPUT:
• a matrix

If $B_{old}$ is the current matrix $B$ and $B_{new}$ is the $B$ matrix of the next dictionary (after the update step), then $B_{new} = B_{old}E$.

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
(continues on next page)```
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.leave(4)
sage: D.E()

\[
\begin{bmatrix}
1 & 1 \\
0 & 3
\end{bmatrix}
\]

\textbf{E\textsubscript{\textit{inverse}}}()

Return the inverse of the matrix \(E()\).

This inverse matrix is computed in a more efficient way than generic inversion.

\textbf{OUTPUT:}

* a matrix

\textbf{EXAMPLES:}

\[
\begin{bmatrix}
1 & -1/3 \\
0 & 1/3
\end{bmatrix}
\]

\textbf{add_row}(\textit{nonbasic\_coefficients, constant, basic\_variable=\text{None}})

Return a dictionary with an additional row based on a given dictionary.

The implementation of this method for revised dictionaries adds a new inequality constraint to the problem, in which the given \textit{basic}\textsubscript{variable} becomes the slack variable. The resulting dictionary (with \textit{basic}\textsubscript{variable} added to the basis) will have the given \textit{nonbasic}\textsubscript{coefficients} and \textit{constant} as a new row.

\textbf{INPUT:}

* \textit{nonbasic\_coefficients}– a list of the coefficients for the new row (with which nonbasic variables are subtracted in the relation for the new basic variable)
* \textit{constant}– the constant term for the new row
* \textit{basic\_variable}– (default: depends on \textit{style()}) a string giving the name of the basic variable of the new row

\textbf{OUTPUT:}

* a \textit{revised dictionary}

\textbf{EXAMPLES:}

\[
\begin{bmatrix}
-1 & 1111 & 3 & 17 \\
8 & 222 & 7 & 6 \\
3 & 7 & 17 & 5 \\
9 & 5 & 7 & 3
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 & 17 & 11 & 27 \\
5/133 & 1/10 & 1/18 & 47/3
\end{bmatrix}
\]
Note: Basic indices are indices of `basic_variables()` in the list of generators of the `coordinate_ring()` of the `problem()` of `self`, they may not coincide with the indices of variables which are parts of their names. (They will for the default indexed names.)

**OUTPUT:**

- a list.

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
```
basic_variables()

Return the basic variables of self.

OUTPUT:

• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.basic_variables()
(x3, x4)

c_B()

Return the \( c_B \) vector, objective coefficients of basic variables.

OUTPUT:

• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.c_B()
(10, 5)

c_N()

Return the \( c_N \) vector, objective coefficients of non-basic variables.

OUTPUT:

• a vector

EXAMPLES:

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.c_N()
(10, 5)
**column_coefficients(v)**

Return the coefficients of a nonbasic variable.

INPUT:

- v – a nonbasic variable of self, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

OUTPUT:

- a vector

EXAMPLES:

```sage
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.column_coefficients(1)
(1, 3)
```

**constant_terms()**

Return constant terms in the relations of self.

OUTPUT:

- a vector

EXAMPLES:

```sage
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.constant_terms()
(1000, 1500)
```

**dictionary()**

Return a regular LP dictionary matching self.

OUTPUT:

- an LP dictionary

EXAMPLES:

```sage
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.dictionary()
LP problem dictionary (use ...)
```

**nonbasic_indices()**

Return the non-basic indices of self.
**Note:** Non-basic indices are indices of `nonbasic_variables()` in the list of generators of the `coordinate_ring()` of the `problem()` of `self`, they may not coincide with the indices of variables which are parts of their names. (They will for the default indexed names.)

**OUTPUT:**

- a list

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.nonbasic_indices()
[1, 2]
```

### nonbasic_variables()

Return non-basic variables of `self`.

**OUTPUT:**

- a vector

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```

### objective_coefficients()

Return coefficients of the objective of `self`.

**OUTPUT:**

- a vector

These are coefficients of non-basic variables when basic variables are eliminated.

**EXAMPLES:**

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_coefficients()
(10, 5)
```

### objective_name()

Return the objective name of `self`.

**OUTPUT:**
• a symbolic expression

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_name()
z
```

**objective_value()**

Return the value of the objective at the basic solution of `self`.

OUTPUT:

• a number

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_value()
0
```

**problem()**

Return the original problem.

OUTPUT:

• an LP problem in standard form

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.problem() is P
True
```

**row_coefficients(v)**

Return the coefficients of the basic variable `v`.

These are the coefficients with which nonbasic variables are subtracted in the relation for `v`.

**INPUT:**

• `v` – a basic variable of `self`, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

**OUTPUT:**

• a vector of coefficients of a basic variable
EXAMPLES:

```python
sage: A = ([−1, 1], [8, 2])
sage: b = (2, 17)
sage: c = (55/10, 21/10)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.row_coefficients("x3")
(-1, 1)
```

We can also use indices of variables:

```python
sage: D.row_coefficients(3)
(-1, 1)
```

Or variable names without quotes after injecting them:

```python
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.row_coefficients(x3)
(-1, 1)
```

**update()**

Update `self` using previously set entering and leaving variables.

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_value()
0
sage: D.enter("x1")
sage: D.leave("x4")
sage: D.update()
sage: D.objective_value()
5000
```

**x_B()**

Return the basic variables of `self`.

OUTPUT:

• a vector

EXAMPLES:

```python
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.basic_variables()
(x3, x4)
```
\textbf{x.N()}

Return non-basic variables of \texttt{self}.

**OUTPUT:**
- a vector

**EXAMPLES:**

\begin{verbatim}
sage: A = ([1, 1], [3, 1]) sage: b = (1000, 1500) sage: c = (10, 5) sage: P = InteractiveLPProblemStandardForm(A, b, c) sage: D = P.revised_dictionary() sage: D.nonbasic_variables() (x1, x2)
\end{verbatim}

\textbf{y()}

Return the \textit{y} vector, the product of \texttt{c.B()} and \texttt{B.inverse()}.

**OUTPUT:**
- a vector

**EXAMPLES:**

\begin{verbatim}
sage: A = ([1, 1], [3, 1]) sage: b = (1000, 1500) sage: c = (10, 5) sage: P = InteractiveLPProblemStandardForm(A, b, c) sage: D = P.revised_dictionary() sage: D.y() (0, 0)
\end{verbatim}

\texttt{sage.numerical.interactive_simplex_method.default_variable_name(\texttt{variable})}

Return default variable name for the current \texttt{style()}.

**INPUT:**
- \texttt{variable} - a string describing requested name

**OUTPUT:**
- a string with the requested name for current style

**EXAMPLES:**

\begin{verbatim}
sage: sage.numerical.interactive_simplex_method.default_variable_name("primal slack \rightarrow") 'x' sage: sage.numerical.interactive_simplex_method.style('Vanderbei') 'Vanderbei' sage: sage.numerical.interactive_simplex_method.default_variable_name("primal slack \rightarrow") 'w' sage: sage.numerical.interactive_simplex_method.style('UAlberta') 'UAlberta'
\end{verbatim}
Construct a random dictionary.

INPUT:

- \( m \) – the number of constraints/basic variables
- \( n \) – the number of decision/non-basic variables
- bound – (default: 5) a bound on dictionary entries
- special_probability – (default: 0.2) probability of constructing a potentially infeasible or potentially optimal dictionary

OUTPUT:

- an LP problem dictionary

EXAMPLES:

```python
sage: from sage.numerical.interactive_simplex_method import random_dictionary
sage: random_dictionary(3, 4)  # indirect doctest
LP problem dictionary (use 'view(...)' or '%display typeset' for details)
```

Set or get the current style of problems and dictionaries.

INPUT:

- new_style – a string or None (default)

OUTPUT:

- a string with current style (same as new_style if it was given)

If the input is not recognized as a valid style, a ValueError exception is raised.

Currently supported styles are:

- ‘UAlberta’ (default): Follows the style used in the Math 373 course on Mathematical Programming and Optimization at the University of Alberta, Edmonton, Canada; based on Chvatal’s book.
  - Objective functions of dictionaries are printed at the bottom.
  - Variable names default to
    - \( z \) for primal objective
    - \( z \) for dual objective
    - \( w \) for auxiliary objective
    - \( x_1, x_2, \ldots, x_n \) for primal decision variables
    - \( x_{n+1}, x_{n+2}, \ldots, x_{n+m} \) for primal slack variables
    - \( y_1, y_2, \ldots, y_m \) for dual decision variables
    - \( y_{m+1}, y_{m+2}, \ldots, y_{m+n} \) for dual slack variables
- ‘Vanderbei’: Follows the style of Robert Vanderbei’s textbook, Linear Programming – Foundations and Extensions.
  - Objective functions of dictionaries are printed at the top.
Variable names default to

- \( \zeta \) for primal objective
- \( x_i \) for dual objective
- \( x_i \) for auxiliary objective
- \( x_1, x_2, \ldots, x_n \) for primal decision variables
- \( w_1, w_2, \ldots, w_m \) for primal slack variables
- \( y_1, y_2, \ldots, y_m \) for dual decision variables
- \( z_1, z_2, \ldots, z_n \) for dual slack variables

EXAMPLES:

```
sage: sage.numerical.interactive_simplex_method.style()
'UAlberta'
sage: sage.numerical.interactive_simplex_method.style('Vanderbei')
'Vanderbei'
sage: sage.numerical.interactive_simplex_method.style('Doesntexist')
Traceback (most recent call last):
...
ValueError: Style must be one of: UAlberta, Vanderbei
sage: sage.numerical.interactive_simplex_method.style('UAlberta')
'UAlberta'
```

`sage.numerical.interactive_simplex_method.variable(v)`

Interpret \( v \) as a variable of \( R \).

**INPUT:**

- \( R \) – a polynomial ring
- \( v \) – a variable of \( R \) or convertible into \( R \), a string with the name of a variable of \( R \) or an index of a variable in \( R \)

**OUTPUT:**

- a variable of \( R \)

**EXAMPLES:**

```
sage: from sage.numerical.interactive_simplex_method \ 
.....: import variable
sage: R = PolynomialRing(QQ, "x3, y5, x5, y")
sage: R.inject_variables()
Defining x3, y5, x5, y
sage: variable(R, "x3")
x3
sage: variable(R, x3)
x3
sage: variable(R, 3)
x3
sage: variable(R, 0)
Traceback (most recent call last):
...
ValueError: there is no variable with the given index
```

(continues on next page)
```
sage: variable(R, 5)
Traceback (most recent call last):
...
ValueError: the given index is ambiguous
sage: variable(R, 2 * x3)
Traceback (most recent call last):
...
ValueError: cannot interpret given data as a variable
sage: variable(R, "z")
Traceback (most recent call last):
...
ValueError: cannot interpret given data as a variable
```
GAUSS-LEGENDRE INTEGRATION FOR VECTOR-VALUED FUNCTIONS

Routine to perform Gauss-Legendre integration for vector-functions.

EXAMPLES:

We verify that \( \int_0^1 nx^{n-1} \, dx = 1 \) for \( n = 1, \ldots, 4 \):

```python
sage: from sage.numerical.gauss_legendre import integrate_vector
sage: prec = 100
sage: K = RealField(prec)
sage: N = 4
sage: V = VectorSpace(K, N)
sage: f = lambda x: V([((n+1)*x^n) for n in range(N)])
sage: I = integrate_vector(f, prec)
sage: max([c.abs() for c in I - V(N*[1])])
0.0000000000000000000
```

AUTHORS:

- Nils Bruin (2017-06-06): initial version
- Linden Disney-Hogg (2021-06-17): documentation and integrate_vector method changes

**Note:** The code here is directly based on mpmath (see http://mpmath.org), but has a highly optimized routine to compute the nodes.

```
sage.numerical.gauss_legendre.estimate_error(results, prec, epsilon)
```

Routine to estimate the error in a list of quadrature approximations.

The method used is based on Borwein, Bailey, and Girgensohn. As mentioned in mpmath: Although not very conservative, this method seems to be very robust in practice.

The routine takes a list of vector results and, under assumption that these vectors approximate a given vector approximately quadratically, gives an estimate of the maximum norm of the error in the last approximation.

**INPUT:**

- `results` – list. List of approximations to estimate the error from. Should be at least length 2.
- `prec` – integer. Binary precision at which computations are happening.
- `epsilon` – multiprecision float. Default error estimate in case of insufficient data.

**OUTPUT:**
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An estimate of the error.

EXAMPLES:

```
sage: from sage.numerical.gauss_legendre import estimate_error
sage: prec = 200
sage: K = RealField(prec)
sage: V = VectorSpace(K, 2)
sage: a = V([1, -1])
sage: b = V([1, 1/2])
sage: L = [a + 2^(-2^i)*b for i in [0..5]]
sage: estimate_error(L, prec, K(2^(-prec)))
2.328235...e-10
```

```
sage.numerical.gauss_legendre.integrate_vector(f, prec, epsilon=None)
```

Integrate a one-argument vector-valued function numerically using Gauss-Legendre.

This function uses the Gauss-Legendre quadrature scheme to approximate the integral \( \int_0^1 f(t) \, dt \).

INPUT:

- \( f \) – callable. Vector-valued integrand.
- \( \text{prec} \) – integer. Binary precision to be used.
- \( \text{epsilon} \) – multiprecision float (default: \( 2^{-(\text{prec}+3)} \)). Target error bound.

OUTPUT:

Vector approximating value of the integral.

EXAMPLES:

```
sage: from sage.numerical.gauss_legendre import integrate_vector
sage: prec = 200
sage: Kreal = RealField(prec)
sage: K = ComplexField(prec)
sage: V = VectorSpace(K, 2)
sage: epsilon = Kreal(2^(-prec + 4))
sage: f = lambda t: V((t, K(exp(2*pi*t*K.0))))
```

We can also use complex-valued integrands:

```
sage: prec = 200
sage: Kreal = RealField(prec)
sage: K = ComplexField(prec)
sage: V = VectorSpace(K, 2)
sage: epsilon = Kreal(2^(-prec + 4))
sage: f = lambda t: V((t, K(exp(2*pi*t*K.0))))
```

(continues on next page)
sage: \texttt{max(c.abs() for c in (I - J)) < epsilon} \\
\texttt{# needs sage.symbolic} \\
True

\begin{verbatim}
sage.numerical.gauss_legendre.integrate_vector_N(f, prec, N=3)
Integrate a one-argument vector-valued function numerically using Gauss-Legendre, setting the number of nodes.
This function uses the Gauss-Legendre quadrature scheme to approximate the integral \(\int_0^1 f(t) \, dt\). It is different from integrate_vector by using a specific number of nodes rather than targeting a specified error bound on the result.

INPUT:

• \texttt{f} – callable. Vector-valued integrand.
• \texttt{prec} – integer. Binary precision to be used.
• \texttt{N} – integer (default: 3). Number of nodes to use.

OUTPUT:

Vector approximating value of the integral.

EXAMPLES:

\begin{verbatim}
sage: from sage.numerical.gauss_legendre import integrate_vector_N
sage: prec = 100
sage: K = RealField(prec)
sage: V = VectorSpace(K,1)
sage: f = lambda t: V([t])
sage: integrate_vector_N(f, prec, 4)
(0.50000000000000000000000000000)
\end{verbatim}
\end{verbatim}

\textbf{Note}: The nodes and weights are calculated in the real field with \texttt{prec} bits of precision. If the vector space in which \texttt{f} takes values is over a field which is incompatible with this field (e.g. a finite field) then a \texttt{TypeError} occurs.

\begin{verbatim}
sage.numerical.gauss_legendre.nodes(prec)
Compute the integration nodes and weights for the Gauss-Legendre quadrature scheme, caching the output.
Works by calling nodes_uncached.

INPUT:

• \texttt{degree} – integer. The number of nodes. Must be 3 or even.
• \texttt{prec} – integer (minimal value 53). Binary precision with which the nodes and weights are computed.

OUTPUT:

A list of (node, weight) pairs.

EXAMPLES:

The nodes for the Gauss-Legendre scheme are roots of Legendre polynomials. The weights can be computed by a straightforward formula (note that evaluating a derivative of a Legendre polynomial isn’t particularly numerically stable, so the results from this routine are actually more accurate than what the values the closed formula produces):
\end{verbatim}
Compute the integration nodes and weights for the Gauss-Legendre quadrature scheme

We use the recurrence relations for Legendre polynomials to compute their values. This is a version of the algorithm that in [Neu2018] is called the REC algorithm.

**INPUT:**

- **degree** – integer. The number of nodes. Must be 3 or even.
- **prec** – integer (minimal value 53). Binary precision with which the nodes and weights are computed.

**OUTPUT:**

A list of (node, weight) pairs.

**EXAMPLES:**

The nodes for the Gauss-Legendre scheme are roots of Legendre polynomials. The weights can be computed by a straightforward formula (note that evaluating a derivative of a Legendre polynomial isn’t particularly numerically stable, so the results from this routine are actually more accurate than what the values the closed formula produces):

**Todo:** It may be worth testing if using the Arb algorithm for finding the nodes and weights in `arb/acb_calc/` has better performance.
11.1 Generic Backend for LP solvers

This class only lists the methods that should be defined by any interface with a LP Solver. All these methods immediately raise `NotImplementedError` exceptions when called, and are obviously meant to be replaced by the solver-specific method. This file can also be used as a template to create a new interface: one would only need to replace the occurrences of "Nonexistent_LP_solver" by the solver's name, and replace `GenericBackend` by `SolverName(GenericBackend)` so that the new solver extends this class.

AUTHORS:
- Nathann Cohen (2010-10): initial implementation
- Risan (2012-02): extension for PPL backend
- Ingolfur Edvardsson (2014-06): extension for CVXOPT backend

class sage.numerical.backends.generic_backend.GenericBackend
Bases: SageObject

add_col(indices, coeffs)
Add a column.

INPUT:
- indices (list of integers) – this list contains the indices of the constraints in which the variable's coefficient is nonzero
- coeffs (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the i-th entry of coeffs corresponds to the coefficient of the variable in the constraint represented by the i-th entry in indices.

Note: indices and coeffs are expected to be of the same length.

EXAMPLES:

```sage
# optional - nonexistent_lp_solver
from sage.numerical.backends.generic_backend import get_solver
p = get_solver(solver="Nonexistent_LP_solver")
p.ncols()
0
p.nrows()
(continues on next page)
```
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(continued from previous page)

```
0
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(list(range(5)), list(range(5)))
sage: p.nrows()
5
```

**add_linear_constraint** *(coefficients, lower_bound, upper_bound, name=None)*

Add a linear constraint.

**INPUT:**

- **coefficients** – an iterable of pairs (i, v). In each pair, i is a variable index (integer) and v is a
  value (element of *base_ring()*).
- **lower_bound** – element of *base_ring()* or None. The lower bound.
- **upper_bound** – element of *base_ring()* or None. The upper bound.
- **name** – string or None. Optional name for this row.

**EXAMPLES:**

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint( zip(range(5), range(5)), 2.0, 2.0)
sage: p.row(0)
([0, 1, 2, 3, 4], [0.0, 1.0, 2.0, 3.0, 4.0])
sage: p.row_bounds(0)
(2.0, 2.0)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1.0, 1.0, name='foo')
sage: p.row_name(1)
'foo'
```

**add_linear_constraint_vector** *(degree, coefficients, lower_bound, upper_bound, name=None)*

Add a vector-valued linear constraint.

**Note:** This is the generic implementation, which will split the vector-valued constraint into components and add these individually. Backends are encouraged to replace it with their own optimized implementation.

**INPUT:**

- **degree** – integer. The vector degree, that is, the number of new scalar constraints.
- **coefficients** – an iterable of pairs (i, v). In each pair, i is a variable index (integer) and v is a vector (real and of length degree).
- **lower_bound** – either a vector or None. The component-wise lower bound.
- **upper_bound** – either a vector or None. The component-wise upper bound.
- **name** – string or None. An optional name for all new rows.

**EXAMPLES:**

---

Chapter 11. Linear Optimization (LP) and Mixed Integer Linear Optimization (MIP) Solver backends
```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: coeffs = ([0, vector([1, 2])), [1, vector([2, 3]))
sage: upper = vector([5, 5])
\sage: lower = vector([0, 0])
\sage: p.add_variables(2)
1
sage: p.add_linear_constraint_vector(2, coeffs, lower, upper, 'foo')
```

**add_linear_constraints**(*number, lower_bound, upper_bound, names=None*)

Add 'number' linear constraints.

**INPUT:**

- *number* (integer) – the number of constraints to add.
- *lower_bound* - a lower bound, either a real value or None
- *upper_bound* - an upper bound, either a real value or None
- *names* - an optional list of names (default: None)

**EXAMPLES:**

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
\sage: p.add_variables(5)
5
\sage: p.add_linear_constraints(5, None, 2)
\sage: p.row(4)
([], [])
\sage: p.row_bounds(4)
(None, 2.0)
```

**add_variable**(*lower_bound=0, upper_bound=None, binary=False, continuous=True, integer=False, obj=None, name=None*)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

**INPUT:**

- *lower_bound* - the lower bound of the variable (default: 0)
- *upper_bound* - the upper bound of the variable (default: None)
- *binary* - True if the variable is binary (default: False).
- *continuous* - True if the variable is continuous (default: True).
- *integer* - True if the variable is integral (default: False).
- *obj* - (optional) coefficient of this variable in the objective function (default: 0.0)
- *name* - an optional name for the newly added variable (default: None)

**OUTPUT:** The index of the newly created variable

**EXAMPLES:**
add_variables \( (n, \text{lower\_bound}=False, \text{upper\_bound}=None, \text{binary}=False, \text{continuous}=True, \\text{integer}=False, \text{obj}=None, \text{names}=None) \)

Add \( n \) variables.

This amounts to adding new columns to the matrix. By default, the variables are both nonnegative and real.

**INPUT:**

- \( n \) - the number of new variables (must be \( > 0 \))
- \( \text{lower\_bound} \) - the lower bound of the variable (default: \( 0 \))
- \( \text{upper\_bound} \) - the upper bound of the variable (default: \( \text{None} \))
- \( \text{binary} \) - \( True \) if the variable is binary (default: \( False \)).
- \( \text{continuous} \) - \( True \) if the variable is binary (default: \( True \)).
- \( \text{integer} \) - \( True \) if the variable is binary (default: \( False \)).
- \( \text{obj} \) - (optional) coefficient of all variables in the objective function (default: \( 0.0 \))
- \( \text{names} \) - optional list of names (default: \( \text{None} \))

**OUTPUT:** The index of the variable created last.

**EXAMPLES:**
sage: p.ncols()
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a','b'])

base_ring()

best_known_objective_bound()

Return the value of the currently best known bound.

This method returns the current best upper (resp. lower) bound on the optimal value of the objective function in a maximization (resp. minimization) problem. It is equal to the output of \texttt{get_objective_value()} if the MILP found an optimal solution, but it can differ if it was interrupted manually or after a time limit (cf \texttt{solver_parameter()}).

\textbf{Note:} Has no meaning unless \texttt{solve} has been called before.

\textbf{EXAMPLES:}

```
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: b = p.new_variable(binary=True)
sage: for u,v in graphs.CycleGraph(5).edges(labels=False):
    ....:     p.add_constraint(b[u]+b[v]<=1)
sage: p.set_objective(p.sum(b[x] for x in range(5)))
sage: p.solve()
2.0
sage: pb = p.get_backend()
sage: pb.get_objective_value()
2.0
sage: pb.best_known_objective_bound()
2.0
```

col_bounds(index)

Return the bounds of a specific variable.

\textbf{INPUT:}

- \texttt{index} (integer) – the variable’s id.

\textbf{OUTPUT:}

A pair \texttt{(lower\_bound, upper\_bound)}. Each of them can be set to \texttt{None} if the variable is not bounded in the corresponding direction, and is a real value otherwise.

\textbf{EXAMPLES:}

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
```
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)

**col_name(index)**

Return the index-th column name

**INPUT:**

- **index** (integer) – the column id
- **name** (char *) – its name. When set to NULL (default), the method returns the current name.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable(name="I am a variable")
1
sage: p.col_name(0)
'I am a variable'
```

**copy()**

Returns a copy of self.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: b = p.new_variable()
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: copy(p).solve()
6.0
```

**get_objective_value()**

Return the value of the objective function.

**Note:** Behavior is undefined unless `solve` has been called before.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
(continues on next page)```
get_relative_objective_gap()

Return the relative objective gap of the best known solution.

For a minimization problem, this value is computed by $(\text{bestinteger} - \text{bestobjective})/(1e - 10 + |\text{bestobjective}|)$, where bestinteger is the value returned by get_objective_value() and bestobjective is the value returned by best_known_objective_bound(). For a maximization problem, the value is computed by $(\text{bestobjective} - \text{bestinteger})/(1e - 10 + |\text{bestobjective}|)$.

Note: Has no meaning unless solve has been called before.

EXAMPLES:

```sage
# optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(2)
sage: p.add_linear_constraint([[0, 1], (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
```

get_variable_value(variable)

Return the value of a variable given by the solver.

Note: Behavior is undefined unless solve has been called before.

EXAMPLES:

```sage
# optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: b = p.new_variable(binary=True)
sage: for u,v in graphs.CycleGraph(5).edges(labels=False):
    ...:     p.add_constraint(b[u]+b[v]<=1)
sage: p.set_objective(p.sum(b[x] for x in range(5)))
sage: p.solve()
2.0
```

(continues on next page)
is_maximization()

Test whether the problem is a maximization

EXAMPLES:

```sage
# optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

is_slack_variable_basic(index)

Test whether the slack variable of the given row is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

INPUT:

• index (integer) – the variable’s id

EXAMPLES:

```sage
# optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(maximization=True,
                        ....: solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_slack_variable_basic(0)
True
sage: b.is_slack_variable_basic(1)
False
```

is_slack_variable_nonbasic_at_lower_bound(index)

Test whether the given variable is nonbasic at lower bound.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

INPUT:
is_variable_basic(index)

Test whether the given variable is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

INPUT:

• index (integer) – the variable’s id

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(maximization=True,
...: solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])

sage: b = p.get_backend()
sage: b.is_variable_basic(0)  # Backend-specific commands to instruct solver to use simplex method here
True
sage: b.is_variable_basic(1)
False
```

is_variable_binary(index)

Test whether the given variable is of binary type.

INPUT:

• index (integer) – the variable’s id

EXAMPLES:
is_variable_continuous(index)
Test whether the given variable is of continuous/real type.

INPUT:
- index (integer) – the variable’s id

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True
```

is_variable_integer(index)
Test whether the given variable is of integer type.

INPUT:
- index (integer) – the variable’s id

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
```

is_variable_nonbasic_at_lower_bound(index)
Test whether the given variable is nonbasic at lower bound.
This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- **index** (integer) – the variable’s id

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(maximization=True,
....:     solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()

# Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_variable_nonbasic_at_lower_bound(1)
True
```

**ncols()**

Return the number of columns/variables.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")

sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2
```

**nrows()**

Return the number of rows/constraints.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")

sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2.0, None)
2
sage: p.nrows()
2
```

**objective_coefficient**(variable, coeff=None)

Set or get the coefficient of a variable in the objective function
INPUT:

- variable (integer) – the variable’s id
- coeff (double) – its coefficient

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0,2)
sage: p.objective_coefficient(0)
2.0
```

**objective_constant_term**(d=None)

Set or get the constant term in the objective function

INPUT:

- d (double) – its coefficient. If *None* (default), return the current value.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.objective_constant_term()
0.0
sage: p.objective_constant_term(42)
sage: p.objective_constant_term()
42.0
```

**problem_name**(name=None)

Return or define the problem’s name

INPUT:

- name (str) – the problem’s name. When set to *None* (default), the method returns the problem’s name.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
# optional - Nonexistent_LP_solver
sage: p.objective_constant_term("There once was a french fry")
# optional - Nonexistent_LP_solver
sage: print(p.objective_constant_term())
# optional - Nonexistent_LP_solver
There once was a french fry
```

**remove_constraint**(i)

Remove a constraint.

INPUT:
• i – index of the constraint to remove.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: v = p.new_variable(nonnegative=True)
sage: x, y = v[0], v[1]
sage: p.add_constraint(2*x + 3*y, max=6)
sage: p.add_constraint(3*x + 2*y, max=6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0
sage: p.remove_constraint(0)
sage: p.solve()
10.0
sage: p.get_values([x,y])
[0.0, 3.0]
```

remove_constraints(constraints)

Remove several constraints.

INPUT:

• constraints – an iterable containing the indices of the rows to remove.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
1
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
```

row(i)

Return a row

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add_linear_constraint method.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

(continues on next page)
row_bounds(index)
Return the bounds of a specific constraint.

INPUT:

- index (integer) – the constraint’s id.

OUTPUT:

A pair (lower_bound, upper_bound). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(range(5)), list(range(5)), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])  ## FIXME: Why backwards?
sage: p.row_bounds(0)
(2.0, 2.0)
```

row_name(index)
Return the index th row name

INPUT:

- index (integer) – the row’s id

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(1, 2, None, names=['Empty constraint 1'])
sage: p.row_name(0)
'Empty constraint 1'
```

set_objective(coeff, d=0.0)
Set the objective function.

INPUT:

- coeff – a list of real values, whose i-th element is the coefficient of the i-th variable in the objective function.
- d (double) – the constant term in the linear function (set to 0 by default)

EXAMPLES:
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")

sage: p.add_variables(5)

sage: p.set_objective([1, 1, 2, 1, 3])

sage: [p.objective_coefficient(x) for x in range(5)]
[1.0, 1.0, 2.0, 1.0, 3.0]

Constants in the objective function are respected:

sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver='Nonexistent_LP_solver')

sage: x,y = p[0], p[1]

sage: p.add_constraint(2*x + 3*y, max=6)

sage: p.add_constraint(3*x + 2*y, max=6)

sage: p.set_objective(x + y + 7)

sage: p.set_integer(x); p.set_integer(y)

sage: p.solve()

9.0

set_sense(sense)

Set the direction (maximization/minimization).

INPUT:

• sense (integer):
  – +1 => Maximization
  – -1 => Minimization

EXAMPLES:

sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")

sage: p.is_maximization()
True

sage: p.set_sense(-1)

sage: p.is_maximization()
False

set_variable_type(variable, vtype)

Set the type of a variable

INPUT:

• variable (integer) – the variable’s id
  
  • vtype (integer):
    – 1 Integer
    – 0 Binary
    – -1 Continuous

EXAMPLES:
```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
sage: p.is_variable_integer(0)
True
```

**set_verbosity**(level)

Set the log (verbosity) level

**INPUT:**

- level (integer) – From 0 (no verbosity) to 3.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.set_verbosity(2)
```

**solve()**

Solve the problem.

**Note:** This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(list(range(5)), list(range(5)))
sage: p.solve()
0
```

```
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
  ...MIPSolverException: ...
```

**solver_parameter**(name, value=None)

Return or define a solver parameter

**INPUT:**

- name (string) – the parameter
- value – the parameter’s value if it is to be defined, or None (default) to obtain its current value.
Note: The list of available parameters is available at `solver_parameter()`.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.solver_parameter("timelimit")
sage: p.solver_parameter("timelimit", 60)
sage: p.solver_parameter("timelimit")
```

**variable_lower_bound(index, value=False)**

Return or define the lower bound on a variable

**INPUT:**

- `index` (integer) – the variable’s id
- `value` – real value, or `None` to mean that the variable has no lower bound. When set to `False` (default), the method returns the current value.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5.0, None)
```

**variable_upper_bound(index, value=False)**

Return or define the upper bound on a variable

**INPUT:**

- `index` (integer) – the variable’s id
- `value` – real value, or `None` to mean that the variable has no upper bound. When set to `False` (default), the method returns the current value.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)
```
write\_lp(name)
Write the problem to a .lp file

INPUT:
• filename (string)

EXAMPLES:

sage:\ # optional - nonexistent\_lp\_solver
sage: from sage.numerical.backends.generic_backend import get\_solver
sage: p = get\_solver(solver="Nonexistent\_LP\_solver")

\[ \begin{align*}
p &\text{.add\_variables}(2) \\
p &\text{.add\_linear\_constraint}([([0, 1], (1, 2]], \text{None}, 3) \\
p &\text{.set\_objective}([2, 5]) \\
p &\text{from tempfile import NamedTemporaryFile} \\
p &\text{with NamedTemporaryFile(suffix=".lp") as } f: \\
\end{align*} \]

write\_mps(name, modern)
Write the problem to a .mps file

INPUT:
• filename (string)

EXAMPLES:

sage:\ # optional - nonexistent\_lp\_solver
sage: from sage.numerical.backends.generic_backend import get\_solver
sage: p = get\_solver(solver="Nonexistent\_LP\_solver")

\[ \begin{align*}
p &\text{.add\_variables}(2) \\
p &\text{.add\_linear\_constraint}([([0, 1], (1, 2]], \text{None}, 3) \\
p &\text{.set\_objective}([2, 5]) \\
p &\text{from tempfile import NamedTemporaryFile} \\
p &\text{with NamedTemporaryFile(suffix=".lp") as } f: \\
\end{align*} \]

zero()

sage.numerical.backends.generic_backend.default\_mip\_solver(solver=None)

Returns/sets the default MILP solver used by Sage

INPUT:
• solver – one of the following:
  – a string indicating one of the available solvers (see MixedIntegerLinearProgram);
  – a callable (typically a subclass of sage.numerical.backends.generic_backend.GenericBackend);
  – None (default), in which case the current default solver is returned; this is either a string or a callable.

OUTPUT:

This function returns the current default solver’s name if solver = None (default). Otherwise, it sets the default solver to the one given. If this solver does not exist, or is not available, a ValueError exception is raised.
EXAMPLES:

```
sage: former_solver = default_mip_solver()
sage: default_mip_solver("GLPK")
'Glpk'
sage: default_mip_solver("PPL")
'Ppl'
sage: default_mip_solver("GUROBI") # random
Traceback (most recent call last):
...
ValueError: Gurobi is not available. Please refer to the documentation to install it.
sage: default_mip_solver("Yeahhhhhhhhhhh")
Traceback (most recent call last):
...
ValueError: 'solver' should be set to ...
sage: default_mip_solver(former_solver)
```

```
sage.numerical.backends.generic_backend.get_solver(constraint_generation=False, solver=None, base_ring=None)
```

Return a solver according to the given preferences

**INPUT:**

- **solver** – one of the following:
  - a string indicating one of the available solvers (see `MixedIntegerLinearProgram`);
  - `None` (default), in which case the default solver is used (see `default_mip_solver()`);
  - or a callable (such as a class), in which case it is called, and its result is returned.

- **base_ring** – If not `None`, request a solver that works over this (ordered) field. If `base_ring` is not a field, its fraction field is used.

  For example, if `base_ring=ZZ` is provided, the solver will work over the rational numbers. This is unrelated to whether variables are constrained to be integers or not.

- **constraint_generation** – Only used when `solver=None`.

  - When set to `True`, after solving the `MixedIntegerLinearProgram`, it is possible to add a constraint, and then solve it again. The effect is that solvers that do not support this feature will not be used. (Coin and SCIP are such solvers.)
  
  - Defaults to `False`.

**See also:**

- `default_mip_solver()` – Returns/Sets the default MIP solver.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver()
```

```
Real Double Field
```

(continues on next page)
Passing a callable as the solver:

```python
sage: from sage.numerical.backends.glpk_backend import GLPKBackend
sage: p = get_solver(solver=GLPKBackend); p
<...sage.numerical.backends.glpk_backend.GLPKBackend...>
```

Passing a callable that customizes a backend:

```python
sage: def glpk_exact_solver():
    ...:     from sage.numerical.backends.generic_backend import get_solver
    ...:     b = get_solver(solver="GLPK")
    ...:     b.solver_parameter("simplex_or_intopt", "exact_simplex_only")
    ...:     return b
sage: codes.bounds.delsarte_bound_additive_hamming_space(11,3,4,solver=glpk_exact_solver)  # long time
```

## 11.2 InteractiveLP Backend

**AUTHORS:**

- Nathann Cohen (2010-10): generic_backend template
- Matthias Koeppe (2016-03): this backend

```python
class sage.numerical.backends.interactivelp_backend.InteractiveLPBackend
    Bases: GenericBackend
```

Chapter 11. Linear Optimization (LP) and Mixed Integer Linear Optimization (MIP) Solver backends
MIP Backend that works with `InteractiveLPProblem`.

This backend should be used only for linear programs over general fields, or for educational purposes. For fast computations with floating point arithmetic, use one of the numerical backends. For exact computations with rational numbers, use backend ‘PPL’.

There is no support for integer variables.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g Sage: p = get_solver(solver = "InteractiveLP")
```

`add_col(indices, coeffs)`
Add a column.

**INPUT:**
- `indices` (list of integers) – this list contains the indices of the constraints in which the variable’s coefficient is nonzero
- `coeffs` (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the i-th entry of `coeffs` corresponds to the coefficient of the variable in the constraint represented by the i-th entry in `indices`.

**Note:** `indices` and `coeffs` are expected to be of the same length.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g Sage: p = get_solver(solver = "InteractiveLP")
g Sage: p.ncols()
g 0
g Sage: p.nrows()
g 0
g Sage: p.add_linear_constraints(5, 0, None)
g Sage: p.add_col(list(range(5)), list(range(5)))
g Sage: p.nrows()
g 5
```

`add_linear_constraint(coefficients, lower_bound, upper_bound, name=None)`
Add a linear constraint.

**INPUT:**
- `coefficients` – an iterable of pairs (i, v). In each pair, i is a variable index (integer) and v is a value (element of `base_ring()`).
- `lower_bound` – element of `base_ring()` or `None`. The lower bound.
- `upper_bound` – element of `base_ring()` or `None`. The upper bound.
- `name` – string or `None`. Optional name for this row.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g Sage: p = get_solver(solver = "InteractiveLP")
```
sage: p.add_variables(5)
4
sage: p.add_linear_constraint( zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1, 1, name='foo')
sage: p.row_name(1)
'foo'

add_variable(lower_bound=0, upper_bound=None, binary=False, continuous=True, integer=False, obj=None, name=None, coefficients=None)

Add a variable.
This amounts to adding a new column to the matrix. By default, the variable is both nonnegative and real.
In this backend, variables are always continuous (real). If integer variables are requested via the parameters binary and integer, an error will be raised.

INPUT:
• lower_bound - the lower bound of the variable (default: 0)
• upper_bound - the upper bound of the variable (default: None)
• binary - True if the variable is binary (default: False).
• continuous - True if the variable is continuous (default: True).
• integer - True if the variable is integral (default: False).
• obj - (optional) coefficient of this variable in the objective function (default: 0)
• name - an optional name for the newly added variable (default: None).
• coefficients – (optional) an iterable of pairs (i, v). In each pair, i is a variable index (integer) and v is a value (element of base_ring()).

OUTPUT: The index of the newly created variable

EXAMPLES:
'x'
sage: p.objective_coefficient(1)
1

**base_ring()**
Return the base ring.

**OUTPUT:**
A ring. The coefficients that the chosen solver supports.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.base_ring()
Rational Field
```

**col_bounds(index)**
Return the bounds of a specific variable.

**INPUT:**
- **index** (integer) – the variable's id.

**OUTPUT:**
A pair \((\text{lower\_bound}, \text{upper\_bound})\). Each of them can be set to \text{None} if the variable is not bounded in the corresponding direction, and is a real value otherwise.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variable(lower_bound=None)
0
sage: p.col_bounds(0)
(None, None)
sage: p.variable_lower_bound(0, 0)
sage: p.col_bounds(0)
(0, None)
```

**col_name(index)**
Return the \(\text{index}\)-th column name

**INPUT:**
- **index** (integer) – the column id
- **name** (char *) – its name. When set to NULL (default), the method returns the current name.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
g from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variable(name="I_am_a_variable")
0
```
dictionary()
Return a dictionary representing the current basis.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver="InteractiveLP")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(11/2 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: d = b.dictionary(); d
LP problem dictionary ...
set(d.basic_variables())
{x1, x3}
sage: d.basic_solution()
(17/8, 0)
```

get_objective_value()
Return the value of the objective function.

Note: Behavior is undefined unless solve has been called before.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2
```

get_variable_value(variable)
Return the value of a variable given by the solver.

Note: Behavior is undefined unless solve has been called before.
EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2
```

`interactive_lp_problem()`

Return the `InteractiveLPProblem` object associated with this backend.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(maximization=True,
sage:       solver="InteractiveLP")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(11/2 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: b.interactive_lp_problem()
LP problem ...
```

`is_maximization()`

Test whether the problem is a maximization

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

`is_slack_variable_basic(index)`

Test whether the slack variable of the given row is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

INPUT:

- `index` (integer) – the variable's id

EXAMPLES:
**is_slack_variable_nonbasic_at_lower_bound(index)**

Test whether the given variable is nonbasic at lower bound.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- index (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver="InteractiveLP")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(11/2 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_slack_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_slack_variable_nonbasic_at_lower_bound(1)
True
```

**is_variable_basic(index)**

Test whether the given variable is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- index (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True, solver="InteractiveLP")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(11/2 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_slack_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_slack_variable_nonbasic_at_lower_bound(1)
True
```

(continues on next page)
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(11/2 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_variable_basic(0)
True
sage: b.is_variable_basic(1)
False

### is_variable_binary(index)

Test whether the given variable is of binary type.

**INPUT:**

- index (integer) – the variable's id

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_binary(0)
False
```

### is_variable_continuous(index)

Test whether the given variable is of continuous/real type.

**INPUT:**

- index (integer) – the variable's id

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True
```

### is_variable_integer(index)

Test whether the given variable is of integer type.

**INPUT:**

- index (integer) – the variable's id

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_integer(0)
True
```
\begin{frame}
\begin{verbatim}
from sage.numerical.backends.generic_backend import get_solver
p = get_solver(solver = "InteractiveLP")
0
p.add_variable()
0
p.is_variable_integer(0)
False
\end{verbatim}

\textbf{is\_variable\_nonbasic\_at\_lower\_bound(index)}

Test whether the given variable is nonbasic at lower bound.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

\textbf{INPUT:}

- \texttt{index} (integer) – the variable’s id

\textbf{EXAMPLES:}

\begin{verbatim}
p = MixedIntegerLinearProgram(maximization=True,
solver="InteractiveLP")
x = p.new_variable(nonnegative=True)
p.add_constraint(-x[0] + x[1] <= 2)
p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
p.set_objective(11/2 * x[0] - 3 * x[1])
b = p.get_backend()
# Backend-specific commands to instruct solver to use simplex method here
b.solve()
0
b.is_variable_nonbasic_at_lower_bound(0)
False
b.is_variable_nonbasic_at_lower_bound(1)
True
\end{verbatim}

\textbf{n\texttt{c}ols()}

Return the number of columns/variables.

\textbf{EXAMPLES:}

\begin{verbatim}
p = MixedIntegerLinearProgram(maximization=True,
solver="InteractiveLP")
0
p.add_variables(2)
1
p.ncols()
2
\end{verbatim}

\textbf{n\texttt{r}ows()}

Return the number of rows/constraints.

\textbf{EXAMPLES:}
\end{frame}
```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
```

```python
sage: p.nrows()
0
```

```python
sage: p.add_linear_constraints(2, 0, None)
```

```python
sage: p.nrows()
2
```

**objective_coefficient**(variable, coeff=None)

Set or get the coefficient of a variable in the objective function

INPUT:

- variable (integer) – the variable’s id
- coeff (double) – its coefficient

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
```

```python
sage: p.objective_coefficient(0)
0
```

```python
sage: p.objective_coefficient(0,2)
```

```python
sage: p.objective_coefficient(0)
2
```

**objective_constant_term**(d=None)

Set or get the constant term in the objective function

INPUT:

- d (double) – its coefficient. If None (default), return the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
```

```python
sage: p.objective_constant_term()
0
```

```python
sage: p.objective_constant_term(42)
```

```python
sage: p.objective_constant_term()
42
```

**problem_name**(name=None)

Return or define the problem’s name

INPUT:

- name (str) – the problem’s name. When set to None (default), the method returns the problem’s name.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
```

```python
sage: p = get_solver(solver = "InteractiveLP")
```

(continues on next page)
remove_constraint(i)

Remove a constraint.

INPUT:

• i – index of the constraint to remove.

EXAMPLES:

```python
sage: p = MixedIntegerLinearProgram(solver="InteractiveLP")
sage: v = p.new_variable(nonnegative=True)
sage: x, y = v[0], v[1]
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: p.solve()
47/5
sage: p.remove_constraint(0)
sage: p.solve()
10
sage: p.get_values([x,y])
[0, 3]
```

row(i)

Return a row

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add_linear_constraint method.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 0, None)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
```

row_bounds(index)

Return the bounds of a specific constraint.

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:
A pair `(lower_bound, upper_bound)`. Each of them can be set to `None` if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row_bounds(0)
(2, 2)
```

**row_name(index)**
Return the `index`th row name

**INPUT:**
- `index` (integer) – the row’s id

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
tage: p = get_solver(solver = "InteractiveLP")
sage: p.add_linear_constraints(1, 2, None, names=['Empty constraint 1'])
sage: p.row_name(0)
'Empty constraint 1'
```

**set_objective(coeff, d=0)**
Set the objective function.

**INPUT:**
- `coeff` – a list of real values, whose `i`-th element is the coefficient of the `i`-th variable in the objective function.
- `d` (real) – the constant term in the linear function (set to `0` by default)

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])
sage: [p.objective_coefficient(x) for x in range(5)]
[1, 1, 2, 1, 3]
```

Constants in the objective function are respected:

```
sage: p = MixedIntegerLinearProgram(solver='InteractiveLP')
sage: x,y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: p.solve()
47/5
```
**set_sense**(*sense*)

Set the direction (maximization/minimization).

**INPUT:**

- *sense* (integer):
  - +1 => Maximization
  - -1 => Minimization

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**set_variable_type**(*variable, vtype*)

Set the type of a variable.

In this backend, variables are always continuous (real). If integer or binary variables are requested via the parameter *vtype*, an error will be raised.

**INPUT:**

- *variable* (integer) – the variable’s id
- *vtype* (integer):
  - 1 Integer
  - 0 Binary
  - -1 Continuous

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,-1)
sage: p.is_variable_continuous(0)
True
```

**set_verbosity**(*level*)

Set the log (verbosity) level

**INPUT:**

- *level* (integer) – From 0 (no verbosity) to 3.

**EXAMPLES:**
```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
```

```python
def solve()
    Solve the problem.
```

**Note:** This method raises `MIPSolverException` exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(list(range(5)), list(range(5)))
sage: p.solve()
0
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
  ... MIPSolverException: ...
```

```python
variable_lower_bound(index, value=False)
```

Return or define the lower bound on a variable

**INPUT:**

- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has no lower bound. When set to False (default), the method returns the current value.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
sage: p.add_variable(lower_bound=None)
0
sage: p.col_bounds(0)
(None, None)
sage: p.variable_lower_bound(0) is None
True
sage: p.variable_lower_bound(0, 0)
0
sage: p.col_bounds(0)
(0, None)
sage: p.variable_lower_bound(0)
0
sage: p.variable_lower_bound(0, None)
sage: p.variable_lower_bound(0) is None
True
```
variable_upper_bound(index, value=False)

Return or define the upper bound on a variable

INPUT:

- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "InteractiveLP")
```

```python
sage: p.add_variable(lower_bound=None)
0
```

```python
sage: p.col_bounds(0)
(None, None)
```

```python
sage: p.variable_upper_bound(0) is None
True
```

```python
sage: p.variable_upper_bound(0, 0)
0
```

```python
sage: p.variable_upper_bound(0, None)
```

```python
sage: p.variable_upper_bound(0) is None
True
```

# 11.3 GLPK Backend

AUTHORS:

- Nathann Cohen (2010-10): initial implementation
- John Perry (2012-01): glp_simplex preprocessing
- John Perry and Raniere Gaia Silva (2012-03): solver parameters
- Christian Kuper (2012-10): Additions for sensitivity analysis

class sage.numerical.backends.glpk_backend.GLPKBackend

Bases: GenericBackend

MIP Backend that uses the GLPK solver.

add_col(indices, coeffs)

Add a column.

INPUT:

- indices (list of integers) – this list contains the indices of the constraints in which the variable’s coefficient is nonzero
- coeffs (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the ith entry of coeffs corresponds to the coefficient of the variable in the constraint represented by the ith entry in indices.
Note: indices and coeffs are expected to be of the same length.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
generate: p = get_solver(solver = "GLPK")

sage: p.ncols()
0

sage: p.nrows()
0

sage: p.add_linear_constraints(5, 0, None)

sage: p.add_col(range(5), range(5))

sage: p.nrows()
5
```

add_linear_constraint(coefficients, lower_bound, upper_bound, name=None)

Add a linear constraint.

**INPUT:**

- coefficients an iterable with (c, v) pairs where c is a variable index (integer) and v is a value (real value).
- lower_bound - a lower bound, either a real value or None
- upper_bound - an upper bound, either a real value or None
- name - an optional name for this row (default: None)

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
generate: p = get_solver(solver = "GLPK")

sage: p.add_variables(5)
4

sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0)

sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])

sage: p.row_bounds(0)
(2.0, 2.0)

sage: p.add_linear_constraint(zip(range(5), range(5)), 1.0, 1.0, name='foo')

sage: p.row_name(1)
'foo'
```

add_linear_constraints(number, lower_bound, upper_bound, names=None)

Add `number` linear constraints.

**INPUT:**

- number (integer) – the number of constraints to add.
- lower_bound - a lower bound, either a real value or None
- upper_bound - an upper bound, either a real value or None
- names - an optional list of names (default: None)

EXAMPLES:
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
4
sage: p.add_variables(5)

sage: p.add_linear_constraints(5, None, 2)

sage: p.row(4)
([], [])

sage: p.row_bounds(4)
(None, 2.0)

sage: p.add_linear_constraints(2, None, 2, names=['foo','bar'])

add_variable(lower_bound=0.0, upper_bound=None, binary=False, continuous=False, integer=False, obj=0.0, name=None)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive, real and the
coefficient in the objective function is 0.0.

INPUT:

• lower_bound - the lower bound of the variable (default: 0)
• upper_bound - the upper bound of the variable (default: None)
• binary - True if the variable is binary (default: False).
• continuous - True if the variable is continuous (default: True).
• integer - True if the variable is integral (default: False).
• obj - (optional) coefficient of this variable in the objective function (default: 0.0)
• name - an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
4
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable(binary=True)
1
sage: p.add_variable(lower_bound=-2.0, integer=True)
2
sage: p.add_variable(continuous=True, integer=True)
Traceback (most recent call last):
... ValueError: ...

sage: p.add_variable(name='x', obj=1.0)
3
sage: p.col_name(3)
'x'

(continues on next page)
sage: p.objective_coefficient(3)
1.0

**add_variables**

```
(add_variables(number, lower_bound=0.0, upper_bound=None, binary=False, continuous=False, integer=False, obj=0.0, names=None)
```

Add `number` new variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive, real and their coefficient in the objective function is 0.0.

**INPUT:**

- `n` - the number of new variables (must be > 0)
- `lower_bound` - the lower bound of the variable (default: 0)
- `upper_bound` - the upper bound of the variable (default: None)
- `binary` - True if the variable is binary (default: False).
- `continuous` - True if the variable is binary (default: True).
- `integer` - True if the variable is binary (default: False).
- `obj` - (optional) coefficient of all variables in the objective function (default: 0.0)
- `names` - optional list of names (default: None)

**OUTPUT:** The index of the variable created last.

**EXAMPLES:**

```sage
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variables(5)
4
sage: p.ncols()
5
sage: p.add_variables(2, lower_bound=-2.0, integer=True, obj=42.0, names=['a','b'])
6
```

**best_known_objective_bound**

```
(best_known_objective_bound())
```

Return the value of the currently best known bound.

This method returns the current best upper (resp. lower) bound on the optimal value of the objective function in a maximization (resp. minimization) problem. It is equal to the output of `get_objective_value()` if the MILP found an optimal solution, but it can differ if it was interrupted manually or after a time limit (cf `solver_parameter()`).

**Note:** Has no meaning unless `solve` has been called before.

**EXAMPLES:**
```
sage: # needs sage.graphs
g = graphs.CubeGraph(9)
p = MixedIntegerLinearProgram(solver="GLPK")
p.solver_parameter("mip_gap_tolerance",100)
b = p.new_variable(binary=True)
p.set_objective(p.sum(b[v] for v in g))
for v in g:
    p.add_constraint(b[v]+p.sum(b[u] for u in g.neighbors(v)) <= 1)
p.add_constraint(b[v] == 1)  # Force an easy non-0 solution
p.solve()  # rel tol 100
1.0
backend = p.get_backend()
backend.best_known_objective_bound()  # random
48.0
```

**col_bounds(index)**

Return the bounds of a specific variable.

**INPUT:**

- `index` (integer) – the variable’s id.

**OUTPUT:**

A pair `(lower_bound, upper_bound)`. Each of them can be set to `None` if the variable is not bounded in the corresponding direction, and is a real value otherwise.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
p = get_solver(solver = "GLPK")
p.add_variable()
0
p.col_bounds(0)
(0.0, None)
p.variable_upper_bound(0, 5)
p.col_bounds(0)
(0.0, 5.0)
```

**col_name(index)**

Return the index th col name

**INPUT:**

- `index` (integer) – the col’s id

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
p = get_solver(solver = "GLPK")
p.add_variable(name='I am a variable')
0
p.col_name(0)
'I am a variable'
```

**eval_tab_col(k)**

Computes a column of the current simplex tableau.
A (column) corresponds to some non-basic variable specified by the parameter \( k \) as follows:

- if \( 0 \leq k \leq m - 1 \), the non-basic variable is \( k \)-th auxiliary variable,
- if \( m \leq k \leq m + n - 1 \), the non-basic variable is \((k - m)\)-th structural variable,

where \( m \) is the number of rows and \( n \) is the number of columns in the specified problem object.

**Note:** The basis factorization must exist and the variable with index \( k \) must not be basic. Otherwise, a `ValueError` is be raised.

**INPUT:**

- \( k \) (integer) – the id of the non-basic variable.

**OUTPUT:**

A pair \((\text{indices}, \text{coeffs})\) where \text{indices} lists the entries whose coefficient is nonzero, and to which \text{coeffs} associates their coefficient in the computed column of the current simplex tableau.

**Note:** Elements in \text{indices} have the same sense as index \( k \). All these variables are basic by definition.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
sage: lp.add_variables(3)
2
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
sage: lp.set_objective([60, 30, 20])

sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)

sage: lp.eval_tab_col(1)
Traceback (most recent call last):
  ... ValueError: basis factorization does not exist
sage: lp.solve()
0
sage: lp.eval_tab_col(1)
([0, 5, 3], [-2.0, 2.0, -0.5])
sage: lp.eval_tab_col(2)
([0, 5, 3], [8.0, -4.0, 1.5])
sage: lp.eval_tab_col(4)
([0, 5, 3], [-2.0, 2.0, -1.25])
sage: lp.eval_tab_col(0)
Traceback (most recent call last):
  ... ValueError: slack variable 0 is basic
sage: lp.eval_tab_col(-1)
Traceback (most recent call last):
  ... ValueError: ...
```

11.3. GLPK Backend
eval_tab_row\(k\)

Computes a row of the current simplex tableau.

A row corresponds to some basic variable specified by the parameter \(k\) as follows:

- if \(0 \leq k \leq m - 1\), the basic variable is \(k\)-th auxiliary variable,
- if \(m \leq k \leq m + n - 1\), the basic variable is \((k - m)\)-th structural variable,

where \(m\) is the number of rows and \(n\) is the number of columns in the specified problem object.

**Note:** The basis factorization must exist and the variable with index \(k\) must be basic. Otherwise, a `ValueError` is be raised.

**INPUT:**

- \(k\) (integer) – the id of the basic variable.

**OUTPUT:**

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient in the computed row of the current simplex tableau.

**Note:** Elements in indices have the same sense as index \(k\). All these variables are non-basic by definition.

**EXAMPLES:**

```python
def from sage.numerical.backends.generic_backend import get_solver
def lp = get_solver(solver = "GLPK")
lp.add_variables(3)
lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
lp.set_objective([60, 30, 20])
lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
lp.eval_tab_row(0)
Traceback (most recent call last):
... ValueError: basis factorization does not exist
lp.solve()
0
lp.eval_tab_row(0)
[[1, 2, 4], [-2.0, 8.0, -2.0]]
lp.eval_tab_row(3)
[[1, 2, 4], [-0.5, 1.5, -1.25]]
lp.eval_tab_row(5)
[[1, 2, 4], [2.0, -4.0, 2.0]]
lp.eval_tab_row(1)
Traceback (most recent call last):
... ValueError: slack variable 1 is not basic
lp.eval_tab_row(-1)
```

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Traceback (most recent call last):
...
ValueError: ...

get_col_dual \(\text{variable}\)

Returns the dual value (reduced cost) of a variable.

The dual value is the reduced cost of a variable. The reduced cost is the amount by which the objective coefficient of a non basic variable has to change to become a basic variable.

INPUT:

- variable – The number of the variable

**Note:** Behaviour is undefined unless solve has been called before. If the simplex algorithm has not been used for solving just a 0.0 will be returned.

### EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
```

```python
sage: p.add_variables(3)
```

```python
2
```

```python
sage: p.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
```

```python
sage: p.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
```

```python
sage: p.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
```

```python
sage: p.set_objective([60, 30, 20])
```

```python
sage: import sage.numerical.backends.glpk_backend as backend
```

```python
sage: p.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
```

```python
sage: p.solve()
0
```

```python
sage: p.get_col_dual(1)
-5.0
```

get_col_stat \(j\)

Retrieve the status of a variable.

**INPUT:**

- \(j\) – The index of the variable

**OUTPUT:**

- Returns current status assigned to the structural variable associated with \(j\)-th column:
  - GLP_BS = 1 basic variable
  - GLP_NL = 2 non-basic variable on lower bound
  - GLP_NU = 3 non-basic variable on upper bound
  - GLP_NF = 4 non-basic free (unbounded) variable
  - GLP_NS = 5 non-basic fixed variable

**EXAMPLES:**
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")

sage: lp.add_variables(3)

sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)

sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)

sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)

sage: lp.set_objective([60, 30, 20])

sage: import sage.numerical.backends.glpk_backend as backend

sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)

sage: lp.solve()

0

sage: lp.get_col_stat(0)
1

sage: lp.get_col_stat(1)
2

sage: lp.get_col_stat(100)

Traceback (most recent call last):
...
ValueError: The variable's index j must satisfy 0 <= j < number_of_variables

get_objective_value()

Returns the value of the objective function.

Note: Behaviour is undefined unless solve has been called before.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver = "GLPK")

sage: p.add_variables(2)

1

sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)

sage: p.set_objective([2, 5])

sage: p.solve()

0

sage: p.get_objective_value()

7.5

sage: p.get_variable_value(0) # abs tol 1e-15
0.0

sage: p.get_variable_value(1)

1.5

get_relative_objective_gap()

Return the relative objective gap of the best known solution.

For a minimization problem, this value is computed by \(\frac{\text{bestinteger} - \text{bestobjective}}{1e-10 + |\text{bestobjective}|}\), where bestinteger is the value returned by \(\text{get_objective_value()}\) and bestobjective is the value returned by \(\text{best_known_objective_bound()}\). For a maximization problem, the value is computed by \(\frac{\text{bestobjective} - \text{bestinteger}}{1e-10 + |\text{bestobjective}|}\).
Numerical Optimization, Release 10.2

**Note:** Has no meaning unless `solve` has been called before.

**EXAMPLES:**

```
sage: # needs sage.graphs
sage: g = graphs.CubeGraph(9)
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("mip_gap_tolerance",100)
sage: b = p.new_variable(binary=True)
sage: p.set_objective(p.sum(b[v] for v in g))
sage: for v in g:
    ....:     p.add_constraint(b[v]+p.sum(b[u] for u in g.neighbors(v)) <= 1)
sage: p.add_constraint(b[v] == 1)  # Force an easy non-0 solution
sage: p.solve()  # rel tol 100
1.0
sage: backend = p.get_backend()
sage: backend.get_relative_objective_gap()  # random
46.99999999999999
```

**get_row_dual**(variable)

Returns the dual value of a constraint.

The dual value of the ith row is also the value of the ith variable of the dual problem.

The dual value of a constraint is the shadow price of the constraint. The shadow price is the amount by which the objective value will change if the constraints bounds change by one unit under the precondition that the basis remains the same.

**INPUT:**

- **variable** – The number of the constraint

**Note:** Behaviour is undefined unless `solve` has been called before. If the simplex algorithm has not been used for solving 0.0 will be returned.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
sage: lp.add_variables(3)
2
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
sage: lp.set_objective([60, 30, 20])
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: lp.solve()
0
sage: lp.get_row_dual(0)  # tolerance 0.00001
0.0
```

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```python
sage: lp.get_row_dual(1)  # tolerance 0.00001
10.0
```

**get_row_prim(i)**

Returns the value of the auxiliary variable associated with i-th row.

**Note:** Behaviour is undefined unless `solve` has been called before.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
```
```
sage: lp.add_variables(3)
```
```
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
```
```
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
```
```
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
```
```
sage: lp.set_objective([60, 30, 20])
```
```
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
```
```
sage: lp.solve()
0
```
```
sage: lp.get_objective_value()
280.0
```
```
sage: lp.get_row_prim(0)
24.0
```
```
sage: lp.get_row_prim(1)
20.0
```
```
sage: lp.get_row_prim(2)
8.0
```

**get_row_stat(i)**

Retrieve the status of a constraint.

**INPUT:**

- i – The index of the constraint

**OUTPUT:**

- Returns current status assigned to the auxiliary variable associated with i-th row:
  - `GLP_BS = 1` basic variable
  - `GLP_NL = 2` non-basic variable on lower bound
  - `GLP_NU = 3` non-basic variable on upper bound
  - `GLP_NF = 4` non-basic free (unbounded) variable
  - `GLP_NS = 5` non-basic fixed variable

**EXAMPLES:**
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
2
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)

sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)

sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)

sage: lp.set_objective([60, 30, 20])

sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)

sage: lp.solve()
0

sage: lp.get_row_stat(0)
1

sage: lp.get_row_stat(1)
3

sage: lp.get_row_stat(-1)
Traceback (most recent call last):
... 
ValueError: The constraint's index i must satisfy 0 <= i < number_of_constraints

get_variable_value(variable)

Returns the value of a variable given by the solver.

Note: Behaviour is undefined unless solve has been called before.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")

1

sage: p.add_variables(2)

3

sage: p.set_objective([2, 5])

0

sage: p.solve()

7.5

sage: p.get_variable_value(0) # abs tol 1e-15

0.0

sage: p.get_variable_value(1)

1.5
True

```python
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**is_slack_variable_basic(index)**

Test whether the slack variable of the given row is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True,
   solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: import sage.numerical.backends.glpk_backend as backend
sage: b.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: b.solve()
0
sage: b.is_slack_variable_basic(0)
True
sage: b.is_slack_variable_basic(1)
False
```

**is_slack_variable_nonbasic_at_lower_bound(index)**

Test whether the slack variable of the given row is nonbasic at lower bound.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True,
   solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: import sage.numerical.backends.glpk_backend as backend
sage: b.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
```
**is_variable_basic(index)**

Test whether the given variable is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True,  
       solver="GLPK")
sage: x = p.new_variable(nonnegative=True)  
sage: p.add_constraint(-x[0] + x[1] <= 2)  
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)  
sage: p.set_objective(5.5 * x[0] - 3 * x[1])  
sage: b = p.get_backend()  
sage: import sage.numerical.backends.glpk_backend as backend  
sage: b.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)  
sage: b.solve()  
0  
sage: b.is_variable_basic(0)  
True  
sage: b.is_variable_basic(1)  
False
```

**is_variable_binary(index)**

Test whether the given variable is of binary type.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver  
sage: p = get_solver(solver = "GLPK")  
sage: p.ncols()  
0  
sage: p.add_variable()  
0  
sage: p.set_variable_type(0,0)  
sage: p.is_variable_binary(0)  
True
```
**is_variable_continuous**(\textit{index})

Test whether the given variable is of continuous/real type.

INPUT:

• \textit{index} (integer) – the variable’s id

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
game  
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True
sage: p.set_variable_type(0,1)
sage: p.is_variable_continuous(0)
False
```}

**is_variable_integer**(\textit{index})

Test whether the given variable is of integer type.

INPUT:

• \textit{index} (integer) – the variable’s id

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
sage: p.is_variable_integer(0)
True
```}

**is_variable_nonbasic_at_lower_bound**(\textit{index})

Test whether the given variable is nonbasic at lower bound. This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

INPUT:

• \textit{index} (integer) – the variable’s id

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(maximization=True,
solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: import sage.numerical.backends.glpk_backend as backend
```
sage: b.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: b.solve()
0
sage: b.is_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_variable_nonbasic_at_lower_bound(1)
True

ncols()

Return the number of columns/variables.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2

nrows()

Return the number of rows/constraints.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2, None)
sage: p.nrows()
2

objective_coefficient(variable, coeff=None)

Set or get the coefficient of a variable in the objective function

INPUT:

• variable (integer) – the variable’s id
• coeff (double) – its coefficient or None for reading (default: None)

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0,2)
print_ranges(filename=None)

Print results of a sensitivity analysis

If no filename is given as an input the results of the sensitivity analysis are displayed on the screen. If a
filename is given they are written to a file.

INPUT:

• filename – (optional) name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero.

Note: This method is only effective if an optimal solution has been found for the lp using the simplex
algorithm. In all other cases an error message is printed.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint(list(zip([0, 1], [1, 2])), None, 3)
1
sage: p.set_objective([2, 5])
1
sage: import sage.numerical.backends.glpk_backend as backend
sage: p.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
1
sage: p.print_ranges()
1
sage: p.solve()
0
sage: from tempfile import NamedTemporaryFile
sage: with NamedTemporaryFile(mode="r+t", suffix=".tmp") as f:
....: p.print_ranges(f.name)
....: for ll in f.readlines():
....: if ll: print(ll)
...
GLPK ... - SENSITIVITY ANALYSIS REPORT
Page 1

Problem:

Objective: 7.5 (MAXimum)

<table>
<thead>
<tr>
<th>No.</th>
<th>Row name</th>
<th>St</th>
<th>Activity</th>
<th>Slack</th>
<th>Lower bound</th>
<th>Activity range</th>
<th>Marginal</th>
<th>Upper bound</th>
<th>range</th>
<th>break point variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NU</td>
<td>3.00000</td>
<td>.</td>
<td>-Inf</td>
<td>.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continues on next page)
GLPK ... - SENSITIVITY ANALYSIS REPORT

Problem:
Objective: 7.5 (MAXimum)

<table>
<thead>
<tr>
<th>No.</th>
<th>Column name</th>
<th>St</th>
<th>Activity</th>
<th>Obj coef</th>
<th>Lower bound</th>
<th>Activity range</th>
<th>Upper bound</th>
<th>Marginal</th>
<th>Activity break point variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NL</td>
<td>.</td>
<td></td>
<td>2.00000</td>
<td>.</td>
<td>-Inf</td>
<td>+Inf</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>BS</td>
<td>1.50000</td>
<td>-Inf</td>
<td>-3.50000</td>
<td>+Inf</td>
<td>3.00000</td>
<td>+Inf</td>
<td>4.00000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2.50000</td>
<td>3.00000</td>
<td>.</td>
<td></td>
<td>-Inf</td>
<td></td>
<td>5.00000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4.00000</td>
<td>6.00000</td>
<td>.</td>
<td></td>
<td>+Inf</td>
<td></td>
<td>1.50000</td>
<td></td>
</tr>
</tbody>
</table>

End of report

**problem_name(name=None)**

Return or define the problem’s name.

**INPUT:**

- name (str) – the problem’s name. When set to None (default), the method returns the problem’s name.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.problem_name("There once was a french fry")
sage: print(p.problem_name())
There once was a french fry
```

**remove_constraint(i)**

Remove a constraint from self.

**INPUT:**

- i – index of the constraint to remove

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x, y = p['x'], p['y']
sage: p.add_constraint(2*x + 3*y <= 6)
sage: p.add_constraint(3*x + 2*y <= 6)
sage: p.add_constraint(x >= 0)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
```
Removing fancy constraints does not make Sage crash:

```python
sage: MixedIntegerLinearProgram(solver = "GLPK").remove_constraint(-2)
Traceback (most recent call last):
 ...
ValueError: The constraint's index i must satisfy 0 <= i < number_of_constraints
```

### remove_constraints(constraints)

Remove several constraints.

**INPUT:**

- `constraints` – an iterable containing the indices of the rows to remove.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x, y = p['x'], p['y']
sage: p.add_constraint(2*x + 3*y <= 6)
sage: p.add_constraint(3*x + 2*y <= 6)
sage: p.add_constraint(x >= 0)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0
sage: p.remove_constraints([0])
sage: p.solve()
10.0
sage: p.get_values([x,y])
[0.0, 3.0]
```

### row(index)

Return a row

**INPUT:**

- `index` (integer) – the constraint’s id.

**OUTPUT:**

A pair `(indices, coeffs)` where `indices` lists the entries whose coefficient is nonzero, and to which `coeffs` associates their coefficient on the model of the `add_linear_constraint` method.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(zip(range(5), range(5))), 2, 2)
sage: p.row(0)
(continues on next page)```
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row_bounds(index)
Return the bounds of a specific constraint.

INPUT:
• index (integer) – the constraint’s id.

OUTPUT:
A pair (lower_bound, upper_bound). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(zip(range(5), range(5))), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)
(2.0, 2.0)
```

row_name(index)
Return the index th row name

INPUT:
• index (integer) – the row’s id

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_linear_constraints(1, 2, None, names=['Empty constraint 1'])
sage: p.row_name(0)
'Empty constraint 1'
```

set_col_stat(j, stat)
Set the status of a variable.

INPUT:
• j – The index of the constraint
• stat – The status to set to

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
sage: lp.add_variables(3)
2
```

(continues on next page)
set_objective(coeff, d=0.0)
Set the objective function.

INPUT:

- coeff - a list of real values, whose ith element is the coefficient of the ith variable in the objective function.
- d (double) – the constant term in the linear function (set to 0 by default)

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])
sage: [p.objective_coefficient(x) for x in range(5)]
[1.0, 1.0, 2.0, 1.0, 3.0]
```

set_row_stat(i, stat)
Set the status of a constraint.

INPUT:

- i – The index of the constraint
- stat – The status to set to

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
sage: lp.add_variables(3)
2
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
sage: lp.set_objective([60, 30, 20])
```
\[\text{set\_sense}(\text{sense})\]
Set the direction (maximization/minimization).

**INPUT:**
- \text{sense} (integer):
  - +1 \Rightarrow \text{Maximization}
  - -1 \Rightarrow \text{Minimization}

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

\[\text{set\_variable\_type}(\text{variable, vtype})\]
Set the type of a variable

**INPUT:**
- \text{variable} (integer) – the variable’s id
- \text{vtype} (integer):
  - 1 \text{ Integer}
  - 0 \text{ Binary}
  - -1 \text{ Real}

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
sage: p.is_variable_integer(0)
True
```
set_verbosity(level)

Set the verbosity level

INPUT:

- level (integer) – From 0 (no verbosity) to 3.

EXAMPLES:

```
sage: p.<x> = MixedIntegerLinearProgram(solver="GLPK")
sage: p.add_constraint(10 * x[0] <= 1)
sage: p.add_constraint(5 * x[1] <= 1)
sage: p.set_objective(x[0] + x[1])
sage: p.solve()
0.30000000000000004
sage: p.get_backend().set_verbosity(3)
sage: p.solver_parameter("simplex_or_intopt", "intopt_only")
sage: p.solve()
GLPK Integer Optimizer...
2 rows, 2 columns, 2 non-zeros
0 integer variables, none of which are binary
Preprocessing...
Objective value = 3.0000000000e-01
INTEGER OPTIMAL SOLUTION FOUND BY MIP PREPROCESSOR
0.30000000000000004
```

```
sage: p.<x> = MixedIntegerLinearProgram(solver="GLPK/exact")
sage: p.add_constraint(10 * x[0] <= 1)
sage: p.add_constraint(5 * x[1] <= 1)
sage: p.set_objective(x[0] + x[1])
sage: p.solve() # tol 1e-14
0.3
sage: p.get_backend().set_verbosity(2)
sage: p.solve() # tol 1e-14
* 2: objval = 0.3 (0)
0.3
sage: p.get_backend().set_verbosity(3)
sage: p.solve() # tol 1e-14
GLP_exact: 2 rows, 2 columns, 2 non-zeros
...
* 2: objval = 0.3 (0)
* 2: objval = 0.3 (0)
OPTIMAL SOLUTION FOUND
0.3
```

solve()

Solve the problem.

Sage uses GLPK’s implementation of the branch-and-cut algorithm (glp_intopt) to solve the mixed-integer linear program. This algorithm can be requested explicitly by setting the solver parameter “simplex_or_intopt” to “intopt_only”. By default, the simplex method will be used first to detect pathological problems that the integer solver cannot handle. If all variables are continuous, the integer algorithm reduces to solving the linear program by the simplex method.

EXAMPLES:
```
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = False)

sage: x, y = lp[0], lp[1]
sage: lp.add_constraint(-2*x + y <= 1)
sage: lp.add_constraint(x - y <= 1)
sage: lp.add_constraint(x + y >= 2)
sage: lp.set_objective(x + y)
sage: lp.set_integer(x)
sage: lp.set_integer(y)
sage: lp.solve()
2.0

sage: lp.get_values([x, y])
[1.0, 1.0]
```

**Note:** This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver = "GLPK")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.solve()
0
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
...
MIPSolverException: ...
```

**Warning:** GLPK’s glp_intopt sometimes fails catastrophically when given a system it cannot solve (github issue #12309). It can loop indefinitely, or just plain segfault. Upstream considers this behavior “essentially innate” to the current design, and suggests preprocessing with glp_simplex, which is what SageMath does by default. Set the simplex_or_intopt solver parameter to glp_intopt_only at your own risk.

**EXAMPLES:**

```
sage: lp = MixedIntegerLinearProgram(solver = "GLPK")
sage: v = lp.new_variable(nonnegative=True)
sage: lp.add_constraint(v[0] -4.0/3 *v[1] +1.0/3 *v[2], max=-1.0/3)
sage: lp.add_constraint(v[0] +0.5 *v[1] -0.5 *v[2] +0.25 *v[3], max=-0.25)
sage: lp.solve()
0.0
sage: lp.add_constraint(v[0] +4.0 *v[1] -v[2] +v[3], max=-1.0)
sage: lp.solve()
Traceback (most recent call last):
...
MIPSolverException: GLPK: Problem has no feasible solution
```
If we switch to “simplex_only”, the integrality constraints are ignored, and we get an optimal solution to the continuous relaxation.

**EXAMPLES:**

```python
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = False)
sage: x, y = lp[0], lp[1]
sage: lp.add_constraint(-2*x + y <= 1)
sage: lp.add_constraint(x - y <= 1)
sage: lp.add_constraint(x + y >= 2)
sage: lp.set_objective(x + y)
sage: lp.set_integer(x)
sage: lp.set_integer(y)
sage: lp.solver_parameter("simplex_or_intopt", "simplex_only")  # use simplex_only

sage: lp.solve()
2.0
```

If one solves a linear program and wishes to access dual information (get_col_dual etc.) or tableau data (get_row_stat etc.), one needs to switch to “simplex_only” before solving.

GLPK also has an exact rational simplex solver. The only access to data is via double-precision floats, which means that rationals in the input data may be rounded before the exact solver sees them. Thus, it is unreasonable to expect that arbitrary LPs with rational coefficients are solved exactly. Once the LP has been read into the backend, it reconstructs rationals from doubles and does solve exactly over the rationals, but results are returned as as doubles.

**EXAMPLES:**

```python
sage: lp.solver_parameter("simplex_or_intopt", "exact_simplex_only")  # use exact simplex only

sage: lp.solve()
2.0
```

If you need the rational solution, you need to retrieve the basis information via get_col_stat and get_row_stat and calculate the corresponding basic solution. Below we only test that the basis information is indeed available. Calculating the corresponding basic solution is left as an exercise.

**EXAMPLES:**

```python
sage: lp.get_backend().get_row_stat(0)
1
sage: lp.get_backend().get_col_stat(0)
1
```

Below we test that integers that can be exactly represented by IEEE 754 double-precision floating point numbers survive the rational reconstruction done by glp_exact and the subsequent conversion to double-precision floating point numbers.

**EXAMPLES:**

```python
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = True)
sage: test = 2^53 - 43
```

(continues on next page)
sage: lp.solver_parameter("simplex_or_intopt", "exact_simplex_only") # use...
˓→ exact simplex only
sage: x = lp[0]
sage: lp.add_constraint(x <= test)
sage: lp.set_objective(x)
sage: lp.solve() == test # yes, we want an exact comparison here
True
sage: lp.get_values(x) == test # yes, we want an exact comparison here
True

Below we test that GLPK backend can detect unboundedness in “simplex_only” mode (github issue #18838).

EXAMPLES:

sage: lp = MixedIntegerLinearProgram(maximization=True, solver = "GLPK")
多重IntegerLinearProgram(maximization=True, solver = "GLPK")
sage: lp.set_objective(lp[0])
sage: lp.solver_parameter("simplex_or_intopt", "simplex_only")
sage: lp.solve()  # not bounded
Traceback (most recent call last):
...
MIPSolverException: GLPK: Problem has unbounded solution
sage: lp.solver_parameter("primal_v_dual", "GLP_DUAL")
sage: lp.solve()  # not bounded
Traceback (most recent call last):
...
MIPSolverException: GLPK: Problem has unbounded solution
sage: lp.solver_parameter("simplex_or_intopt", "simplex_then_intopt")
sage: lp.solve()  # not bounded
Traceback (most recent call last):
...
MIPSolverException: GLPK: The LP (relaxation) problem has no dual feasible...
˓→ solution
sage: lp.solver_parameter("simplex_or_intopt", "intopt_only")
sage: lp.solve()  # not bounded
Traceback (most recent call last):
...
MIPSolverException: GLPK: The LP (relaxation) problem has no dual feasible...
˓→ solution
sage: lp.set_max(lp[1],5)
sage: lp.solve()  # bounded
5.0

Solving a LP within the acceptable gap. No exception is raised, even if the result is not optimal. To do this, we try to compute the maximum number of disjoint balls (of diameter 1) in a hypercube:

sage: # needs sage.graphs
多重needs sage.graphs
sage: g = graphs.CubeGraph(9)
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: p.solver_parameter("mip_gap_tolerance",100)
sage: b = p.new_variable(binary=True)
sage: p.set_objective(p.sum(b[v] for v in g))
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(continued from previous page)

```python
sage: for v in g:
    ....:     p.add_constraint(b[v]+p.sum(b[u] for u in g.neighbors(v)) <= 1)
sage: p.add_constraint(b[v] == 1) # Force an easy non-0 solution
sage: p.solve() # rel tol 100
1
```

Same, now with a time limit:

```python
sage: # needs sage.graphs
sage: p.solver_parameter("mip_gap_tolerance",1)
sage: p.solver_parameter("timelimit",3.0)
sage: p.solve() # rel tol 100
1
```

`solver_parameter(name, value=None)`

Returns or defines a solver parameter

**INPUT:**

- `name` (string) – the parameter
- `value` – the parameter’s value if it is to be defined, or `None` (default) to obtain its current value.

You can supply the name of a parameter and its value using either a string or a `glp_` constant (which are defined as Cython variables of this module).

In most cases, you can use the same name for a parameter as that given in the GLPK documentation, which is available by downloading GLPK from [http://www.gnu.org/software/glpk/](http://www.gnu.org/software/glpk/). The exceptions relate to parameters common to both methods; these require you to append `_simplex` or `_intopt` to the name to resolve ambiguity, since the interface allows access to both.

We have also provided more meaningful names, to assist readability.

Parameter names are specified in lower case. To use a constant instead of a string, prepend `glp_` to the name. For example, both `glp_gmi_cuts` or "gmi_cuts" control whether to solve using Gomory cuts.

Parameter values are specified as strings in upper case, or as constants in lower case. For example, both `glp_on` and "GLP_ON" specify the same thing.

Naturally, you can use `True` and `False` in cases where `glp_on` and `glp_off` would be used.

A list of parameter names, with their possible values:

**General-purpose parameters:**
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>timelimit</td>
<td>specify the time limit IN SECONDS. This affects both simplex and intopt.</td>
</tr>
<tr>
<td>timelimit_simplex and</td>
<td>specify the time limit IN MILLISECONDS. (This is glpk’s default.)</td>
</tr>
<tr>
<td>timelimit_intopt</td>
<td></td>
</tr>
<tr>
<td>simplex_or_intopt</td>
<td>specify which solution routines in GLPK to use. Set this to either simplex_only, exact_simplex_only, intopt_only, or simplex_then_intopt (the default). The simplex_then_intopt option does some extra work, but avoids hangs/crashes in GLPK on problems with no solution; SageMath will try simplex first, then perform integer optimization only if a solution of the LP relaxation exists. If you know that your system is not pathological, one of the other options will be faster.</td>
</tr>
<tr>
<td>verbosity_intopt and</td>
<td>one of GLP_MSG_OFF, GLP_MSG_ERR, GLP_MSG_ON, or GLP_MSG_ALL. The default is GLP_MSG_OFF.</td>
</tr>
<tr>
<td>verbosity_simplex</td>
<td></td>
</tr>
<tr>
<td>output_frequency_intopt</td>
<td>the output frequency, in milliseconds. Default is 5000.</td>
</tr>
<tr>
<td>output_frequency_simplex</td>
<td></td>
</tr>
<tr>
<td>output_delay_intopt</td>
<td>the output delay, in milliseconds, regarding the use of the simplex method on the LP relaxation. Default is 10000.</td>
</tr>
<tr>
<td>output_delay_simplex</td>
<td></td>
</tr>
</tbody>
</table>

intopt-specific parameters:
### Numerical Optimization, Release 10.2

#### Branching
- GLP_BR_FFV first fractional variable
- GLP_BR_LFV last fractional variable
- GLP_BR_MFV most fractional variable
- GLP_BR_DTH Driebeck-Tomlin heuristic (default)
- GLP_BR_PCH hybrid pseudocost heuristic

#### Backtracking
- GLP_BT_DFS depth first search
- GLP_BT_BFS breadth first search
- GLP_BT_BLB best local bound (default)
- GLP_BT_BPH best projection heuristic

#### Preprocessing
- GLP_PP_NONE
- GLP_PP_ROOT preprocessing only at root level
- GLP_PP_ALL (default)

#### Feasibility Pump
- GLP_ON or GLP_OFF (default)

#### Gomory Cuts
- GLP_ON or GLP_OFF (default)

#### Mixed Integer Rounding Cuts
- GLP_ON or GLP_OFF (default)

#### Mixed Cover Cuts
- GLP_ON or GLP_OFF (default)

#### Clique Cuts
- GLP_ON or GLP_OFF (default)

#### Absolute Tolerance
- (double) used to check if optimal solution to LP relaxation is integer feasible. GLPK manual advises, “do not change... without detailed understanding of its purpose.”

#### Relative Tolerance
- (double) used to check if objective value in LP relaxation is not better than best known integer solution. GLPK manual advises, “do not change... without detailed understanding of its purpose.”

#### MIP Gap Tolerance
- (double) relative mip gap tolerance. Default is 0.0.

#### Presolve Intopt
- GLP_ON (default) or GLP_OFF.

#### Binarize
- GLP_ON or GLP_OFF (default)

### Simplex-Specific Parameters:
### primal_v_dual

- GLP_PRIMAL (default)
- GLP_DUAL
- GLP_DUALP

### pricing

- GLP_PT_STD standard (textbook)
- GLP_PT_PSE projected steepest edge (default)

### ratio_test

- GLP_RT_STD standard (textbook)
- GLP_RT_HAR Harris’ two-pass ratio test (default)

### tolerance_primal

(double) tolerance used to check if basic solution is primal feasible. GLPK manual advises, “do not change… without detailed understanding of its purpose.”

### tolerance_dual

(double) tolerance used to check if basic solution is dual feasible. GLPK manual advises, “do not change… without detailed understanding of its purpose.”

### tolerance_pivot

(double) tolerance used to choose pivot. GLPK manual advises, “do not change… without detailed understanding of its purpose.”

### obj_lower_limit

(double) lower limit of the objective function. The default is -DBL_MAX.

### obj_upper_limit

(double) upper limit of the objective function. The default is DBL_MAX.

### iteration_limit

(int) iteration limit of the simplex algorithm. The default is INT_MAX.

### presolve_simplex

GLP_ON or GLP_OFF (default).

---

**Note:** The coverage for GLPK’s control parameters for simplex and integer optimization is nearly complete. The only thing lacking is a wrapper for callback routines.

To date, no attempt has been made to expose the interior point methods.

---

**EXAMPLES:**

```python
gp = get_solver(solver = "GLPK")
gp.solver_parameter("timelimit", 60)
gp.solver_parameter("timelimit")
60.0
```

- Don’t forget the difference between `timelimit` and `timelimit_intopt`

```python
gp.solver_parameter("timelimit_intopt")
60000
```

If you don’t care for an integer answer, you can ask for an LP relaxation instead. The default solver performs integer optimization, but you can switch to the standard simplex algorithm through the `glp_simplex_or_intopt` parameter.

**EXAMPLES:**
```python
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = False)
sage: x, y = lp[0], lp[1]
sage: lp.add_constraint(-2*x + y <= 1)
sage: lp.add_constraint(x - y <= 1)
sage: lp.add_constraint(x + y >= 2)
sage: lp.set_integer(x); lp.set_integer(y)
sage: lp.set_objective(x + y)
sage: lp.solve()
2.0
sage: lp.get_values([x,y])
[1.0, 1.0]
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_→only)
sage: lp.solve()
2.0
sage: lp.get_values([x,y])
[1.5, 0.5]
```

You can get GLPK to spout all sorts of information at you. The default is to turn this off, but sometimes (debugging) it’s very useful:

```python
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_→then_intopt)
sage: lp.solver_parameter(backend.glp_mir_cuts, backend.glp_on)
sage: lp.solver_parameter(backend.glp_msg_lev_intopt, backend.glp_msg_all)
sage: lp.solver_parameter(backend.glp_mir_cuts)
1
```

If you actually try to solve `lp`, you will get a lot of detailed information.

### variable_lower_bound(index, value=False)

Return or define the lower bound on a variable

**INPUT:**

- **index** (integer) – the variable’s id
- **value** – real value, or None to mean that the variable has not lower bound. When set to False (default), the method returns the current value.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5.0, None)
```

### variable_upper_bound(index, value=False)

Return or define the upper bound on a variable
INPUT:

- **index** (integer) – the variable’s id
- **value** – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
```

```python
sage: p.add_variable() 0
```

```python
sage: p.col_bounds(0)
(0.0, None)
```

```python
sage: p.variable_upper_bound(0, 5)
```

```python
sage: p.col_bounds(0)
(0.0, 5.0)
```

**warm_up()**

Warm up the basis using current statuses assigned to rows and cols.

OUTPUT:

- Returns the warming up status
  - 0 The operation has been successfully performed.
  - GLP_EBADB The basis matrix is invalid.
  - GLP_ESING The basis matrix is singular within the working precision.
  - GLP_ECOND The basis matrix is ill-conditioned.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
```

```python
sage: lp.add_variables(3)
2
```

```python
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [8, 6, 1])), None, 48)
```

```python
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [4, 2, 1.5])), None, 20)
```

```python
sage: lp.add_linear_constraint(list(zip([0, 1, 2], [2, 1.5, 0.5])), None, 8)
```

```python
sage: lp.set_objective([60, 30, 20])
```

```python
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
```

```python
sage: lp.solve() 0
```

```python
sage: lp.get_objective_value()
280.0
```

```python
sage: lp.set_row_stat(0,3)
```

```python
sage: lp.set_col_stat(1,1)
```

```python
sage: lp.warm_up() 0
```

**write_lp**(*filename*)

Write the problem to a .lp file

INPUT:
• filename (string)

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
sage: p.set_objective([2, 5])
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix='.lp') as f:
    ...
    _ = p.write_lp(f.name)
    ...
    len(f.readlines())
...
9 lines were written
9
```

**write_mps(filename, modern)**

Write the problem to a .mps file

INPUT:

• filename (string)

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
sage: p.set_objective([2, 5])
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix="mps") as f:
    ...
    _ = p.write_mps(f.name, 2)
    ...
    len(f.readlines())
...
17 records were written
17
```

### 11.4 GLPK/exact Backend (simplex method in exact rational arithmetic)

**AUTHORS:**

• Matthias Koeppe (2016-03)

**class** `sage.numerical.backends.generic_backend.GLPKExactBackend`

Bases: `GLPKBackend`

MIP Backend that runs the GLPK solver in exact rational simplex mode.

The only access to data is via double-precision floats, which means that rationals in the input data may be rounded before the exact solver sees them. Thus, it is unreasonable to expect that arbitrary LPs with rational coefficients...
are solved exactly. Once the LP has been read into the backend, it reconstructs rationals from doubles and does solve exactly over the rationals, but results are returned as doubles.

There is no support for integer variables.

\texttt{add\_variable(\textit{lower\_bound}=0.0, \textit{upper\_bound}=\textit{None}, \textit{binary}=False, \textit{continuous}=False, \textit{integer}=False, \textit{obj}=0.0, \textit{name}=\textit{None})}

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both nonnegative and real.

In this backend, variables are always continuous (real). If integer variables are requested via the parameters \texttt{binary} and \texttt{integer}, an error will be raised.

**INPUT:**

- \texttt{lower\_bound} - the lower bound of the variable (default: 0)
- \texttt{upper\_bound} - the upper bound of the variable (default: \textit{None})
- \texttt{binary} - True if the variable is binary (default: False).
- \texttt{continuous} - True if the variable is continuous (default: True).
- \texttt{integer} - True if the variable is integer (default: False).
- \texttt{obj} - (optional) coefficient of this variable in the objective function (default: 0.0)
- \texttt{name} - an optional name for the newly added variable (default: \textit{None}).

**OUTPUT:** The index of the newly created variable

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK/exact")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable()
1
sage: p.add_variable(lower_bound=-2.0)
2
sage: p.add_variable(continuous=True)
3
sage: p.add_variable(name='x', obj=1.0)
4
sage: p.objective_coefficient(4)
1.0
```

\texttt{add\_variables(\textit{number}, \textit{lower\_bound}=0.0, \textit{upper\_bound}=\textit{None}, \textit{binary}=False, \textit{continuous}=False, \textit{integer}=False, \textit{obj}=0.0, \textit{names}=\textit{None})}

Add \textit{number} variables.

This amounts to adding new columns to the matrix. By default, the variables are both nonnegative and real.

In this backend, variables are always continuous (real). If integer variables are requested via the parameters \texttt{binary} and \texttt{integer}, an error will be raised.
INPUT:

- \( n \) - the number of new variables (must be \( > 0 \))
- lower_bound - the lower bound of the variable (default: 0)
- upper_bound - the upper bound of the variable (default: None)
- binary - True if the variable is binary (default: False).
- continuous - True if the variable is binary (default: True).
- integer - True if the variable is binary (default: False).
- obj - (optional) coefficient of all variables in the objective function (default: 0.0)
- names - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK/exact")
sage: p.ncols()
0
sage: p.add_variables(5)
4
sage: p.ncols()
5
sage: p.add_variables(2, lower_bound=-2.0, obj=42.0, names=['a', 'b'])
6
```

**set_variable_type** (variable, vtype)

Set the type of a variable.

In this backend, variables are always continuous (real). If integer or binary variables are requested via the parameter vtype, an error will be raised.

INPUT:

- variable (integer) – the variable's id
- vtype (integer):
  - 1 Integer
  - 0 Binary
  - -1 Real

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK/exact")
sage: p.add_variables(5)
4
sage: p.set_variable_type(3, -1)
sage: p.set_variable_type(3, -2)
Traceback (most recent call last):
... ValueError: ...
```
11.5 GLPK Backend for access to GLPK graph functions

AUTHORS:

• Christian Kuper (2012-11): Initial implementation

11.5.1 Methods index

Graph creation and modification operations:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>add_vertex()</td>
<td>Adds an isolated vertex to the graph.</td>
</tr>
<tr>
<td>add_vertices()</td>
<td>Adds vertices from an iterable container of vertices.</td>
</tr>
<tr>
<td>set_vertex_demand()</td>
<td>Sets the vertex parameters.</td>
</tr>
<tr>
<td>set_vertices_demand()</td>
<td>Sets the parameters of selected vertices.</td>
</tr>
<tr>
<td>get_vertex()</td>
<td>Returns a specific vertex as a dict object.</td>
</tr>
<tr>
<td>get_vertices()</td>
<td>Returns a dictionary of the dictionaries associated to each vertex.</td>
</tr>
<tr>
<td>vertices()</td>
<td>Returns a list of all vertices.</td>
</tr>
<tr>
<td>delete_vertex()</td>
<td>Removes a vertex from the graph.</td>
</tr>
<tr>
<td>delete_vertices()</td>
<td>Removes vertices from the graph.</td>
</tr>
<tr>
<td>add_edge()</td>
<td>Adds an edge between vertices u and v.</td>
</tr>
<tr>
<td>add_edges()</td>
<td>Adds edges to the graph.</td>
</tr>
<tr>
<td>get_edge()</td>
<td>Returns an edge connecting two vertices.</td>
</tr>
<tr>
<td>edges()</td>
<td>Returns a list of all edges in the graph.</td>
</tr>
<tr>
<td>delete_edge()</td>
<td>Deletes an edge from the graph.</td>
</tr>
<tr>
<td>delete_edges()</td>
<td>Deletes edges from the graph.</td>
</tr>
</tbody>
</table>

Graph writing operations:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>write_graph()</td>
<td>Writes the graph to a plain text file.</td>
</tr>
<tr>
<td>write_ccdata()</td>
<td>Writes the graph to a text file in DIMACS format.</td>
</tr>
<tr>
<td>write_mincost()</td>
<td>Writes the mincost flow problem data to a text file in DIMACS format.</td>
</tr>
<tr>
<td>write_maxflow()</td>
<td>Writes the maximum flow problem data to a text file in DIMACS format.</td>
</tr>
</tbody>
</table>

Network optimization operations:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>mincost_okalg()</td>
<td>Finds solution to the mincost problem with the out-of-kilter algorithm.</td>
</tr>
<tr>
<td>maxflow_ffalg()</td>
<td>Finds solution to the maxflow problem with Ford-Fulkerson algorithm.</td>
</tr>
<tr>
<td>cpp()</td>
<td>Solves the critical path problem of a project network.</td>
</tr>
</tbody>
</table>

11.5.2 Classes and methods

class sage.numerical.backends.glpk_graph_backend.GLPKGraphBackEnd

    Bases: object

GLPK Backend for access to GLPK graph functions

The constructor can either be called without arguments (which results in an empty graph) or with arguments to read graph data from a file.

INPUT:

• data – a filename or a Graph object.
• format – when data is a filename, specifies the format of the data read from a file. The format parameter is a string and can take values as described in the table below.

**Format parameters:**

<table>
<thead>
<tr>
<th>Format</th>
<th>Description</th>
</tr>
</thead>
</table>
| plain  | Read data from a plain text file containing the following information:
|        | `nv` `na`  |
|        | `i[1]` `j[1]` |
|        | ...       |
|        | `i[na]` `j[na]` |
|        | where:    |
|        | • `nv` is the number of vertices (nodes); |
|        | • `na` is the number of arcs; |
|        | • `i[k]`, `k = 1, ... , na`, is the index of tail vertex of arc `k`; |
|        | • `j[k]`, `k = 1, ... , na`, is the index of head vertex of arc `k`. |

| dimacs | Read data from a plain ASCII text file in DIMACS format. A description of the DIMACS format can be found at [http://dimacs.rutgers.edu/Challenges/](http://dimacs.rutgers.edu/Challenges/). |
| mincost | Reads the mincost flow problem data from a text file in DIMACS format |
| maxflow | Reads the maximum flow problem data from a text file in DIMACS format |

**Note:** When data is a Graph, the following restrictions are applied.

• vertices – the value of the demand of each vertex (see `set_vertex_demand()`) is obtained from the numerical value associated with the key “rhs” if it is a dictionary.

• edges – The edge values used in the algorithms are read from the edges labels (and left undefined if the edge labels are equal to `None`). To be defined, the labels must be dict objects with keys “low”, “cap” and “cost”. See `get_edge()` for details.

**EXAMPLES:**

The following example creates an empty graph:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```

The following example creates an empty graph, adds some data, saves the data to a file and loads it:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertices([None, None])
['0', '1']
sage: a = gbe.add_edge('0', '1')
sage: import tempfile
sage: with tempfile.NamedTemporaryFile() as f:
    ....:   _ = gbe.write_graph(f.name)
    ....:   gbe1 = GLPKGraphBackend(f.name, "plain")
Writing graph to ...
4 lines were written
Reading graph from ...
```

(continues on next page)
The following example imports a Sage Graph and then uses it to solve a maxflow problem:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: g = graphs.PappusGraph()
```

```python
sage: for ed in g.edges(sort=False):
....:     g.set_edge_label(ed[0], ed[1], {"cap":1})
```

```python
sage: gbe = GLPKGraphBackend(g)
```

```python
sage: gbe.maxflow_ffalg('1', '2')
3.0
```

**add_edge**(u, v, params=None)

Adds an edge between vertices u and v.

Allows adding an edge and optionally providing parameters used by the algorithms. If a vertex does not exist it is created.

**INPUT:**

- u – The name (as str) of the tail vertex
- v – The name (as str) of the head vertex
- params – An optional dict containing the edge parameters used for the algorithms. The following keys are used:
  - low – The minimum flow through the edge
  - cap – The maximum capacity of the edge
  - cost – The cost of transporting one unit through the edge

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```

```python
sage: gbe.add_edge("A", "B", {"low":0.0, "cap":10.0, "cost":5})
```

```python
sage: gbe.vertices()
['A', 'B']
```

```python
sage: for ed in gbe.edges():
....:     print((ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']))
('A', 'B', 10.0, 5.0, 0.0)
```

```python
sage: gbe.add_edge("B", "C", {"low":0.0, "cap":10.0, "cost":5'})
Traceback (most recent call last):
... TypeError: Invalid edge parameter.
```

**add_edges**(edges)

Adds edges to the graph.

**INPUT:**

- edges – An iterable container of pairs of the form (u, v), where u is name (as str) of the tail vertex and v is the name (as str) of the head vertex or an iterable container of triples of the form (u, v, params) where params is a dict as described in add_edge.

**EXAMPLES:**

11.5. GLPK Backend for access to GLPK graph functions
```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```
```
sage: edges = [('A', 'B', {'low':0.0, 'cap':10.0, 'cost':5})]
```
```
sage: edges.append(('B', 'C'))
```
```
sage: gbe.add_edges(edges)
```
```
sage: for ed in gbe.edges():
    ....:     print((ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']))
```
```
('A', 'B', 10.0, 5.0, 0.0)
('B', 'C', 0.0, 0.0, 0.0)
```
```
sage: edges = [('C', 'D', {'low':0.0, 'cap':10.0, 'cost':5})]
```
```
sage: edges.append(('C', 'E', 5))
```
```
sage: gbe.add_edges(edges)
```
```
Traceback (most recent call last):
  ...:
```
```
sage: for ed in gbe.edges():
    ....:     print((ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']))
```
```
('A', 'B', 10.0, 5.0, 0.0)
('B', 'C', 0.0, 0.0, 0.0)
('C', 'D', 10.0, 5.0, 0.0)
```
```
```
```
```
```
```
add_vertex(name=None)
```
**Adds** an isolated vertex to the graph.

If the vertex already exists, nothing is done.

**INPUT:**

- name – str of max 255 chars length. If no name is specified, then the vertex will be represented by the string representation of the ID of the vertex or - if this already exists - a string representation of the least integer not already representing a vertex.

**OUTPUT:**

If no name is passed as an argument, the new vertex name is returned. None otherwise.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```
```
sage: gbe.add_vertex() '0'
```
```
sage: gbe.add_vertex("2")
```
```
sage: gbe.add_vertex() '1'
```
```
add_vertices(vertices)
```
**Adds** vertices from an iterable container of vertices.

Vertices that already exist in the graph will not be added again.

**INPUT:**

- vertices – iterator of vertex labels (str). A label can be None.

**OUTPUT:**

Generated names of new vertices if there is at least one None value present in vertices. None otherwise.

Chapter 11. Linear Optimization (LP) and Mixed Integer Linear Optimization (MIP) Solver backends
EXAMPLES:

```python
from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
gbe = GLPKGraphBackend()
vertices = [None for i in range(3)]
gbe.add_vertices(vertices)
['0', '1', '2']
gbe.add_vertices(['A', 'B', None])
['5']
gbe.add_vertices(['A', 'B', 'C'])
sage: gbe.vertices()
['0', '1', '2', 'A', 'B', '5', 'C']
```

cpp()
Solves the critical path problem of a project network.

OUTPUT:
The length of the critical path of the network

EXAMPLES:

```python
from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
gbe = GLPKGraphBackend()
vertices = [None for i in range(3)]
gbe.add_vertices(vertices)
['0', '1', '2']
gbe.set_vertex_demand('0', 3)
gbe.set_vertex_demand('1', 1)
gbe.set_vertex_demand('2', 4)
a = gbe.add_edge('0', '2')
a = gbe.add_edge('1', '2')
sage: a.cpp()
7.0
sage: v = gbe.get_vertex('1')
sage: 1, v['rhs'], v['es'], v['ls']
# abs tol 1e-6
(1, 1.0, 0.0, 2.0)
```

delete_edge(u, v, params=None)
Deletes an edge from the graph.
If an edge does not exist it is ignored.

INPUT:

- `u` – The name (as str) of the tail vertex of the edge
- `v` – The name (as str) of the tail vertex of the edge
- `params` – An optional dict containing the edge parameters (see `add_edge()`). If this parameter is not provided, all edges connecting `u` and `v` are deleted. Otherwise only edges with matching parameters are deleted.

See also:

delete_edges()
```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()

sage: edges = [('A', 'B', {'low':0.0, 'cap':10.0, 'cost':5})]
sage: edges.append(('A', 'B', {'low':0.0, 'cap':15.0, 'cost':10}))
sage: edges.append(('B', 'C', {'low':0.0, 'cap':20.0, 'cost':1}))
sage: edges.append(('B', 'C', {'low':0.0, 'cap':10.0, 'cost':20}))
sage: gbe.add_edges(edges)

sage: gbe.delete_edge('A', 'B')

sage: gbe.delete_edge('B', 'C', {'low':0.0, 'cap':10.0, 'cost':20})

sage: gbe.edges()[0][0], gbe.edges()[0][1], gbe.edges()[0][2]['cost']
('B', 'C', 1.0)
```

delete_edges(edges)

Deletes edges from the graph.

Non existing edges are ignored.

INPUT:

- edges – An iterable container of edges.

See also:

delete_edge()

EXAMPLES:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()

sage: edges = [('A', 'B', {'low':0.0, 'cap':10.0, 'cost':5})]

sage: edges.append(('A', 'B', {'low':0.0, 'cap':15.0, 'cost':10}))

sage: edges.append(('B', 'C', {'low':0.0, 'cap':20.0, 'cost':1}))

sage: edges.append(('B', 'C', {'low':0.0, 'cap':10.0, 'cost':20}))

sage: gbe.add_edges(edges)

sage: gbe.delete_edges(edges[1:])

sage: len(gbe.edges())
1

sage: gbe.edges()[0][0], gbe.edges()[0][1], gbe.edges()[0][2]['cap']
('A', 'B', 10.0)
```

delete_vertex(vert)

Removes a vertex from the graph.

Trying to delete a non existing vertex will raise an exception.

INPUT:

- vert – The name (as str) of the vertex to delete.

EXAMPLES:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()

sage: verts = ['A', 'D']

sage: gbe.add_vertices(verts)

sage: gbe.delete_vertex('A')

sage: gbe.vertices()
('C', 'D')
```
```
delete_vertices(verts)

Removes vertices from the graph.

Trying to delete a non-existing vertex will raise an exception.

**INPUT:**
- verts – iterable container containing names (as str) of the vertices to delete

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ["A", "B", "C", "D"]
sage: gbe.add_vertices(verts)
sage: v_d = ["A", "B"]
sage: gbe.delete_vertices(v_d)
sage: gbe.vertices()
['C', 'D']
sage: gbe.delete_vertices(["C", "A"])
Traceback (most recent call last):
... RuntimeError: Vertex A does not exist.
sage: gbe.vertices()
['C', 'D']
```

**edges()**

Returns a list of all edges in the graph

**OUTPUT:**

A list of triples representing the edges of the graph.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(('B', 'C'))
sage: gbe.add_edges(edges)
sage: for ed in gbe.edges():
    ....:     print((ed[0], ed[1], ed[2]['cost']))
('A', 'B', 5.0)
('B', 'C', 0.0)
```

**get_edge(u, v)**

Returns an edge connecting two vertices.

**Note:** If multiple edges connect the two vertices only the first edge found is returned.
INPUT:

- \( u \) – Name (as \texttt{str}) of the tail vertex
- \( v \) – Name (as \texttt{str}) of the head vertex

OUTPUT:

A triple describing if edge was found or \texttt{None} if not. The third value of the triple is a \texttt{dict} containing the following edge parameters:

- \( \text{low} \) – The minimum flow through the edge
- \( \text{cap} \) – The maximum capacity of the edge
- \( \text{cost} \) – The cost of transporting one unit through the edge
- \( x \) – The actual flow through the edge after solving

EXAMPLES:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [('A', 'B'), ('A', 'C'), ('B', 'C')]
```

```python
sage: gbe.add_edges(edges)
```
**get_vertices(verts)**

Returns a dictionary of the dictionaries associated to each vertex.

**INPUT:**

- verts – iterable container of vertices

**OUTPUT:**

A list of pairs (vertex, properties) where properties is a dictionary containing the numerical values associated with a vertex. For more information, see the documentation of GLPKGraphBackend.get_vertex().

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ['A', 'B']
sage: gbe.add_vertices(verts)
sage: sorted(gbe.get_vertices(verts)['B'].items())
[('cut', 0), ('es', 0.0), ('ls', 0.0), ('pi', 0.0), ('rhs', 0.0)]
sage: gbe.get_vertices(['C', 'D'])
{ }
```

**maxflow_ffalg**(μ=None, v=None)

Finds solution to the maxflow problem with Ford-Fulkerson algorithm.

**INPUT:**

- u – Name (as str) of the tail vertex. Default is None.
- v – Name (as str) of the head vertex. Default is None.

If u or v are None, the currently stored values for the head or tail vertex are used. This behavior is useful when reading maxflow data from a file. When calling this function with values for u and v, the head and tail vertex are stored for later use.

**OUTPUT:**

The solution to the maxflow problem, i.e. the maximum flow.

**Note:**

- If the source or sink vertex does not exist, an IndexError is raised.
- If the source and sink are identical, a ValueError is raised.
- This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: v = gbe.add_vertices([None for i in range(5)])
sage: edges = ((0, 1, 2), (0, 2, 3), (1, 2, 3), (1, 3, 4),
            (3, 4, 1), (2, 4, 2))
sage: for a in edges:
    ....:     edge = gbe.add_edge(str(a[0]), str(a[1]), {"cap":a[2]})
```

(continues on next page)
### mincost_okalg

Finds solution to the mincost problem with the out-of-kilter algorithm.

The out-of-kilter algorithm requires all problem data to be integer valued.

**OUTPUT:**

The solution to the mincost problem, i.e. the total cost, if operation was successful.

---

**Note:** This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

**EXAMPLES:**

```python
gbe = GLPKGraphBackend() vertices = (35, 50, 40, -45, -20, -30, -30) vs = gbe.add_vertices([None for i in range(len(vertices))]) v_dict = {} for i, v in enumerate(vs): v_dict[v] = vertices[i] gbe.set_vertices_demand(list(v_dict.items())) cost = ((8, 6, 10, 9), (9, 12, 13, 7), (14, 9, 16, 5))
```
**set_vertex_demand***(vertex, demand)***

Sets the demand of the vertex in a mincost flow algorithm.

**INPUT:**

- **vertex** – Name of the vertex
- **demand** – the numerical value representing demand of the vertex in a mincost flow algorithm (it could be for instance −1 to represent a sink, or 1 to represent a source and 0 for a neutral vertex). This can either be an int or float value.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = [None for i in range(3)]
sage: gbe.add_vertices(vertices)
['0', '1', '2']
sage: gbe.set_vertex_demand('0', 2)
sage: gbe.get_vertex('0')['rhs']
2.0
sage: gbe.set_vertex_demand('3', 2)
Traceback (most recent call last):
  ... KeyError: 'Vertex 3 does not exist.'
```

**set_vertices_demand***(pairs)***

Sets the parameters of selected vertices.

**INPUT:**

- **pairs** – A list of pairs (vertex, demand) associating a demand to each vertex. For more information, see the documentation of **set_vertex_demand()**.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = [None for i in range(3)]
sage: gbe.add_vertices(vertices)
['0', '1', '2']
sage: gbe.set_vertices_demand([(0, 2), (1, 3), (3, 4)])
sage: sorted(gbe.get_vertex('1').items())
[('cut', 0), ('es', 0.0), ('ls', 0.0), ('pi', 0.0), ('rhs', 3.0)]
```

**vertices()**

Returns the list of all vertices

**Note:** Changing elements of the list will not change anything in the the graph.

**Note:** If a vertex in the graph does not have a name / label it will appear as None in the resulting list.

**EXAMPLES:**
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ['A', 'B', 'C']
sage: gbe.add_vertices(verts)
sage: a = gbe.vertices(); a
['A', 'B', 'C']
sage: a.pop(0)
'A'
sage: gbe.vertices()
['A', 'B', 'C']

write_ccdata(fname)

Writes the graph to a text file in DIMACS format.

Writes the data to plain ASCII text file in DIMACS format. A description of the DIMACS format can be found at http://dimacs.rutgers.edu/Challenges/.

INPUT:

- fname – full name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero

EXAMPLES:

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: a = gbe.add_edge("0", "1")

import tempfile
sage: with tempfile.NamedTemporaryFile() as f:
......: gbe.write_ccdata(f.name)
Writing graph to ...
6 lines were written

write_graph(fname)

Writes the graph to a plain text file

INPUT:

- fname – full name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero

EXAMPLES:

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()

import tempfile
sage: with tempfile.NamedTemporaryFile() as f:
......: gbe.write_graph(f.name)
Writing graph to ...

write_maxflow(fname)

Writes the maximum flow problem data to a text file in DIMACS format.

INPUT:
  • fname – Full name of file

OUTPUT:
Zero if successful, otherwise non-zero

EXAMPLES:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: import tempfile
sage: with tempfile.NamedTemporaryFile() as f:
    ....:     gbe.write_maxflow(f.name)
    Traceback (most recent call last):
    ...:
    OSError: Cannot write empty graph
sage: gbe.add_vertices([None for i in range(2)])
['0', '1']
sage: a = gbe.add_edge('0', '1')
sage: gbe.maxflow_ffalg('0', '1')
0.0
sage: with tempfile.NamedTemporaryFile() as f:
    ....:     gbe.write_maxflow(f.name)
    Writing maximum flow problem data to ...
    6 lines were written
0
```

write_mincost(fname)

Writes the mincost flow problem data to a text file in DIMACS format.

INPUT:
  • fname – Full name of file

OUTPUT:
Zero if successful, otherwise nonzero

EXAMPLES:

```python
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: a = gbe.add_edge("0", "1")
```
11.6 PPL Backend

AUTHORS:

- Risan (2012-02): initial implementation
- Jeroen Demeyer (2014-08-04) allow rational coefficients for constraints and objective function (github issue #16755)

class sage.numerical.backends.ppl_backend.PPLBackend

Bases: GenericBackend

MIP Backend that uses the exact MIP solver from the Parma Polyhedra Library.

add_col(indices, coeffs)

Add a column.

INPUT:

- indices (list of integers) – this list contains the indices of the constraints in which the variable’s coefficient is nonzero
- coeffs (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the ith entry of coeffs corresponds to the coefficient of the variable in the constraint represented by the ith entry in indices.

Note: indices and coeffs are expected to be of the same length.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
0
sage: p.add_col(list(range(5)), list(range(5)))
1
sage: p.nrows()
5
```

add_linear_constraint(coefficients, lower_bound, upper_bound, name=None)

Add a linear constraint.

INPUT:

- coefficients – an iterable with (c,v) pairs where c is a variable index (integer) and v is a value (real value).
- lower_bound – a lower bound, either a real value or None
- upper_bound – an upper bound, either a real value or None
- name – an optional name for this row (default: None)

EXAMPLES:
```
sage: p = MixedIntegerLinearProgram(solver="PPL")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(x[0]/2 + x[1]/3 <= 2/5)
sage: p.set_objective(x[1])
sage: p.solve()
6/5
sage: p.add_constraint(x[0] - x[1] >= 1/10)
sage: p.solve()
21/50
sage: p.set_max(x[0], 1/2)
sage: p.set_min(x[1], 3/8)
sage: p.solve()
2/5
sage: from sage.numerical.backends.generic_backend import get_solver
def solve(self):
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
def add_linear_constraint(self, number, lower_bound, upper_bound, names=None):
    """Add constraints.
    INPUT:
    • number (integer) -- the number of constraints to add.
    • lower_bound -- a lower bound, either a real value or None
    • upper_bound -- an upper bound, either a real value or None
    • names -- an optional list of names (default: None)
    EXAMPLES:
    ""
    sage: from sage.numerical.backends.generic_backend import get_solver
def add_variable(self, lower_bound=0, upper_bound=None, binary=False, continuous=False, integer=False, obj=0, name=None):
    """Add a variable.
    This amounts to adding a new column to the matrix. By default, the variable is both positive and real.
```

11.6. PPL Backend
It has not been implemented for selecting the variable type yet.

INPUT:

- lower_bound – the lower bound of the variable (default: 0)
- upper_bound – the upper bound of the variable (default: None)
- binary – True if the variable is binary (default: False).
- continuous – True if the variable is continuous (default: True).
- integer – True if the variable is integral (default: False).
- obj – (optional) coefficient of this variable in the objective function (default: 0)
- name – an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable(lower_bound=-2)
1
sage: p.add_variable(name='x',obj=2/3)
2
sage: p.col_name(2)
'x'
sage: p.objective_coefficient(2)
2/3
sage: p.add_variable(integer=True)
3
```

add_variables(n, lower_bound=0, upper_bound=None, binary=False, continuous=True, integer=False, obj=0, names=None)

Add n variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive and real.

It has not been implemented for selecting the variable type yet.

INPUT:

- n – the number of new variables (must be > 0)
- lower_bound – the lower bound of the variable (default: 0)
- upper_bound – the upper bound of the variable (default: None)
- binary – True if the variable is binary (default: False).
- continuous – True if the variable is continuous (default: True).
- integer – True if the variable is integral (default: False).
- obj – (optional) coefficient of all variables in the objective function (default: 0)
• names – optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

```
sage: p.ncols()
0
```

```
sage: p.add_variables(5)
4
```

```
sage: p.ncols()
5
```

```
sage: p.add_variables(2, lower_bound=-2.0, obj=42.0, names=['a','b'])
6
```

**base_ring()**

**col_bounds(index)**

Return the bounds of a specific variable.

INPUT:

- • index (integer) – the variable’s id.

OUTPUT:

A pair (lower_bound, upper_bound). Each of them can be set to None if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

```
sage: p.col_bounds(0)
(0, None)
```

```
sage: p.variable_upper_bound(0, 5)
```

```
sage: p.col_bounds(0)
(0, 5)
```

**col_name(index)**

Return the index th col name

INPUT:

- • index (integer) – the col’s id
- • name (char *) – its name. When set to NULL (default), the method returns the current name.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

```
sage: p.add_variable(name="I am a variable")
```

```
sage: p.col_name(0)
'I am a variable'
```
**get_objective_value()**

Return the exact value of the objective function.

**Note:** Behaviour is undefined unless `solve` has been called before.

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver="PPL")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(5/13*x[0] + x[1]/2 == 8/7)
sage: p.set_objective(5/13*x[0] + x[1]/2)
sage: p.solve()
8/7

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2
```

**get_variable_value(variable)**

Return the value of a variable given by the solver.

**Note:** Behaviour is undefined unless `solve` has been called before.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2
```
init_mip()
Converting the matrix form of the MIP Problem to PPL MIP_Problem.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="PPL")
```

is_maximization()
Test whether the problem is a maximization

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

is_variable_binary(index)
Test whether the given variable is of binary type.

INPUT:
- index (integer) – the variable’s id

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

is_variable_continuous(index)
Test whether the given variable is of continuous/real type.

INPUT:
- index (integer) – the variable’s id

EXAMPLES:
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True

is_variable_integer(index)
Test whether the given variable is of integer type.

INPUT:

• index (integer) – the variable’s id

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_integer(0)
False

ncols()
Return the number of columns/variables.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2

nrows()
Return the number of rows/constraints.

EXAMPLES:

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2.0, None)
sage: p.nrows()
2

objective_coefficient(variable, coeff=None)
Set or get the coefficient of a variable in the objective function.

INPUT:

• variable (integer) – the variable’s id
• coeff (integer) – its coefficient

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
gsfage: p = get_solver(solver = "PPL")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0
sage: p.objective_coefficient(0, 2)
2
```

**problem_name**(name=None)

Return or define the problem’s name

INPUT:

• name (str) – the problem’s name. When set to None (default), the method returns the problem’s name.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
gsfage: p = get_solver(solver = "PPL")
sage: p.problem_name("There once was a french fry")
sage: print(p.problem_name())
There once was a french fry
```

**row**(i)

Return a row

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add_linear_constraint method.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
gsfage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row_bounds**(index)

Return the bounds of a specific constraint.

INPUT:

• index (integer) – the constraint’s id.
OUTPUT:

A pair \((\text{lower\_bound}, \text{upper\_bound})\). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
dsage: p = get_solver(solver = "PPL")
dsage: p.add_variables(5)
4
dsage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
dsage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
dsage: p.row_bounds(0)
(2, 2)
```

**row_name(index)**

Return the index th row name

INPUT:

- index (integer) – the row’s id

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
dsage: p = get_solver(solver = "PPL")
dsage: p.add_linear_constraints(1, 2, None, names=["Empty constraint 1"])
dsage: p.row_name(0)
'Empty constraint 1'
```

**set_objective(coeff, d=0)**

Set the objective function.

INPUT:

- coeff – a list of real values, whose ith element is the coefficient of the ith variable in the objective function.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver="PPL")
dsage: x = p.new_variable(nonnegative=True)
dsage: p.add_constraint(x[0]*5 + x[1]/11 <= 6)
dsage: p.set_objective(x[0])
dsage: p.solve()
6/5
sage: p.set_objective(x[0]/2 + 1)
sage: p.show()
Maximization:
  1/2 x_0 + 1
Constraints:
  constraint_0: 5 x_0 + 1/11 x_1 <= 6
Variables:
  x_0 is a continuous variable (min=0, max=+oo)
```

(continues on next page)
x_1 is a continuous variable (min=0, max=+oo)
sage: p.solve()
8/5

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])

\[p.\text{objective\_coefficient}(x) \text{ for } x \text{ in } \text{range}(5)]
\[1, 1, 2, 1, 3\]

**set_sense**(sense)
Set the direction (maximization/minimization).

**INPUT:**

- sense (integer):
  - +1 => Maximization
  - -1 => Minimization

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**set_variable_type**(variable, vtype)
Set the type of a variable.

**INPUT:**

- variable (integer) – the variable’s id
- vtype (integer):
  - 1 Integer
  - 0 Binary
  - -1 Continuous

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.set_variable_type(0,1)
sage: p.is_variable_integer(0)
True
sage: p.set_variable_type(3,0)
```

(continues on next page)
set_verbosity(level)

Set the log (verbosity) level. Not Implemented.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.set_verbosity(0)
```

solve()

Solve the problem.

Note: This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the solver was not able to find it, etc...)

EXAMPLES:

A linear optimization problem:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(list(range(5)), list(range(5)))
sage: p.solve()
0
```

An unbounded problem:

```python
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
  ...
MIPSolverException: ...
```

An integer optimization problem:

```python
sage: p = MixedIntegerLinearProgram(solver='PPL')
sage: x = p.new_variable(integer=True, nonnegative=True)
sage: p.add_constraint(2*x[0] + 3*x[1], max = 6)
sage: p.add_constraint(3*x[0] + 2*x[1], max = 6)
sage: p.set_objective(x[0] + x[1] + 7)
sage: p.solve()
9
```
variable_lower_bound\( (index, value=False) \)

Return or define the lower bound on a variable

INPUT:

- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has not lower bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

```python
go
sage: p.add_variable()
```

```python
sage: p.col_bounds(0)
```

```python
(0, None)
```

```python
sage: p.variable_lower_bound(0, 5)
```

```python
sage: p.col_bounds(0)
```

```python
(5, None)
```

```python
sage: p.variable_lower_bound(0, None)
```

```python
sage: p.col_bounds(0)
```

```python
(None, None)
```


variable_upper_bound\( (index, value=False) \)

Return or define the upper bound on a variable

INPUT:

- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
```

```python
go
sage: p.add_variable()
```

```python
sage: p.col_bounds(0)
```

```python
(0, None)
```

```python
sage: p.variable_upper_bound(0, 5)
```

```python
sage: p.col_bounds(0)
```

```python
(0, 5)
```

```python
sage: p.variable_upper_bound(0, None)
```

```python
sage: p.col_bounds(0)
```

```python
(0, None)
```

zero()
11.7 CVXOPT Backend

AUTHORS:

- Ingolfur Edvardsson (2014-05): initial implementation

class sage.numerical.backends.cvxopt_backend.CVXOPTBackend
    Bases: GenericBackend

MIP Backend that uses the CVXOPT solver.
There is no support for integer variables.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver="CVXOPT")
```

`add_col(indices, coeffs)`

Add a column.

INPUT:

- `indices` (list of integers) – this list contains the indices of the constraints in which the variable’s coefficient is nonzero
- `coeffs` (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the ith entry of coeffs corresponds to the coefficient of the variable in the constraint represented by the ith entry in indices.

**Note:** indices and coeffs are expected to be of the same length.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
```

```
sage: p.add_col(range(5), range(5))
sage: p.nrows()
5
```

`add_linear_constraint(coefficients, lower_bound, upper_bound, name=None)`

Add a linear constraint.

INPUT:

- `coefficients` an iterable with (c, v) pairs where c is a variable index (integer) and v is a value (real value).
- `lower_bound` - a lower bound, either a real value or None
- `upper_bound` - an upper bound, either a real value or None
- `name` - an optional name for this row (default: None)

EXAMPLES:
```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2.00000000000000, 2.00000000000000)
sage: p.add_linear_constraint(zip(range(5), range(5)), 1.0, 1.0, name='foo')
sage: p.row_name(-1)
'foo'
```

**add_variable**(*lower_bound*=0.0, *upper_bound*=None, *binary*=False, *continuous*=True, *integer*=False, *obj*=None, *name*=None)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real. Variable types are always continuous, and thus the parameters `binary`, `integer`, and `continuous` have no effect.

**INPUT:**

- *lower_bound* - the lower bound of the variable (default: 0)
- *upper_bound* - the upper bound of the variable (default: None)
- *binary* - True if the variable is binary (default: False).
- *continuous* - True if the variable is continuous (default: True).
- *integer* - True if the variable is integer (default: False).
- *obj* - (optional) coefficient of this variable in the objective function (default: 0.0)
- *name* - an optional name for the newly added variable (default: None).

**OUTPUT:** The index of the newly created variable

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable()
1
sage: p.add_variable(lower_bound=-2.0)
2
sage: p.add_variable(continuous=True)
3
sage: p.add_variable(name='x', obj=1.0)
4
sage: p.col_name(3)
(continues on next page)
```
### col_bound(index)

Return the bounds of a specific variable.

**INPUT:**

- `index` (integer) – the variable's id.

**OUTPUT:**

A pair `(lower_bound, upper_bound)`. Each of them can be set to `None` if the variable is not bounded in the corresponding direction, and is a real value otherwise.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variable() 0
sage: p.col_bounds(0) (0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0) (0.0, 5)
```

### col_name(index)

Return the index th col name

**INPUT:**

- `index` (integer) – the col's id
- `name` (char *) – its name. When set to NULL (default), the method returns the current name.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variable(name="I am a variable") 0
sage: p.col_name(0) 'I am a variable'
```

### get_objective_value()

Return the value of the objective function.

**Note:** Behaviour is undefined unless `solve` has been called before.

**EXAMPLES:**
```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="cvxopt")
1
sage: p.add_variables(2)

sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)

sage: p.set_objective([2, 5])

sage: p.solve()

0

sage: N(p.get_objective_value(),4)
7.5

sage: N(p.get_variable_value(0),4)
3.6e-7

sage: N(p.get_variable_value(1),4)
1.5
```

**get_variable_value**(variable)

Return the value of a variable given by the solver.

**Note:** Behaviour is undefined unless `solve` has been called before.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")

sage: p.add_variables(2)

sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)

sage: p.set_objective([2, 5])

sage: p.solve()

0

sage: N(p.get_objective_value(),4)
7.5

sage: N(p.get_variable_value(0),4)
3.6e-7

sage: N(p.get_variable_value(1),4)
1.5
```

**is_maximization**()

Test whether the problem is a maximization

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver="CVXOPT")

sage: p.is_maximization()
True

sage: p.set_sense(-1)

sage: p.is_maximization()
False
```

**is_variable_binary**(index)

Test whether the given variable is of binary type. CVXOPT does not allow integer variables, so this is a bit moot.

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INPUT:
- **index** (integer) – the variable’s id

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
0
```

```python
sage: p.add_variable()
0
```

```python
sage: p.set_variable_type(0,0)
Traceback (most recent call last):
... ValueError: ...
```

```python
sage: p.is_variable_binary(0)
False
```

### is_variable_continuous(index)

Test whether the given variable is of continuous/real type. CVXOPT does not allow integer variables, so this is a bit moot.

INPUT:
- **index** (integer) – the variable’s id

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
0
```

```python
sage: p.add_variable()
0
```

```python
sage: p.is_variable_continuous(0)
True
```

```python
sage: p.set_variable_type(0,1)
Traceback (most recent call last):
... ValueError: ...
```

```python
sage: p.is_variable_continuous(0)
True
```

### is_variable_integer(index)

Test whether the given variable is of integer type. CVXOPT does not allow integer variables, so this is a bit moot.

INPUT:
- **index** (integer) – the variable’s id

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
```

(continues on next page)
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,-1)
sage: p.set_variable_type(0,1)
Traceback (most recent call last):
...
ValueError: ...
sage: p.is_variable_integer(0)
False

**ncols()**

Return the number of columns/variables.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2
```

**nrows()**

Return the number of rows/constraints.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.nrows()
0
sage: p.add_variables(5)
4
sage: p.add_linear_constraints(2, 2.0, None)
sage: p.nrows()
2
```

**objective_coefficient**(variable, coeff=None)

Set or get the coefficient of a variable in the objective function

**INPUT:**

- variable (integer) – the variable’s id
- coeff (double) – its coefficient

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variable()
0
```

(continues on next page)
problem_name(name=None)

Return or define the problem’s name

INPUT:

• name (str) – the problem’s name. When set to None (default), the method returns the problem’s name.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.problem_name()
''
sage: p.problem_name("There once was a french fry")
sage: print(p.problem_name())
There once was a french fry
```

row(i)

Return a row

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add_linear_constraint method.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(zip(range(5), range(5))), 2, 2)
```

```
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
```

```
sage: p.row_bounds(0)
(2, 2)
```

row_bounds(index)

Return the bounds of a specific constraint.

INPUT:

• index (integer) – the constraint’s id.

OUTPUT:

A pair (lower_bound, upper_bound). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.
EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(zip(range(5), range(5))), 2, 2)
```

```python
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row_name(index)**

Return the `index` th row name

**INPUT:**

- `index` (integer) – the row’s id

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_linear_constraints(1, 2, None, names=["Empty constraint 1")
```

```python
sage: p.row_name(0)
'Empty constraint 1'
```

**set_objective(coeff, d=0.0)**

Set the objective function.

**INPUT:**

- `coeff` – a list of real values, whose `ith` element is the coefficient of the `ith` variable in the objective function.
- `d` (double) – the constant term in the linear function (set to `0` by default)

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])
```

```python
sage: [p.objective_coefficient(x) for x in range(5)]
[1, 1, 2, 1, 3]
```

**set_sense(sense)**

Set the direction (maximization/minimization).

**INPUT:**

- `sense` (integer):
  - `+1` => Maximization
  - `−1` => Minimization

**EXAMPLES:**
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False

**set_variable_type**(variable, vtype)

Set the type of a variable.

**EXAMPLES:**

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="cvxopt")
sage: p.add_variables(5)
4
sage: p.set_variable_type(3, -1)
sage: p.set_variable_type(3, -2)
Traceback (most recent call last):
...  
ValueError: ...

**set_verbosity**(level)

Does not apply for the cvxopt solver

**solve**()

Solve the problem.

**Note:** This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(solver="cvxopt", maximization=False)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(-4*x[0] - 5*x[1])
sage: p.add_constraint(2*x[0] + x[1] <= 3)
sage: p.add_constraint(2*x[1] + x[0] <= 3)
sage: N(p.solve(), digits=2)
-9.0
```

```python
sage: p = MixedIntegerLinearProgram(solver="cvxopt", maximization=False)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + 2*x[1])
sage: p.add_constraint(-5*x[0] + x[1] <= 7)
sage: p.add_constraint(-5*x[0] + x[1] >= 7)
sage: p.add_constraint(x[0] + x[1] >= 26)
sage: p.add_constraint(x[0] >= 3)
sage: p.add_constraint(x[1] >= 4)
sage: N(p.solve(), digits=4)
48.83
```

(continues on next page)
When the optimal solution is not unique, CVXOPT as an interior point solver gives a different type of solution compared to the solvers that use the simplex method.

In the following example, the top face of the cube is optimal, and CVXOPT gives the center point of the top face, whereas the other tested solvers return a vertex:

```python
sage: c = MixedIntegerLinearProgram(solver="cvxopt")
```

```python
sage: p = MixedIntegerLinearProgram(solver="ppl")
```

```python
g = MixedIntegerLinearProgram()
```

```python
sage: xc = c.new_variable(nonnegative=True)
```

```python
sage: xp = p.new_variable(nonnegative=True)
```

```python
g = MixedIntegerLinearProgram()
```

```python
sage: xg = g.new_variable(nonnegative=True)
```

```python
sage: c.set_objective(xc[2])
```

```python
sage: p.set_objective(xp[2])
```

```python
g.set_objective(xg[2])
```

```python
sage: c.add_constraint(xc[0] <= 100)
```

```python
sage: c.add_constraint(xc[1] <= 100)
```

```python
sage: c.add_constraint(xc[2] <= 100)
```

```python
sage: p.add_constraint(xp[0] <= 100)
```

```python
sage: p.add_constraint(xp[1] <= 100)
```

```python
```

```python
sage: g.add_constraint(xg[0] <= 100)
```

```python
sage: g.add_constraint(xg[1] <= 100)
```

```python
sage: g.add_constraint(xg[2] <= 100)
```

```python
sage: N(c.solve(), digits=4)
```

```python
100.0
```

```python
sage: N(c.get_values(xc[0]), digits=3)
```

```python
50.0
```

```python
sage: N(c.get_values(xc[1]), digits=3)
```

```python
50.0
```

```python
sage: N(c.get_values(xc[2]), digits=4)
```

```python
100.0
```

```python
sage: N(p.solve(), digits=4)
```

```python
100.0
```

```python
sage: N(p.get_values(xp[0]), 2)
```

```python
0.00
```

```python
sage: N(p.get_values(xp[1]), 2)
```

```python
0.00
```

```python
sage: N(p.get_values(xp[2]), digits=4)
```

```python
100.0
```

```python
sage: N(g.solve(), digits=4)
```

(continues on next page)
sage: N(g.get_values(xg[0]), 2)
0.00
sage: N(g.get_values(xg[1]), 2)
0.00
sage: N(g.get_values(xg[2]), digits=4)
100.0

solver_parameter
(name, value=None)
Return or define a solver parameter

INPUT:

* name (string) – the parameter
* value – the parameter’s value if it is to be defined, or None (default) to obtain its current value.

Note: The list of available parameters is available at solver_parameter().

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.solver_parameter("show_progress")
False
sage: p.solver_parameter("show_progress", True)
sage: p.solver_parameter("show_progress")
True
```

variable_lower_bound
(index, value=False)
Return or define the lower bound on a variable

INPUT:

* index (integer) – the variable’s id
* value – real value, or None to mean that the variable has no lower bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
```

```python
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
(5, None)
```

variable_upper_bound
(index, value=False)
Return or define the upper bound on a variable

INPUT:
• index (integer) – the variable’s id
• value – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLES:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5)
```

Sage also supports, via optional packages, CBC (COIN-OR), CPLEX (ILOG), and Gurobi. In order to find out how to use them in Sage, please refer to the Thematic Tutorial on Linear Programming.

The following backend is used for debugging and testing purposes.

## 11.8 Logging Backend

It records, for debugging and unit testing purposes, all calls to backend methods in one of three ways.

See [LoggingBackendFactory](#) for more information.

```python
class sage.numerical.backends.logging_backend.LoggingBackend(backend, printing=True, doctest=None, test_method=None, base_ring=None)
```

Bases: [GenericBackend](#)

See [LoggingBackendFactory](#) for documentation.

EXAMPLES:

```
sage: import sage.numerical.backends.logging_backend
sage: from sage.numerical.backends.logging_backend import LoggingBackend
sage: from sage.numerical.backends.generic_backend import get_solver
sage: b = get_solver(solver = "GLPK")
sage: lb = LoggingBackend(backend=b)
sage: lb.add_variable(obj=42, name='Helloooooo')
# p.add_variable(obj=42, name='Helloooooo')
# result: 0
0
sage: lb.add_variable(obj=1789)
# p.add_variable(obj=1789)
# result: 1
1
```

**add_col(indices, coeffs)**

Add a column.

**INPUT:**
• **indices** (list of integers) – this list contains the indices of the constraints in which the variable’s coefficient is nonzero

• **coeffs** (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the i-th entry of **coeffs** corresponds to the coefficient of the variable in the constraint represented by the i-th entry in **indices**.

**Note:** **indices** and **coeffs** are expected to be of the same length.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
0
sage: p.add_col(list(range(5)), list(range(5)))
5
```

**add_linear_constraint** (*coefficients, lower_bound, upper_bound, name=None*)

Add a linear constraint.

**INPUT:**

• **coefficients** – an iterable of pairs (i, v). In each pair, i is a variable index (integer) and v is a value (element of **base_ring**).

• **lower_bound** – element of **base_ring** or None. The lower bound.

• **upper_bound** – element of **base_ring** or None. The upper bound.

• **name** – string or None. Optional name for this row.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint( zip(range(5), range(5)), 2.0, 2.0)
0
sage: p.row(0)
([0, 1, 2, 3, 4], [0.0, 1.0, 2.0, 3.0, 4.0])
sage: p.row_bounds(0)
(2.0, 2.0)
```

**add_linear_constraint_vector** (*degree, coefficients, lower_bound, upper_bound, name=None*)

Add a vector-valued linear constraint.

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(5, 0, None)
0
sage: p.add_col(list(range(5)), list(range(5)))
5
```

**add_linear_constraint_vector** (*degree, coefficients, lower_bound, upper_bound, name=None*)

Add a vector-valued linear constraint.
Note: This is the generic implementation, which will split the vector-valued constraint into components and add these individually. Backends are encouraged to replace it with their own optimized implementation.

**INPUT:**
- `degree` – integer. The vector degree, that is, the number of new scalar constraints.
- `coefficients` – an iterable of pairs `(i, v)`. In each pair, `i` is a variable index (integer) and `v` is a vector (real and of length `degree`).
- `lower_bound` – either a vector or `None`. The component-wise lower bound.
- `upper_bound` – either a vector or `None`. The component-wise upper bound.
- `name` – string or `None`. An optional name for all new rows.

**EXAMPLES:**
```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
...: coefs = ([0, vector([1, 2])], [1, vector([2, 3])])
...: upper = vector([5, 5])
...: lower = vector([0, 0])
...: p.add_variables(2)
1
...: p.add_linear_constraint_vector(2, coefs, lower, upper, 'foo')
```

### add_linear_constraints(number, lower_bound, upper_bound, names=None)
Add *number* linear constraints.

**INPUT:**
- `number` (integer) – the number of constraints to add.
- `lower_bound` - a lower bound, either a real value or `None`
- `upper_bound` - an upper bound, either a real value or `None`
- `names` - an optional list of names (default: `None`)

**EXAMPLES:**
```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
...: p.add_variables(5)
5
...: p.add_linear_constraints(5, None, 2)
...: p.row(4)
([], [])
...: p.row_bounds(4)
(2.0)
```

### add_variable(*args, **kwdsargs)
Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

**INPUT:**
add_variables(*args, **kwdargs)

Add n variables.

This amounts to adding new columns to the matrix. By default, the variables are both nonnegative and real.

INPUT:

- **n** - the number of new variables (must be > 0)
- **lower_bound** - the lower bound of the variable (default: 0)
- **upper_bound** - the upper bound of the variable (default: None)
- **binary** - True if the variable is binary (default: False).
- **continuous** - True if the variable is continuous (default: True).
- **integer** - True if the variable is integral (default: False).
- **obj** - (optional) coefficient of all variables in the objective function (default: 0.0)
- **name** - an optional name for the newly added variable (default: None).
• names - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```
sage: p.ncols()
0
```

```
sage: p.add_variables(5)
4
```

```
sage: p.ncols()
5
```

```
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a','b'])
6
```

```
base_ring()
```

Return the base ring.

The backend’s base ring can be overridden. It is best to run the tests with GLPK and override the base ring to QQ. Then default input to backend methods, prepared by MixedIntegerLinearProgram, depends on the base ring. This way input will be rational and so suitable for both exact and inexact methods; whereas output will be float and will thus trigger assertAlmostEqual() tests.

EXAMPLES:

```
sage: import sage.numerical.backends.logging_backend
sage: from sage.numerical.backends.logging_backend import LoggingBackend
sage: from sage.numerical.backends.generic_backend import get_solver
```

```
sage: b = get_solver(solver = "GLPK")
```

```
sage: lb = LoggingBackend(backend=b)
```

```
sage: lb.base_ring()
Real Double Field
```

```
sage: from sage.rings.rational_field import QQ
```

```
sage: lb = LoggingBackend(backend=b, base_ring=QQ)
```

```
sage: lb.base_ring()
Rational Field
```

```
best_known_objective_bound()
```

Return the value of the currently best known bound.

This method returns the current best upper (resp. lower) bound on the optimal value of the objective function in a maximization (resp. minimization) problem. It is equal to the output of get_objective_value() if the MILP found an optimal solution, but it can differ if it was interrupted manually or after a time limit (cf solver_parameter()).

**Note:** Has no meaning unless solve has been called before.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
```

```
sage: b = p.new_variable(binary=True)
```

(continues on next page)
sage: for u,v in graphs.CycleGraph(5).edges(labels=False):
....:     p.add_constraint(b[u]+b[v]<=1)

sage: p.set_objective(p.sum(b[x] for x in range(5)))
sage: p.solve()
2.0
sage: pb = p.get_backend()
sage: pb.get_objective_value()
2.0
sage: pb.best_known_objective_bound()
2.0

category()
col_bounds(index)

Return the bounds of a specific variable.

INPUT:

• index (integer) – the variable's id.

OUTPUT:

A pair (lower_bound, upper_bound). Each of them can be set to None if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLES:

sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)

sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)

col_name(index)

Return the index-th column name

INPUT:

• index (integer) – the column id

• name (char *) – its name. When set to NULL (default), the method returns the current name.

EXAMPLES:

sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable(name="I am a variable")
1
sage: p.col_name(0)
'I am a variable'
copy()
Returns a copy of self.

EXAMPLES:

```plaintext
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: b = p.new_variable()
sage: p.set_objective(b[1] + b[2])
sage: copy(p).solve()
6.0
```

dump(filename, compress=True)
Same as self.save(filename, compress)

dumps(compress=True)
Dump self to a string s, which can later be reconstituted as self using loads(s).

There is an optional boolean argument compress which defaults to True.

EXAMPLES:

```plaintext
sage: from sage.misc.persist import comp
sage: O = SageObject()
\sage: p_comp = O.dumps()
sage: p_uncomp = O.dumps(compress=False)
sage: comp.decompress(p_comp) == p_uncomp
True
sage: import pickletools
sage: pickletools.dis(p_uncomp)
0: \x80 PROTO 2
2: c GLOBAL 'sage.structure.sage_object SageObject'
41: q BINPUT ...
43: ) EMPTY_TUPLE
44: \x81 NEWOBJ
45: q BINPUT ...
47: . STOP
highest protocol among opcodes = 2
```

get_custom_name()
Return the custom name of this object, or None if it is not renamed.

EXAMPLES:

```plaintext
sage: P.<x> = QQ[]
sage: P.get_custom_name() is None
True
sage: P.rename('A polynomial ring')
sage: P.get_custom_name()
'A polynomial ring'
sage: P.reset_name()
sage: P.get_custom_name() is None
True
```
get_objective_value()

Return the value of the objective function.

**Note:** Behavior is undefined unless `solve` has been called before.

**EXAMPLES:**

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(2)
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
7.5
sage: p.get_variable_value(0)
0.0
sage: p.get_variable_value(1)
1.5
```

get_relative_objective_gap()

Return the relative objective gap of the best known solution.

For a minimization problem, this value is computed by 
\[(\text{bestinteger} - \text{bestobjective})/(1e - 10 + |\text{bestobjective}|),\]
where `bestinteger` is the value returned by `get_objective_value()` and `bestobjective` is the value returned by `best_known_objective_bound()`. For a maximization problem, the value is computed by 
\[(\text{bestobjective} - \text{bestinteger})/(1e - 10 + |\text{bestobjective}|).\]

**Note:** Has no meaning unless `solve` has been called before.

**EXAMPLES:**

```
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
sage: b = p.new_variable(binary=True)
sage: for u,v in graphs.CycleGraph(5).edges(labels=False):
    p.add_constraint(b[u]+b[v]<=1)
sage: p.set_objective(p.sum(b[x] for x in range(5)))
sage: p.solve()
2.0
sage: pb = p.get_backend()
sage: pb.get_objective_value()
2.0
sage: pb.get_relative_objective_gap()
0.0
```

get_variable_value(variable)

Return the value of a variable given by the solver.
Note: Behavior is undefined unless `solve` has been called before.

**EXAMPLES:**

```sage
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
7.5
sage: p.get_variable_value(0)
0.0
sage: p.get_variable_value(1)
1.5
```

**is_maximization(**`*args`, **`**kwdargs`)**

Test whether the problem is a maximization

**EXAMPLES:**

```sage
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**is_slack_variable_basic(**`*args`, **`**kwdargs`)**

Test whether the slack variable of the given row is basic.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```sage
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(maximization=True, 
....: solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
```

(continues on next page)
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_slack_variable_basic(0)
True
sage: b.is_slack_variable_basic(1)
False

is_slack_variable_nonbasic_at_lower_bound(*args, **kwdargs)
Test whether the given variable is nonbasic at lower bound.
This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise
an exception will be raised.
INPUT:
• index (integer) – the variable’s id
EXAMPLES:

sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(maximization=True,
.....: solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_slack_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_slack_variable_nonbasic_at_lower_bound(1)
True

is_variable_basic(*args, **kwdargs)
Test whether the given variable is basic.
This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise
an exception will be raised.
INPUT:
• index (integer) – the variable’s id
EXAMPLES:
is_variable_binary(*args, **kwdargs)

Test whether the given variable is of binary type.

INPUT:

- index (integer) – the variable's id

EXAMPLES:

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,0)
True
sage: p.is_variable_binary(0)
True
```

is_variable_continuous(*args, **kwdargs)

Test whether the given variable is of continuous/real type.

INPUT:

- index (integer) – the variable's id

EXAMPLES:

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
True
sage: p.is_variable_continuous(0)
True
```

is_variable_integer(*args, **kwdargs)

Test whether the given variable is of integer type.

INPUT:

- index (integer) – the variable's id
EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0, 1)
0
sage: p.is_variable_integer(0)
True
```

```python
is_variable_nonbasic_at_lower_bound(*args, **kwargs)
```

Test whether the given variable is nonbasic at lower bound.

This assumes that the problem has been solved with the simplex method and a basis is available. Otherwise an exception will be raised.

**INPUT:**

- `index` (integer) – the variable’s id

**EXAMPLES:**

```python
sage: p = MixedIntegerLinearProgram(maximization=True,
.....: solver="Nonexistent_LP_solver")
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(-x[0] + x[1] <= 2)
sage: p.add_constraint(8 * x[0] + 2 * x[1] <= 17)
sage: p.set_objective(5.5 * x[0] - 3 * x[1])
sage: b = p.get_backend()
sage: # Backend-specific commands to instruct solver to use simplex method here
sage: b.solve()
0
sage: b.is_variable_nonbasic_at_lower_bound(0)
False
sage: b.is_variable_nonbasic_at_lower_bound(1)
True
```

```python
ncols(*args, **kwargs)
```

Return the number of columns/variables.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2
```
**nrows**: `(*args, **kwdsargs)`

Return the number of rows/constraints.

**Examples:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2.0, None)
sage: p.nrows()
2
```

**objective_coefficient**: `(variable, coeff=None)`

Set or get the coefficient of a variable in the objective function.

**Input:**

- `variable` (integer) – the variable’s id
- `coeff` (double) – its coefficient

**Examples:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0, 2)
0.0
sage: p.objective_coefficient(0)
2.0
```

**objective_constant_term**: `(d=None)`

Set or get the constant term in the objective function.

**Input:**

- `d` (double) – its coefficient. If `None` (default), return the current value.

**Examples:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.objective_constant_term()
0.0
sage: p.objective_constant_term(42)
42.0
sage: p.objective_constant_term()
42.0
```

**parent**

Return the type of `self` to support the coercion framework.

**Examples:**
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```python
sage: t = log(sqrt(2) - 1) + log(sqrt(2) + 1); t
# \needs sage.symbolic
\log(sqrt(2) + 1) + \log(sqrt(2) - 1)
sage: u = t.maxima_methods() # \needs sage.symbolic

\needs sage.symbolic
<sage.symbolic.maxima_wrapper.MaximaWrapper>

\problem_name(name=None)
Return or define the problem’s name

**INPUT:**

- name (str) – the problem’s name. When set to None (default), the method returns the problem’s name.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
\problem: p.problem_name("There once was a french fry") # optional - Nonexistent_LP_solver
\print: p.problem_name() # optional - Nonexistent_LP_solver
```

```
There once was a french fry
```

\remove_constraint(i)
Remove a constraint.

**INPUT:**

- i – index of the constraint to remove.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver="Nonexistent_LP_solver")
\v: p.add_variable(nonnegative=True)
\x, y: v[0], v[1]
\sage: p.add_constraint(2*x + 3*y, max=6)
\sage: p.add_constraint(3*x + 2*y, max=6)
\sage: p.set_objective(x + y + 7)
\sage: p.solver()
\9.0
\sage: p.remove_constraint(0)
\sage: p.solver()
\10.0
\sage: p.get_values([x, y])
[0.0, 3.0]
```

\remove_constraints(constraints)
Remove several constraints.

**INPUT:**

• constraints – an iterable containing the indices of the rows to remove.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0, 2), (1, 3)], None, 6)
sage: p.add_linear_constraint([(0, 3), (1, 2)], None, 6)
sage: p.remove_constraints([0, 1])
```

rename(x=None)
Change self so it prints as x, where x is a string.
If x is None, the existing custom name is removed.

Note: This is only supported for Python classes that derive from SageObject.

EXAMPLES:

```
sage: x = PolynomialRing(QQ, 'x', sparse=True).gen()
sage: g = x^3 + x - 5
sage: g
x^3 + x - 5
sage: g.rename('a polynomial')
sage: g
a polynomial
sage: g + x
x^3 + 2*x - 5
sage: h = g^100
sage: str(h)[:20]
'x^300 + 100*x^298 - '
sage: h.rename('x^300 + ...')
sage: h
x^300 + ...
sage: g.rename(None)
sage: g
x^3 + x - 5
```

Real numbers are not Python classes, so rename is not supported:

```
sage: a = 3.14
sage: type(a)  # needs sage.rings.real_mpfr
<... 'sage.rings.real_mpfr.RealLiteral'>
sage: a.rename('pi')  # needs sage.rings.real_mpfr
Traceback (most recent call last):
  ...
NotImplementedError: object does not support renaming: 3.14000000000000
```
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**Note:** The reason C-extension types are not supported by default is if they were then every single one would have to carry around an extra attribute, which would be slower and waste a lot of memory.

To support them for a specific class, add a `cdef public _SageObject__custom_name` attribute.

```python
reset_name()
```

Remove the custom name of an object.

**EXAMPLES:**

```python
sage: P.<x> = QQ[]
sage: P
Univariate Polynomial Ring in x over Rational Field
sage: P.rename('A polynomial ring')
sage: P
A polynomial ring
sage: P.reset_name()
sage: P
Univariate Polynomial Ring in x over Rational Field
```

```python
row(i)
```

Return a row

**INPUT:**

- `index` (integer) – the constraint’s id.

**OUTPUT:**

A pair `(indices, coeffs)` where `indices` lists the entries whose coefficient is nonzero, and to which `coeffs` associates their coefficient on the model of the `add_linear_constraint` method.

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0]) #FIXME: Why backwards?
sage: p.row_bounds(0)
(2.0, 2.0)
```

```python
row_bounds(index)
```

Return the bounds of a specific constraint.

**INPUT:**

- `index` (integer) – the constraint’s id.

**OUTPUT:**

A pair `(lower_bound, upper_bound)`. Each of them can be set to `None` if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

**EXAMPLES:**

Chapter 11. Linear Optimization (LP) and Mixed Integer Linear Optimization (MIP) Solver backends
```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(list(range(5)), list(range(5)), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0]) ## FIXME: Why backwards?
sage: p.row_bounds(0)
(2.0, 2.0)
```

**row_name(index)**

Return the index th row name

INPUT:
- • index (integer) – the row’s id

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(1, 2, None, names=['Empty constraint 1'])
sage: p.row_name(0)
'Empty constraint 1'
```

**save(filename=None, compress=True)**

Save self to the given filename.

EXAMPLES:

```python
sage: # needs sage.symbolic
sage: x = SR.var("x")
sage: f = x^3 + 5
sage: from tempfile import NamedTemporaryFile
sage: with NamedTemporaryFile(suffix=",sobj") as t:
....: f.save(t.name)
....: load(t.name)
x^3 + 5
```

**set_objective(coeff, d=0.0)**

Set the objective function.

INPUT:
- • coeff – a list of real values, whose i-th element is the coefficient of the i-th variable in the objective function.
- • d (double) – the constant term in the linear function (set to 0 by default)

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

(continues on next page)
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])

sage: [p.objective_coefficient(x) for x in range(5)]
[1.0, 1.0, 2.0, 1.0, 3.0]

Constants in the objective function are respected:

sage: # optional - nonexistent_lp_solver
sage: p = MixedIntegerLinearProgram(solver='Nonexistent_LP_solver')

sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max=6)
sage: p.add_constraint(3*x + 2*y, max=6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0

\textbf{set\_sense}(\textit{sense})

Set the direction (maximization/minimization).

INPUT:

* sense (integer):
  - +1 => Maximization
  - -1 => Minimization

EXAMPLES:

sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver='Nonexistent_LP_solver')

sage: p.is_maximization()
True

sage: p.set_sense(-1)

sage: p.is_maximization()
False

\textbf{set\_variable\_type}(\textit{variable, vtype})

Set the type of a variable

INPUT:

* variable (integer) – the variable’s id
* vtype (integer):
  - 1 Integer
  - 0 Binary
  - −1 Continuous

EXAMPLES:
```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
def get_solver(solver):
    return None
```

```python
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
True
```

```python
set_verbosity(level)
Set the log (verbosity) level

INPUT:

- level (integer) – From 0 (no verbosity) to 3.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.set_verbosity(2)
```

```python
solve(*args, **kwds)
Solve the problem.

**Note:** This method raises MIPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.objective_coefficient(0,1)
0
```

```python
sage: p.solve()
Traceback (most recent call last):
  ... MIPSolverException: ...
```

```python
solver_parameter(name, value=None)
Return or define a solver parameter

INPUT:

- name (string) – the parameter
- value – the parameter’s value if it is to be defined, or None (default) to obtain its current value.
```
Note: The list of available parameters is available at `solver_parameter()`.

EXAMPLES:

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.solver_parameter("timelimit")
sage: p.solver_parameter("timelimit", 60)
sage: p.solver_parameter("timelimit")
```

`variable_lower_bound(index, value=False)`
Return or define the lower bound on a variable

INPUT:
- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has not lower bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5.0, None)
```

`variable_upper_bound(index, value=False)`
Return or define the upper bound on a variable

INPUT:
- index (integer) – the variable’s id
- value – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLES:

```python
sage: # optional - nonexistent_LP_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)
```
**write_lp(name)**

Write the problem to a .lp file

INPUT:

- filename (string)

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.add_variables(2)
```

```python
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
```

```python
sage: p.set_objective([2, 5])
```

```python
sage: from tempfile import NamedTemporaryFile
sage: with NamedTemporaryFile(suffix=":.lp") as f:
    ....:     p.write_lp(f.name)
```

**write_mps(name, modern)**

Write the problem to a .mps file

INPUT:

- filename (string)

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.add_variables(2)
```

```python
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
```

```python
sage: p.set_objective([2, 5])
```

```python
sage: from tempfile import NamedTemporaryFile
sage: with NamedTemporaryFile(suffix=":.lp") as f:
    ....:     p.write_lp(f.name)
```

Factory that constructs a LoggingBackend for debugging and testing.

An instance of it can be passed as the solver argument of sage.numerical.backends.generic_backend.get_solver() and MixedIntegerLinearProgram.

EXAMPLES:

Assume that we have the following function that does some computation using MixedIntegerLinearProgram (or MIP backend methods), and suppose we have observed that it works with the GLPK backend, but not with the COIN backend:
sage: def compute_something(solver='GLPK'):
    ....:     from sage.numerical.mip import MIPSolverException
    ....:     mip = MixedIntegerLinearProgram(solver=solver)
    ....:     lb = mip.get_backend()
    ....:     lb.add_variable(obj=42, name='Helloooooo')
    ....:     lb.add_variable(obj=1789)
    ....:     try:
    ....:         lb.solve()
    ....:     except MIPSolverException:
    ....:         return 4711
    ....:     else:
    ....:         return 91

We can investigate what the backend methods are doing by running a `LoggingBackend` in its in-terminal logging mode:

sage: import sage.numerical.backends.logging_backend
sage: from sage.numerical.backends.logging_backend import LoggingBackendFactory
sage: compute_something(solver = LoggingBackendFactory(solver='GLPK'))
# p = get_solver(solver='GLPK')
# p.add_variable(obj=42, name='Helloooooo')
# result: 0
# p.add_variable(obj=1789)
# result: 1
# p.solve()
# exception: GLPK: The LP (relaxation) problem has no dual feasible solution
4711

By replacing ‘GLPK’ by ‘COIN’ above, we can then compare the two logs and see where they differ.

Imagine that we have now fixed the bug in the COIN backend, and we want to add a doctest that documents this fact. We do not want to call `compute_something` in the doctest, but rather just have a sequence of calls to backend methods.

We can have the doctest autogenerated by running a `LoggingBackend` in its doctest-writing mode:

sage: fname = tmp_filename()
sage: compute_something(solver = LoggingBackendFactory(solver='GLPK',
    ....:     printing=False,
    ....:     doctest_file=fname))
4711
sage: with open(fname) as f:
    ....:     for line in f.readlines(): _ = sys.stdout.write('|{}|'.format(line))
    | sage: p = get_solver(solver='GLPK')
    | sage: p.add_variable(obj=42, name='Helloooooo')
    | 0
    | sage: p.add_variable(obj=1789)
    | 1
    | sage: p.solve()
    | Traceback (most recent call last):
    | ...
    | MIPSolverException: GLPK: The LP (relaxation) problem has no dual feasible solution

We then copy from the generated file and paste into the source code of the COIN backend.
If this test seems valuable enough that all backends should be tested against it, we should create a test method instead of a docstring.

We can have the test method autogenerated by running a `LoggingBackend` in its test-method-writing mode:

```python
sage: fname = tmp_filename()
sage: compute_something(solver=LoggingBackendFactory(solver='GLPK', printing=False,
....: test_method_file=fname,
....: test_method='something'))
4711
sage: with open(fname) as f:
....:     for line in f.readlines(): _ = sys.stdout.write('|{}|'.format(line))
....:
    @classmethod
    def _test_something(cls, tester=None, **options):
        ...
        Run tests on ...
        TESTS:
        sage: from sage.numerical.backends.generic_backend import GenericBackend
        sage: p = GenericBackend()
        sage: p._test_something()
        Traceback (most recent call last):
        ...
        NotImplementedError
        ...
        p = cls()  # fresh instance of the backend
        if tester is None:
            tester = p._tester(**options)
        tester.assertEqual(p.add_variable(obj=42, name='Helloooooo'), 0)
        tester.assertEqual(p.add_variable(obj=1789), 1)
        with tester.assertRaises(MIPSolverException) as cm:
            p.solve()
```

We then copy from the generated file and paste into the source code of the generic backend, where all test methods are defined.

If `test_method_file` is not provided, a default output file name will be computed from `test_method`.  

11.8. Logging Backend
Chapter 11. Linear Optimization (LP) and Mixed Integer Linear Optimization (MIP) Solver backends
SEMIDEFINITE OPTIMIZATION (SDP) SOLVER BACKENDS

12.1 Generic Backend for SDP solvers

This class only lists the methods that should be defined by any interface with a SDP Solver. All these methods immediately raise NotImplemented exceptions when called, and are obviously meant to be replaced by the solver-specific method. This file can also be used as a template to create a new interface: one would only need to replace the occurrences of "Nonexistent SDP_solver" by the solver's name, and replace GenericSDPBackend by SolverName(GenericSDPBackend) so that the new solver extends this class.

AUTHORS:
• Ingolfur Edvardsson (2014-07): initial implementation

class sage.numerical.backends.generic_sdp_backend.GenericSDPBackend
    Bases: object

    add_linear_constraint(coefficients, name=None)

    Add a linear constraint.

    INPUT:
    • coefficients an iterable with (c, v) pairs where c is a variable index (integer) and v is a value (real value).
    • lower_bound - a lower bound, either a real value or None
    • upper_bound - an upper bound, either a real value or None
    • name - an optional name for this row (default: None)

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0)  # optional - Nonexistent_LP_solver
sage: p.row(0)  # optional - Nonexistent_LP_solver
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)  # optional - Nonexistent_LP_solver
(2.0, 2.0)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1.0, 1.0, name='foo')
```

(continues on next page)
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(continued from previous page)

```
sage: p.row_name(-1)
"foo"
```

```
add_linear_constraints(number, names=None)

Add constraints.

**INPUT:**

- `number` (integer) – the number of constraints to add.
- `lower_bound` - a lower bound, either a real value or None
- `upper_bound` - an upper bound, either a real value or None
- `names` - an optional list of names (default: None)

**EXAMPLES:**

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
5
sage: p.add_linear_constraints(5, None, 2)
sage: p.row(4)
([], [])
sage: p.row_bounds(4)
(None, 2.0)
```

```
add_variable(obj=0.0, name=None)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

**INPUT:**

- `obj` - (optional) coefficient of this variable in the objective function (default: 0.0)
- `name` - an optional name for the newly added variable (default: None).

**OUTPUT:** The index of the newly created variable

**EXAMPLES:**

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable(name='x', obj=1.0)
3
sage: p.col_name(3)
'x'
```

(continues on next page)
add_variables(n, names=None)

Add \( n \) variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive and real.

INPUT:

- \( n \) - the number of new variables (must be \( > 0 \))
- \( \text{obj} \) - (optional) coefficient of all variables in the objective function (default: 0.0)
- \( \text{names} \) - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.ncols()
0
sage: p.add_variables(5)
4
sage: p.ncols()
5
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a', 'b'])
6
```

base_ring()

The base ring

col_name(index)

Return the \( \text{index} \) th col name

INPUT:

- \( \text{index} \) (integer) – the col’s id
- \( \text{name} \) (char *) – its name. When set to NULL (default), the method returns the current name.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable(name="I am a variable")
1
sage: p.col_name(0)
'I am a variable'
```

dual_variable(i, sparse=False)

The \( i \)-th dual variable

Available after self.solve() is called, otherwise the result is undefined
• index (integer) – the constraint’s id.

OUTPUT:

The matrix of the \( i \)-th dual variable

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: p = SemidefiniteProgram(maximization=False, solver="Nonexistent_LP_solver")
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
```

```python
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.solve()

-3.0
```

```python
sage: B = p.get_backend()
sage: x = p.get_values(x).values()
```

```python
-(a3*B.dual_variable(0)).trace()-(b3*B.dual_variable(1)).trace()

-3.0
```

```python
sage: g = sum((B.slack(j)*B.dual_variable(j)).trace() for j in range(2)); g

0.0
```

\textbf{get_objective_value()}

Return the value of the objective function.

\textbf{Note:} Behaviour is undefined unless \texttt{solve} has been called before.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
```

```python
sage: p.add_variables(2)

2
```

```python
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
```

```python
sage: p.solve()

0
```

```python
sage: p.get_objective_value()

7.5
```

```python
sage: p.get_variable_value(0)

0.0
```

```python
sage: p.get_variable_value(1)

1.5
```

\textbf{get_variable_value(\texttt{variable})}

Return the value of a variable given by the solver.
Note: Behaviour is undefined unless solve has been called before.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
2
sage: p.add_linear_constraint([[0, 1], (1, 2)], None, 3)
7.5
sage: p.set_objective([2, 5])
0
sage: p.solve()
0
sage: p.get_objective_value()
7.5
sage: p.get_variable_value(0)
0.0
sage: p.get_variable_value(1)
1.5
```

`is_maximization()`

Test whether the problem is a maximization

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
True
sage: p.set_sense(-1)
False
```

`ncols()`

Return the number of columns/variables.

EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
2
sage: p.add_variables(2)
2
sage: p.ncols()
2
```

`nrows()`

Return the number of rows/constraints.

EXAMPLES:
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2.0, None)
2

**objective_coefficient**(variable, coeff=None)

Set or get the coefficient of a variable in the objective function

INPUT:

- **variable** (integer) – the variable’s id
- **coeff** (double) – its coefficient

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variable()
1
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0,2)

```

**problem_name**(name=None)

Return or define the problem’s name

INPUT:

- **name** (str) – the problem’s name. When set to NULL (default), the method returns the problem’s name.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.problem_name("There once was a french fry")
sage: print(p.problem_name())
There once was a french fry

```

**row**(i)

Return a row

INPUT:

- **index** (integer) – the constraint’s id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add_linear_constraint method.
EXAMPLES:

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
5
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
```

```python
sage: p.row(0)
(\[4, 3, 2, 1\], \[4.0, 3.0, 2.0, 1.0\])
sage: p.row_bounds(0)
(2.0, 2.0)
```

**row_name(index)**

Return the index-th row name

**INPUT:**

- index (integer) – the row’s id

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(1, 2, None, name="Empty constraint 1")
sage: p.row_name(0)
'Empty constraint 1'
```

**set_objective(coeff, d=0.0)**

Set the objective function.

**INPUT:**

- coeff – a list of real values, whose ith element is the coefficient of the ith variable in the objective function.
- d (double) – the constant term in the linear function (set to 0 by default)

**EXAMPLES:**

```python
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_variables(5)
5
sage: p.set_objective([1, 1, 2, 1, 3])
sage: [p.objective_coefficient(x) for x in range(5)]
[1.0, 1.0, 2.0, 1.0, 3.0]
```

Constants in the objective function are respected.

**set_sense(sense)**

Set the direction (maximization/minimization).

**INPUT:**

- sense (integer):
- +1 => Maximization
- −1 => Minimization

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
```

```
sage: p.is_maximization()
False
```

```
sage: slack(i, sparse=False)
Slack of the 𝑖-th constraint
Available after self.solve() is called, otherwise the result is undefined
• index (integer) – the constraint’s id.

OUTPUT:
The matrix of the slack of the 𝑖-th constraint

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: p = SemidefiniteProgram(maximization=False, solver="Nonexistent_LP_solver")
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.solve()
-3.0
sage: B = p.get_backend()
sage: B1 = B.slack(1); B1
[0.0 0.0]
[0.0 0.0]
sage: B1.is_positive_definite()
True
sage: x = p.get_values(x).values()
sage: x[0]*b1 + x[1]*b2 - b3 + B1
[0.0 0.0]
[0.0 0.0]
```

```
solve()
Solve the problem.
```
Note: This method raises \texttt{SDPSolverException} exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.solve()
0
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
 ... 
SDPSolverException: ...
```

\texttt{solver PARAMETER}(name, value=None)

Return or define a solver parameter

INPUT:

- name (string) – the parameter
- value – the parameter's value if it is to be defined, or \texttt{None} (default) to obtain its current value.

Note: The list of available parameters is available at \texttt{solver PARAMETER()}.

EXAMPLES:

```
sage: # optional - nonexistent_lp_solver
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="Nonexistent_LP_solver")
sage: p.solver parameter("timelimit")
sage: p.solver parameter("timelimit", 60)
sage: p.solver parameter("timelimit")
```

\texttt{zero}()

Zero of the base ring

\texttt{sage.numerical.backends.generic_sdp backend.default_sdp solver}(solver=None)

Return/set the default SDP solver used by Sage

INPUT:

- solver – one of the following:
  - the string “\texttt{CVXOPT}”, to make the use of the CVXOPT solver (see the \texttt{CVXOPT} web site) the default;
  - a subclass of \texttt{sage.numerical.backends.generic_sdp_backend.GenericSDPBackend}, to make it the default; or
  - \texttt{None} (default), in which case the current default solver (a string or a class) is returned.
OUTPUT:

This function returns the current default solver (a string or a class) if solver = None (default). Otherwise, it sets the default solver to the one given. If this solver does not exist, or is not available, a ValueError exception is raised.

EXAMPLES:

```python
sage: former_solver = default_sdp_solver()
sage: default_sdp_solver("Cvxopt")
sage: default_sdp_solver()
'Cvxopt'
sage: default_sdp_solver("Yeahhhhhhhhhhh")
Traceback (most recent call last):
... ValueError: 'solver' should be set to ...
sage: default_sdp_solver(former_solver)
sage: from sage.numerical.backends.generic_sdp_backend import GenericSDPBackend
sage: class my_sdp_solver(GenericSDPBackend): pass
sage: default_sdp_solver(my_sdp_solver)
sage: default_sdp_solver() is my_sdp_solver
True
```

`sage.numerical.backends.generic_sdp_backend.get_solver(solver=None, base_ring=None)`

Return a solver according to the given preferences.

INPUT:

- solver – one of the following:
  - the string "CVXOPT", designating the use of the CVXOPT solver (see the CVXOPT web site);
  - a subclass of `sage.numerical.backends.generic_sdp_backend.GenericSDPBackend`;
  - None (default), in which case the default solver is used (see `default_sdp_solver()`);

See also:

- `default_sdp_solver()` – Returns/sets the default SDP solver.

EXAMPLES:

```python
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver()
```

Passing a class:

```python
sage: from sage.numerical.backends.generic_sdp_backend import GenericSDPBackend
sage: class MockSDPBackend(GenericSDPBackend):
...: def solve(self):
...: raise RuntimeError("SDP is too slow")
```

```python
sage: P = SemidefiniteProgram(solver=MockSDPBackend)
sage: P.solve()
Traceback (most recent call last):
... RuntimeError: SDP is too slow
```
12.2 CVXOPT SDP Backend

AUTHORS:

- Ingolfur Edvardsson (2014-05): initial implementation
- Dima Pasechnik (2015-12): minor fixes

class sage.numerical.backends.cvxopt_sdp_backend.CVXOPTSDPBackend

Bases: MatrixSDPBackend
Cython constructor

EXAMPLES:

```
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver='CVXOPT')
```

dual_variable(i, sparse=False)

The i-th dual variable

Available after self.solve() is called, otherwise the result is undefined

- index (integer) – the constraint’s id.

OUTPUT:

The matrix of the i-th dual variable

EXAMPLES:

```
sage: p = SemidefiniteProgram(maximization=False, solver='cvxopt')
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.solve() # tol 1e-08
-3.0

sage: B = p.get_backend()
sage: x = p.get_values(x).values()
sage: -(a3*B.dual_variable(0)).trace() - (b3*B.dual_variable(1)).trace() # tol 1e-07
-3.0
```

get_objective_value()

Return the value of the objective function.

Note: Behaviour is undefined unless solve has been called before.
EXAMPLES:

```python
sage: p = SemidefiniteProgram(solver="cvxopt", maximization=False)
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1] + x[2])
sage: a1 = matrix([[-7., -11.], [-11., 3.]])
sage: a2 = matrix([[7., -18.], [-18., 8.]])
sage: a3 = matrix([[-2., -8.], [-8., 1.]])
sage: a4 = matrix([[33., -9.], [-9., 26.]])
sage: b1 = matrix([[[-21., -11., -1.], [-11., 10., -10.], [0., 8., 5.]]])
sage: b2 = matrix([[0., -10., -10.], [10., -10., -10.], [16., -10., -3.]])
sage: b3 = matrix([[[-5., 2., -17.], [2., -6., 8.], [-17., 8., 6.]]])
sage: p.add_constraint(a1*x[0] + a2*x[1] + a3*x[2] <= a4)
```

```python
sage: N(p.solve(), digits=4)
-3.154
```
• index (integer) – the constraint’s id.

OUTPUT:
The matrix of the slack of the $i$-th constraint

EXAMPLES:

```
sage: p = SemidefiniteProgram(maximization = False, solver='cvxopt')
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1])
sage: a1 = matrix([[1, 2.], [2., 3.]])
sage: a2 = matrix([[3, 4.], [4., 5.]])
sage: a3 = matrix([[5, 6.], [6., 7.]])
sage: b1 = matrix([[1, 1.], [1., 1.]])
sage: b2 = matrix([[2, 2.], [2., 2.]])
sage: b3 = matrix([[3, 3.], [3., 3.]])
sage: p.add_constraint(a1*x[0] + a2*x[1] <= a3)
sage: p.add_constraint(b1*x[0] + b2*x[1] <= b3)
sage: p.solve()  # tol 1e-08
-3.0
```

```
sage: B = p.get_backend()
sage: B1 = B.slack(1); B1  # tol 1e-08
[0.0 0.0]
[0.0 0.0]
sage: B1.is_positive_definite()
True
```

```
sage: x = sorted(p.get_values(x).values())
sage: x[0]*b1 + x[1]*b2 - b3 + B1  # tol 1e-09
[0.0 0.0]
[0.0 0.0]
```

```
solve()
Solve the problem.

Note: This method raises SDPSolverException exceptions when the solution cannot be computed for any reason (none exists, or the LP solver was not able to find it, etc…)
```

EXAMPLES:

```
sage: p = SemidefiniteProgram(solver="cvxopt", maximization=False)
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1] + x[2])
sage: a1 = matrix([-7., -11.], [-11., 3.])
sage: a2 = matrix([7., -18.], [-18., 8.])
sage: a3 = matrix([-2., -8.], [-8., 1.])
sage: a4 = matrix([33., -9.], [-9., 26.])
sage: b1 = matrix([-21., -11., 0.], [-11., 10., 8.], [0., 8., 5.])
sage: b2 = matrix([0., 10., 16.], [10., -10., -10.], [16., -10., 3.])
sage: b3 = matrix([-5., 2., -17.], [2., -6., 8.], [-17., 8., 6.])
sage: p.add_constraint(a1*x[0] + a3*x[2] <= a4)
sage: N(p.solve(), digits=4)
(continues on next page)
```

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-3.225

```python
sage: p = SemidefiniteProgram(solver="cvxopt", maximization=False)
sage: x = p.new_variable()
sage: p.set_objective(x[0] - x[1] + x[2])
sage: a1 = matrix([[7., -11.], [-11., 3.]])
sage: a2 = matrix([[8., -18.], [-18., 8.]])
sage: a3 = matrix([[-7., -11.], [-11., 3.]])
sage: b1 = matrix([[-21., -11., 0.], [-11., 10., 8.], [0., 8., 5.]]
```

```python
sage: a4 = matrix([[33., -9.], [-9., 26.]])
sage: b2 = matrix([[0., 10., 16.], [10., -10., -10.], [16., -10., 3.]])
sage: b3 = matrix([[-5., 2., -17.], [2., -6., 8.], [-17., 8., 6.]])
```

```python
sage: p.add_constraint(a1*x[0] + a2*x[1] + a3*x[2] <= a4)
sage: N(p.solve(), digits=4)
-3.154
```

### solver_parameter(name, value=None)

Return or define a solver parameter

**INPUT:**

- name (string) – the parameter
- value – the parameter’s value if it is to be defined, or None (default) to obtain its current value.

**Note:** The list of available parameters is available at `solver_parameter()`.

**EXAMPLES:**

```python
sage: from sage.numerical.backends.generic_sdp_backend import get_solver
sage: p = get_solver(solver="CVXOPT")
sage: p.solver_parameter("show_progress")
False
sage: p.solver_parameter("show_progress", True)
True
```

For more details on CVXOPT, see [CVXOPT documentation](http://cvxopt.org/).
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