# CONTENTS

## 1 General
- 1.1 2D Plotting ......................................................... 1
- 1.2 Text in plots ......................................................... 141
- 1.3 Colors ............................................................ 154
- 1.4 Animated plots ....................................................... 165

## 2 Function and Data Plots
- 2.1 Complex Plots ....................................................... 177
- 2.2 Contour Plots ......................................................... 185
- 2.3 Density Plots ......................................................... 254
- 2.4 Plotting fields ....................................................... 261
- 2.5 Streamline Plots .................................................... 271
- 2.6 Scatter Plots ......................................................... 281
- 2.7 Step function plots ............................................... 282
- 2.8 Histograms .......................................................... 282
- 2.9 Bar Charts ........................................................... 285

## 3 Plots of Other Mathematical Objects
- 3.1 Graph Plotting ....................................................... 287
- 3.2 Matrix Plots .......................................................... 321

## 4 Basic Shapes
- 4.1 Arcs of circles and ellipses ....................................... 329
- 4.2 Arrows ................................................................. 331
- 4.3 Bezier Paths .......................................................... 343
- 4.4 Circles ................................................................. 352
- 4.5 Disks ................................................................. 364
- 4.6 Ellipses ............................................................... 366
- 4.7 Line Plots .............................................................. 373
- 4.8 Points ................................................................. 378
- 4.9 Polygons .............................................................. 383
- 4.10 Arcs in hyperbolic geometry .................................... 400
- 4.11 Polygons and triangles in hyperbolic geometry ............. 401
- 4.12 Regular polygons in the upper half model for hyperbolic plane ......................................................... 406

## 5 Infrastructure and Low-Level Functions
- 5.1 Graphics objects .................................................... 411
- 5.2 Graphics arrays and insets ....................................... 438
- 5.3 Plotting primitives .................................................. 458
- 5.4 Plotting utilities ..................................................... 460
6 Indices and Tables 463

Python Module Index 465

Index 467
CHAPTER ONE

GENERAL

1.1 2D Plotting

Sage provides extensive 2D plotting functionality. The underlying rendering is done using the matplotlib Python library.

The following graphics primitives are supported:

- `arrow()` - an arrow from a min point to a max point.
- `circle()` - a circle with given radius
- `ellipse()` - an ellipse with given radii and angle
- `arc()` - an arc of a circle or an ellipse
- `disk()` - a filled disk (i.e. a sector or wedge of a circle)
- `line()` - a line determined by a sequence of points (this need not be straight!)
- `point()` - a point
- `text()` - some text
- `polygon()` - a filled polygon

The following plotting functions are supported:

- `plot()` - plot of a function or other Sage object (e.g., elliptic curve).
- `parametric_plot()`
- `implicit_plot()`
- `polar_plot()`
- `region_plot()`
- `list_plot()`
- `scatter_plot()`
- `bar_chart()`
- `contour_plot()`
- `density_plot()`
- `plot_vector_field()`
- `plot_slope_field()`
- `matrix_plot()`
- `complex_plot()`
• `graphics_array()`
• `multi_graphics()`
• The following log plotting functions:
  – `plot_loglog()`
  – `plot_semilogx()` and `plot_semilogy()`
  – `list_plot_loglog()`
  – `list_plot_semilogx()` and `list_plot_semilogy()`

The following miscellaneous Graphics functions are included:
• `Graphics()`
• `is_Graphics()`
• `hue()`

Type `?` after each primitive in Sage for help and examples.

EXAMPLES:

We draw a curve:

```
sage: plot(x^2, (x,0,5))
```
Graphics object consisting of 1 graphics primitive
We draw a circle and a curve:

```
sage: circle((1,1), 1) + plot(x^2, (x,0,5))
Graphics object consisting of 2 graphics primitives
```

Notice that the aspect ratio of the above plot makes the plot very tall because the plot adopts the default aspect ratio of the circle (to make the circle appear like a circle). We can change the aspect ratio to be what we normally expect for a plot by explicitly asking for an 'automatic' aspect ratio:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), aspect_ratio='automatic')
```

The aspect ratio describes the apparently height/width ratio of a unit square. If you want the vertical units to be twice as big as the horizontal units, specify an aspect ratio of 2:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), aspect_ratio=2)
```

The `figsize` option adjusts the figure size. The default figsize is 4. To make a figure that is roughly twice as big, use `figsize=8`:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), figsize=8)
```

You can also give separate horizontal and vertical dimensions. Both will be measured in inches:
However, do not make the figsize too big (e.g. one dimension greater than 327 or both in the mid-200s) as this will lead to errors or crashes. See `show()` for full details.

Note that the axes will not cross if the data is not on both sides of both axes, even if it is quite close:

```sage
sage: plot(x^3, (x,1,10))
Graphics object consisting of 1 graphics primitive
```

When the labels have quite different orders of magnitude or are very large, scientific notation (the $e$ notation for powers of ten) is used:

```sage
sage: plot(x^2, (x,480,500))  # no scientific notation
Graphics object consisting of 1 graphics primitive
```

```sage
sage: plot(x^2, (x,300,500))  # scientific notation on y-axis
Graphics object consisting of 1 graphics primitive
```

But you can fix your own tick labels, if you know what to expect and have a preference:

```sage
sage: plot(x^2, (x,300,500), ticks=[100,50000])
Graphics object consisting of 1 graphics primitive
```
1.1. 2D Plotting
1.1. 2D Plotting
To change the ticks on one axis only, use the following notation:

```
sage: plot(x^2, (x,300,500), ticks=[None,50000])
```

You can even have custom tick labels along with custom positioning.

```
sage: plot(x^2, (x,0,3), ticks=[[1,2.5],pi/2], tick_formatter=[["x_1","x_2"],pi])
```

We construct a plot involving several graphics objects:

```
sage: G = plot(cos(x), (x, -5, 5), thickness=5, color='green', title='A plot')
sage: P = polygon([[1,2], [5,6], [5,0]], color='red')
sage: G + P
```

Next we construct the reflection of the above polygon about the \( y \)-axis by iterating over the list of first-coordinates of the first graphic element of \( P \) (which is the actual Polygon; note that \( P \) is a Graphics object, which consists of a single polygon):

```
sage: Q = polygon([(-x,y) for x,y in P[0]], color='blue')
sage: Q  # show it
```
We combine together different graphics objects using “+”:

```
sage: H = G + P + Q
sage: print(H)
Graphics object consisting of 3 graphics primitives
sage: type(H)
<class 'sage.plot.graphics.Graphics'>
sage: H[1]
Polygon defined by 3 points
sage: list(H[1])
[(1.0, 2.0), (5.0, 6.0), (5.0, 0.0)]
sage: H  # show it
Graphics object consisting of 3 graphics primitives
```

We can put text in a graph:

```
sage: L = [[cos(pi*i/100)^3,sin(pi*i/100)] for i in range(200)]
sage: p = line(L, rgbcolor=(1/4,1/8,3/4))
sage: tt = text('A Bulb', (1.5, 0.25))
sage: tx = text('x axis', (1.5,-0.2))
```

(continues on next page)
sage: ty = text('y axis', (0.4,0.9))
sage: g = p + tt + tx + ty
sage: g.show(xmin=-1.5, xmax=2, ymin=-1, ymax=1)

We can add a graphics object to another one as an inset:

sage: g1 = plot(x^2*sin(1/x), (x, -2, 2), axes_labels=['$x$', '$y$'])
sage: g2 = plot(x^2*sin(1/x), (x, -0.3, 0.3), axes_labels=['$x$', '$y$'],
....: frame=True)
sage: g1.inset(g2, pos=(0.15, 0.7, 0.25, 0.25))
Multiographics with 2 elements

We can add a title to a graph:

sage: plot(x^2, (x,-2,2), title='A plot of $x^2$')
Graphics object consisting of 1 graphics primitive

We can set the position of the title:

sage: plot(x^2, (-2,2), title='Plot of $x^2$', title_pos=(0.5,-0.05))
Graphics object consisting of 1 graphics primitive

We plot the Riemann zeta function along the critical line and see the first few zeros:
A plot of $x^2$
Plot of $x^2$
sage: i = CDF.0  # define i this way for maximum speed.
sage: p1 = plot(lambda t: arg(zeta(0.5+t*i)), 1, 27, rgbcolor=(0.8,0,0))
sage: p2 = plot(lambda t: abs(zeta(0.5+t*i)), 1, 27, color=hue(0.7))
sage: print(p1 + p2)  # display it
Graphics object consisting of 2 graphics primitives

Note: Not all functions in Sage are symbolic. When plotting non-symbolic functions they should be wrapped in lambda:

sage: plot(lambda x: fibonacci(round(x)), (x,1,10))
Graphics object consisting of 1 graphics primitive

Many concentric circles shrinking toward the origin:

sage: show(sum(circle((i,0), i, hue=sin(i/10)) for i in [10,9,9,...,0])) # long time

Here is a pretty graph:
```python
sage: g = Graphics()
sage: for i in range(60):
    ....:     p = polygon([[i*cos(i),i*sin(i)], (0,i), (i,0)],
    ....:                       color=hue(i/40+0.4), alpha=0.2)
    ....:     g = g + p
sage: g.show(dpi=200, axes=False)
```

Another graph:

```python
sage: x = var('x')
sage: P = plot(sin(x)/x, -4, 4, color='blue') + 
    ....:     plot(x*cos(x), -4, 4, color='red') + 
    ....:     plot(tan(x), -4, 4, color='green')
sage: P.show(ymin=-pi, ymax=pi)
```

PYX EXAMPLES: These are some examples of plots similar to some of the plots in the PyX (http://pyx.sourceforge.net) documentation:

Symbolline:

```python
sage: y(x) = x*sin(x^2)
sage: v = [(x, y(x)) for x in [-3,-2.95,..,3]]
sage: show(points(v, rgbcolor=(0.2,0.6, 0.1), pointsize=30) + plot(spline(v), -3.1, 3))
```
1.1. 2D Plotting
Cycliclink:

```sage
sage: g1 = plot(cos(20*x)*exp(-2*x), 0, 1)
sage: g2 = plot(2*exp(-30*x) - exp(-3*x), 0, 1)
sage: show(graphics_array([g1, g2], 2, 1))
```

Pi Axis:

```sage
sage: g1 = plot(sin(x), 0, 2*pi)
sage: g2 = plot(cos(x), 0, 2*pi, linestyle="--")
sage: (g1+g2).show(ticks=pi/6, tick_formatter=pi)  # long time # show their sum, nicely formatted
```

An illustration of integration:

```sage
sage: f(x) = (x-3)*(x-5)*(x-7)+40
sage: P = line([(2,0),(2,f(2))], color='black')
sage: P += line([(8,0),(8,f(8))], color='black')
sage: P += polygon([(2,0),(2,f(2)) + [x, f(x)] for x in [2,2.1,...,8]] + [(8,0),(2,0)],
               rgbcolor=(0.8,0.8,0.8),aspect_ratio='automatic')
sage: P += text(r"\int_{a}^{b} f(x) dx", (5, 20), fontsize=16, color='black')
sage: P += plot(f, (1, 8.5), thickness=3)
sage: P  # show the result
```

Graphics object consisting of 5 graphics primitives
1.1. 2D Plotting

$$\int_a^b f(x) \, dx$$
NUMERICAL PLOTTING:

Sage includes Matplotlib, which provides 2D plotting with an interface that is a likely very familiar to people doing numerical computation. You can use `plt.clf()` to clear the current image frame and `plt.close()` to close it. For example,

```
sage: import pylab as plt
sage: t = plt.arange(0.0, 2.0, 0.01)
sage: s = sin(2*pi*t)
sage: P = plt.plot(t, s, linewidth=1.0)
sage: xl = plt.xlabel('time (s)')
sage: yl = plt.ylabel('voltage (mV)')
sage: t = plt.title('About as simple as it gets, folks')
sage: plt.grid(True)
sage: plt.savefig(os.path.join(SAGE_TMP, 'sage.png'))
sage: plt.clf()
sage: plt.savefig(os.path.join(SAGE_TMP, 'blank.png'))
sage: plt.close()
sage: plt.imshow([[1,2],[0,1]])
<matplotlib.image.AxesImage object at ...>
```

We test that `imshow` works as well, verifying that trac ticket #2900 is fixed (in Matplotlib).

```
sage: plt.imshow([[0.0,0.0,0.0]])
<matplotlib.image.AxesImage object at ...>
sage: plt.savefig(os.path.join(SAGE_TMP, 'foo.png'))
```

Since the above overwrites many Sage plotting functions, we reset the state of Sage, so that the examples below work!

```
sage: reset()
```

See [http://matplotlib.sourceforge.net](http://matplotlib.sourceforge.net) for complete documentation about how to use Matplotlib.

AUTHORS:

- Alex Clemesha and William Stein (2006-04-10): initial version
- David Joyner: examples
- Alex Clemesha (2006-05-04) major update
- William Stein (2006-05-29): fine tuning, bug fixes, better server integration
- Alex Clemesha (2006-09-29): added contour_plot, frame axes, misc polishing
- Robert Miller (2006-10-30): tuning, NetworkX primitive
- Alex Clemesha (2006-11-25): added plot_vector_field, matrix_plot, arrow, bar_chart, Axes class usage (see axes.py)
- Bobby Moretti and William Stein (2008-01): Change plot to specify ranges using the (varname, min, max) notation.
- William Stein (2008-01-19): raised the documentation coverage from a miserable 12 percent to a ‘wopping’ 35 percent, and fixed and clarified numerous small issues.
- Jason Grout (2009-09-05): shifted axes and grid functionality over to matplotlib; fixed a number of smaller issues.
- Jason Grout (2010-10): rewrote aspect ratio portions of the code
• Jeroen Demeyer (2012-04-19): move parts of this file to graphics.py (trac ticket #12857)
• Aaron Lauve (2016-07-13): reworked handling of ‘color’ when passed a list of functions; now more in-line with other CAS’s. Added list functionality to linestyle and legend_label options as well. (trac ticket #12962)
• Eric Gourgoulhon (2019-04-24): add multi_graphics() and insets

sage.plot.plot.SelectiveFormatter(formatter, skip_values)
This matplotlib formatter selectively omits some tick values and passes the rest on to a specified formatter.

EXAMPLES:
This example is almost straight from a matplotlib example.

```
sage: from sage.plot.plot import SelectiveFormatter
sage: import matplotlib.pyplot as plt
sage: import numpy
sage: fig=plt.figure()
```
```
sage: ax=fig.add_subplot(111)
```
```
sage: t = numpy.arange(0.0, 2.0, 0.01)
```
```
sage: s = numpy.sin(2*numpy.pi*t)
```
```
sage: p = ax.plot(t, s)
```
```
sage: formatter=SelectiveFormatter(ax.xaxis.get_major_formatter(),skip_values=[0,1])
```
```
sage: ax.xaxis.set_major_formatter(formatter)
```
```
sage: fig.savefig(os.path.join(SAGE_TMP, 'test.png'))
```

sage.plot.plot.adaptive_refinement(f, p1, p2, adaptive_tolerance=0.01, adaptive_recursion=0.01, level=5, excluded=0)
The adaptive refinement algorithm for plotting a function $f$. See the docstring for plot for a description of the algorithm.

INPUT:
• $f$ – a function of one variable
• $p_1, p_2$ – two points to refine between
• adaptive_recursion – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.
• adaptive_tolerance – (default: 0.01); how large a relative difference should be before the adaptive refinement code considers it significant; see documentation for generate_plot_points for more information. See the documentation for plot() for more information on how the adaptive refinement algorithm works.
• excluded – (default: False); also return locations where it has been discovered that the function is not defined ($y$-value will be ‘$NaN$’ in this case)

OUTPUT:
A list of points to insert between $p_1$ and $p_2$ to get a better linear approximation between them. If excluded, also $x$-values for which the calculation failed are given with ‘$NaN$’ as $y$-value.

sage.plot.plot.generate_plot_points(f, xrange, plot_points, adaptive_tolerance=5, adaptive_recursion=0.01, randomize=5, initial_points=True, excluded=None)
Calculate plot points for a function $f$ in the interval $xrange$. The adaptive refinement algorithm is also automatically invoked with a relative adaptive tolerance of adaptive_tolerance; see below.

INPUT:
• $f$ – a function of one variable
• $p_1, p_2$ – two points to refine between
• plot_points – (default: 5); the minimal number of plot points. (Note however that in any actual plot a number is passed to this, with default value 200.)

• adaptive_recursion – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.

• adaptive_tolerance – (default: 0.01); how large the relative difference should be before the adaptive refinement code considers it significant. If the actual difference is greater than adaptive_tolerance*delta, where delta is the initial subinterval size for the given xrange and plot_points, then the algorithm will consider it significant.

• initial_points – (default: None); a list of x-values that should be evaluated.

• excluded – (default: False); add a list of discovered x-values, for which f is not defined

OUTPUT:

• a list of points (x, f(x)) in the interval xrange, which approximate the function f.

• if excluded a tuple consisting of the above and a list of x-values at which f is not defined

sage.plot.plot.graphics_array(array, nrows=None, ncols=None)

Plot a list of lists (or tuples) of graphics objects on one canvas, arranged as an array.

INPUT:

• array – either a list of lists of Graphics elements or a single list of Graphics elements

• nrows, ncols – (optional) integers. If both are given then the input array is flattened and turned into an nrows x ncols array, with blank graphics objects padded at the end, if necessary. If only one is specified, the other is chosen automatically.

OUTPUT: an instance of GraphicsArray

EXAMPLES:

Make some plots of sin functions:

```
sage: f(x) = sin(x)
sage: g(x) = sin(2*x)
sage: h(x) = sin(4*x)
sage: p1 = plot(f, (-2*pi, 2*pi), color=hue(0.5))  # long time
sage: p2 = plot(g, (-2*pi, 2*pi), color=hue(0.9))  # long time
sage: p3 = parametric_plot((f,g), (0, 2*pi), color=hue(0.6))  # long time
sage: p4 = parametric_plot((f,h), (0, 2*pi), color=hue(1.0))  # long time
```

Now make a graphics array out of the plots:

```
sage: graphics_array(((p1, p2), (p3, p4)))  # long time
```

Graphics Array of size 2 x 2

One can also name the array, and then use show() or save():

```
sage: ga = graphics_array(((p1, p2), (p3, p4)))  # long time
sage: ga.show()  # long time; same output as above
```

Here we give only one row:

```
sage: p1 = plot(sin, (-4, 4))
sage: p2 = plot(cos, (-4, 4))
sage: ga = graphics_array([p1, p2])
```
1.1. 2D Plotting
It is possible to use `figsize` to change the size of the plot as a whole:

```python
sage: L = [plot(sin(k*x), (x,-pi,pi)) for k in [1..3]]
sage: ga = graphics_array(L)
sage: ga.show(figsize=[5,3])  # smallish and compact
sage: ga.show(figsize=[5,7])  # tall and thin; long time
sage: ga.show(figsize=4)  # width=4 inches, height fixed from default aspect ratio
```

Specifying only the number of rows or the number of columns computes the other dimension automatically:

```python
sage: ga = graphics_array([plot(sin)] * 10, nrows=3)
sage: ga.nrows(), ga.ncols()
3, 4
sage: ga = graphics_array([plot(sin)] * 10, ncols=3)
sage: ga.nrows(), ga.ncols()
4, 3
```
The options like fontsize, scale or frame passed to individual plots are preserved:

```
sage: p1 = plot(sin(x^2), (x, 0, 6),
.....:     axes_labels=[r'$\theta$', r'$\sin(\theta^2)$'], fontsize=16)
sage: p2 = plot(x^3, (x, 1, 100), axes_labels=[r'$x$', r'$y$'],
.....:     scale='semilogy', frame=True, gridlines='minor')
sage: ga = graphics_array([p1, p2])
sage: ga.show()
```

See also:

`GraphicsArray` for more examples

```
sage.plot.plot.list_plot(data, plotjoined=False, aspect_ratio='automatic', **kwargs)
```

`list_plot` takes either a list of numbers, a list of tuples, a numpy array, or a dictionary and plots the corresponding points.

If given a list of numbers (that is, not a list of tuples or lists), `list_plot` forms a list of tuples \((i, x_i)\) where \(i\) goes from 0 to \(\text{len(data)}-1\) and \(x_i\) is the \(i\)-th data value, and puts points at those tuple values.

`list_plot` will plot a list of complex numbers in the obvious way; any numbers for which \(\text{CC}()\) makes sense will work.

`list_plot` also takes a list of tuples \((x_i, y_i)\) where \(x_i\) and \(y_i\) are the \(i\)-th values representing the \(x\) and \(y\)-values, respectively.

If given a dictionary, `list_plot` interprets the keys as \(x\)-values and the values as \(y\)-values.

The plotjoined=True option tells `list_plot` to plot a line joining all the data.
1.1. 2D Plotting
For other keyword options that the list_plot function can take, refer to `plot()`.

It is possible to pass empty dictionaries, lists, or tuples to list_plot. Doing so will plot nothing (returning an empty plot).

EXAMPLES:

```
sage: list_plot([i^2 for i in range(5)])  # long time
```
Graphics object consisting of 1 graphics primitive

Here are a bunch of random red points:

```
sage: r = [(random(),random()) for _ in range(20)]
sage: list_plot(r, color='red')
```
Graphics object consisting of 1 graphics primitive

This gives all the random points joined in a purple line:

```
sage: list_plot(r, plotjoined=True, color='purple')
```
Graphics object consisting of 1 graphics primitive

You can provide a numpy array:
1.1. 2D Plotting
sage: import numpy
sage: list_plot(numpy.arange(10))
Graphics object consisting of 1 graphics primitive

Plot a list of complex numbers:

sage: list_plot([1, I, pi + I/2, CC(.25, .25)])
Graphics object consisting of 1 graphics primitive

sage: list_plot([exp(I*theta) for theta in [0, .2..pi]])
Graphics object consisting of 1 graphics primitive

Note that if your list of complex numbers are all actually real, they get plotted as real values, so this

sage: list_plot([CDF(1), CDF(1/2), CDF(1/3)])
Graphics object consisting of 1 graphics primitive

is the same as list_plot([1, 1/2, 1/3]) – it produces a plot of the points (0, 1), (1, 1/2), and (2, 1/3).

If you have separate lists of $x$ values and $y$ values which you want to plot against each other, use the zip command to make a single list whose entries are pairs of $(x, y)$ values, and feed the result into list_plot:
```python
sage: x_coords = [cos(t)^3 for t in srange(0, 2*pi, 0.02)]
sage: y_coords = [sin(t)^3 for t in srange(0, 2*pi, 0.02)]
sage: list_plot(list(zip(x_coords, y_coords)))
```
Graphics object consisting of 1 graphics primitive

If instead you try to pass the two lists as separate arguments, you will get an error message:

```python
sage: list_plot(x_coords, y_coords)
Traceback (most recent call last):
...  
TypeError: The second argument 'plotjoined' should be boolean (True or False). If you meant to plot two lists 'x' and 'y' against each other, use 'list_plot(list(zip(x,y)))'.
```

Dictionaries with numeric keys and values can be plotted:

```python
```
Graphics object consisting of 1 graphics primitive

Plotting in logarithmic scale is possible for 2D list plots. There are two different syntaxes available:
```python
sage: yl = [2**k for k in range(20)]
sage: list_plot(yl, scale='semilogy')  # long time # log axis on vertical
Graphics object consisting of 1 graphics primitive
```

```python
sage: list_plot_semilogy(yl)  # same
Graphics object consisting of 1 graphics primitive
```

**Warning:** If `plotjoined` is `False` then the axis that is in log scale must have all points strictly positive. For instance, the following plot will show no points in the figure since the points in the horizontal axis starts from \((0, 1)\). Further, matplotlib will display a user warning.

```python
sage: list_plot(yl, scale='loglog')  # both axes are log
doctest:warning
...
Graphics object consisting of 1 graphics primitive
```

Instead this will work. We drop the point \((0, 1)\):

```python
sage: list_plot(list(zip(range(1,len(yl)), yl[1:])), scale='loglog')  # long time
Graphics object consisting of 1 graphics primitive
```

We use `list_plot_loglog()` and plot in a different base.
We can also change the scale of the axes in the graphics just before displaying:

```python
sage: G = list_plot(yl) # long time
sage: G.show(scale=('semilogy', 2)) # long time
```

```python
sage.plot.plot.list_plot_loglog(data, plotjoined=False, base=10, **kwds)
```

Plot the data in ‘loglog’ scale, that is, both the horizontal and the vertical axes will be in logarithmic scale.

**INPUT:**

- `base` – (default: 10); the base of the logarithm. This must be greater than 1. The base can be also given as a list or tuple `(basex, basey)`. `basex` sets the base of the logarithm along the horizontal axis and `basey` sets the base along the vertical axis.

For all other inputs, look at the documentation of `list_plot()`.

**EXAMPLES:**

```python
sage: yl = [5**k for k in range(10)]; xl = [2**k for k in range(10)]
sage: list_plot_loglog(list(zip(xl, yl))) # long time # plot in loglog scale with
˓→base 10
```

Graphics object consisting of 1 graphics primitive
Chapter 1. General
```python
sage: list_plot_loglog(list(zip(xl, yl)), base=2.1) # long time # with base 2.1 on both axes
Graphics object consisting of 1 graphics primitive
```

```
2.1^20
   |
2.1^17
   |
2.1^14
   |
2.1^11
   |
2.1^8
   |
2.1^5
   |
2.1^2
   |
--------+--------+--------+--------+--------
2.1^1   2.1^3   2.1^5   2.1^7
```

```python
sage: list_plot_loglog(list(zip(xl, yl)), base=(2,5)) # long time
Graphics object consisting of 1 graphics primitive
```

**Warning:** If `plotjoined` is `False` then the axis that is in log scale must have all points strictly positive. For instance, the following plot will show no points in the figure since the points in the horizontal axis starts from `(0, 1)`.

```python
sage: yl = [2**k for k in range(20)]
sage: list_plot_loglog(yl)
Graphics object consisting of 1 graphics primitive
```

Instead this will work. We drop the point `(0, 1)`:

```python
sage: list_plot_loglog(list(zip(range(1,len(yl)), yl[1:])))
Graphics object consisting of 1 graphics primitive
```

---

`sage.plot.plot.list_plot_semilogx(data, plotjoined=False, base=10, **kwds)`

Plot data in ‘semilogx’ scale, that is, the horizontal axis will be in logarithmic scale.
INPUT:

- base – (default: 10); the base of the logarithm. This must be greater than 1.

For all other inputs, look at the documentation of list_plot().

EXAMPLES:

```sage
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogx(list(zip(yl,yl))):
Graphics object consisting of 1 graphics primitive
```

```
0 10
100
1000
1500
2000
100 101 102 103
```

**Warning:** If plotjoined is False then the horizontal axis must have all points strictly positive. Otherwise the plot will come up empty. For instance the following plot contains a point at (0, 1).

```sage
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogx(yl)
# plot empty due to (0,1)
Graphics object consisting of 1 graphics primitive
```

We remove (0, 1) to fix this:

```sage
sage: list_plot_semilogx(list(zip(range(1, len(yl)), yl[1:])))
Graphics object consisting of 1 graphics primitive
```
sage: list_plot_semilogx([(1,2),(3,4),(3,-1),(25,3)], base=2) # with base 2
Graphics object consisting of 1 graphics primitive

Plot data in ‘semilogy’ scale, that is, the vertical axis will be in logarithmic scale.

INPUT:

• base – (default: 10); the base of the logarithm. This must be greater than 1.

For all other inputs, look at the documentation of `list_plot()`.

EXAMPLES:

sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogy(yl) # plot in semilogy scale, base 10
Graphics object consisting of 1 graphics primitive

**Warning:** If `plotjoined` is `False` then the vertical axis must have all points strictly positive. Otherwise the plot will come up empty. For instance the following plot contains a point at (1,0). Further, matplotlib will display a user warning.

sage: xl = [2**k for k in range(12)]; yl = range(len(xl))
sage: list_plot_semilogy(list(zip(xl,yl))) # plot empty due to (1,0)
doctest:warning
...
Graphics object consisting of 1 graphics primitive

We remove (1,0) to fix this:

```
sage: list_plot_semilogy(list(zip(xl[1:],yl[1:])))
```
Graphics object consisting of 1 graphics primitive

```
sage: list_plot_semilogy([2, 4, 6, 8, 16, 31], base=2) # with base 2
```
Graphics object consisting of 1 graphics primitive

```
sage.plot.plot.minmax_data(xdata, ydata, dict=False)
```

Return the minimums and maximums of `xdata` and `ydata`.

If `dict` is False, then minmax_data returns the tuple `(xmin, xmax, ymin, ymax)`; otherwise, it returns a dictionary whose keys are ‘xmin’, ‘xmax’, ‘ymin’, and ‘ymax’ and whose values are the corresponding values.

EXAMPLES:

```
sage: from sage.plot.plot import minmax_data
sage: minmax_data([], [])
(-1, 1, -1, 1)
```

(continues on next page)
sage: minmax_data([-1, 2], [4, -3])
(-1, 2, -3, 4)
sage: minmax_data([1, 2], [4, -3])
(1, 2, -3, 4)
sage: d = minmax_data([-1, 2], [4, -3], dict=True)
sage: list(sorted(d.items()))
[('xmax', 2), ('xmin', -1), ('ymax', 4), ('ymin', -3)]
sage: d = minmax_data([1, 2], [3, 4], dict=True)
sage: list(sorted(d.items()))
[('xmax', 2), ('xmin', 1), ('ymax', 4), ('ymin', 3)]

sage.plot.plot.multi_graphics(graphics_list)
Plot a list of graphics at specified positions on a single canvas.

If the graphics positions define a regular array, use graphics_array() instead.

INPUT:

* graphics_list – a list of graphics along with their positions on the canvas; each element of graphics_list is either
  - a pair (graphics, position), where graphics is a Graphics object and position is the 4-tuple (left, bottom, width, height) specifying the location and size of the graphics on the canvas, all quantities being in fractions of the canvas width and height
  - or a single Graphics object; its position is then assumed to occupy the whole canvas, except for some padding; this corresponds to the default position (left, bottom, width, height) = (0.125, 0.11, 0.775, 0.77)

OUTPUT: an instance of MultiGraphics

EXAMPLES:

multi_graphics is to be used for plot arrangements that cannot be achieved with graphics_array(), for instance:

sage: g1 = plot(sin(x), (x, -10, 10), frame=True)
sage: g2 = EllipticCurve([0,0,1,-1,0]).plot(color='red', thickness=2,
\                                   axes_labels=[r'$x$', r'$y$']) \n\
\                                   + text(r"$y^2 + y = x^3 - x$", (1.2, 2), color='red')
sage: g3 = matrix_plot(matrix([[1,3,5,1], [2,4,5,6], [1,3,5,7]]))
sage: G = multi_graphics([(g1, (0.125, 0.65, 0.775, 0.3)),
\                                   (g2, (0.125, 0.11, 0.4, 0.4)),
\                                   (g3, (0.55, 0.18, 0.4, 0.3))])
sage: G
Multigraphics with 3 elements

An example with a list containing a graphics object without any specified position (the graphics, here g3, occupies then the whole canvas):

sage: G = multi_graphics([g3, (g1, (0.4, 0.4, 0.2, 0.2))])
sage: G
Multigraphics with 2 elements

See also:

MultiGraphics for more examples
$y^2 + y = x^3 - x$
sage.plot.plot.parametric_plot(funcs, aspect_ratio=1.0, *args, **kwargs)
Plot a parametric curve or surface in 2d or 3d.

parametric_plot() takes two or three functions as a list or a tuple and makes a plot with the first function giving the \(x\) coordinates, the second function giving the \(y\) coordinates, and the third function (if present) giving the \(z\) coordinates.

In the 2d case, parametric_plot() is equivalent to the plot() command with the option parametric=True. In the 3d case, parametric_plot() is equivalent to parametric_plot3d(). See each of these functions for more help and examples.

INPUT:

- \texttt{funcs} – 2 or 3-tuple of functions, or a vector of dimension 2 or 3.
- \texttt{other options} – passed to plot() or parametric_plot3d()

EXAMPLES: We draw some 2d parametric plots. Note that the default aspect ratio is 1, so that circles look like circles.

```
sage: t = var('t')
sage: parametric_plot( (cos(t), sin(t)), (t, 0, 2*pi))
Graphics object consisting of 1 graphics primitive
```

```
sage: parametric_plot( (sin(t), sin(2*t)), (t, 0, 2*pi), color=hue(0.6) )
Graphics object consisting of 1 graphics primitive
```

1.1. 2D Plotting

55
sage: parametric_plot((1, t), (t, 0, 4))
Graphics object consisting of 1 graphics primitive

Note that in parametric_plot, there is only fill or no fill.

sage: parametric_plot((t, t^2), (t, -4, 4), fill=True)
Graphics object consisting of 2 graphics primitives

A filled Hypotrochoid:

sage: parametric_plot([cos(x) + 2 * cos(x/4), sin(x) - 2 * sin(x/4)], (x,0, 8*pi), fill=True)
Graphics object consisting of 2 graphics primitives

sage: parametric_plot( (5*cos(x), 5*sin(x), x), (x, -12, 12), plot_points=150, color="red") # long time
Graphics3d Object

sage: y=var('y')
sage: parametric_plot( (5*cos(x), x*y, cos(x*y)), (x, -4,4), (y,-4,4)) # long time'
Graphics3d Object
Chapter 1. General
Plotting in logarithmic scale is possible with 2D plots. The keyword aspect_ratio will be ignored if the scale is not 'loglog' or 'linear':

```sage
sage: p = parametric_plot((x, x**2), (x, 1, 10))
sage: p.show(scale='semilogy', aspect_ratio='automatic')
```

We can also change the scale of the axes in the graphics just before displaying. In this case, the aspect_ratio must be specified as 'automatic' if the scale is set to 'semilogx' or 'semilogy'. For other values of the scale parameter, any aspect_ratio can be used, or the keyword need not be provided:

```sage
sage: p = parametric_plot((x, x**2), (x, 1, 10))
sage: p.show(scale='semilogy', aspect_ratio='automatic')
```
sage.plot.plot.plot(funcs, alpha=1, thickness=1, fill=False, fillcolor='automatic', fillalpha=0.5, plot_points=200, adaptive_tolerance=0.01, adaptive_recursion=5, detect_poles=False, exclude=None, legend_label=None, aspect_ratio='automatic', *args, **kwds)

Use plot by writing

plot(X, ...)

where X is a Sage object (or list of Sage objects) that either is callable and returns numbers that can be coerced to floats, or has a plot method that returns a GraphicPrimitive object.

There are many other specialized 2D plot commands available in Sage, such as plot_slope_field, as well as various graphics primitives like Arrow; type sage.plot.plot? for a current list.

Type plot.options for a dictionary of the default options for plots. You can change this to change the defaults for all future plots. Use plot.reset() to reset to the default options.

PLOT OPTIONS:

- plot_points – (default: 200); the minimal number of plot points.
- adaptive_recursion – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.
- adaptive_tolerance – (default: 0.01); how large a difference should be before the adaptive refinement code considers it significant. See the documentation further below for more information, starting at “the algorithm used to insert”.
- base – (default: 10); the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.
- scale – string (default: "linear"); scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy".

The scale can be also be given as single argument that is a list or tuple (scale, base) or (scale, basex, basey).

The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when Graphics is initialized.
- xmin – starting x value in the rendered figure. This parameter is passed directly to the show procedure and it could be overwritten.
- xmax – ending x value in the rendered figure. This parameter is passed directly to the show procedure and it could be overwritten.
- ymin – starting y value in the rendered figure. This parameter is passed directly to the show procedure and it could be overwritten.
- ymax – ending y value in the rendered figure. This parameter is passed directly to the show procedure and it could be overwritten.
- detect_poles – (default: False) If set to True poles are detected. If set to “show” vertical asymptotes are drawn.
- legend_label – a (TeX) string serving as the label for X in the legend. If X is a list, then this option can be a single string, or a list or dictionary with strings as entries/values. If a dictionary, then keys are taken from range(len(X)).
• If the scale is "linear", then irrespective of what base is set to, it will default to 10 and will remain unused.

• If you want to limit the plot along the horizontal axis in the final rendered figure, then pass the xmin and xmax keywords to the show() method. To limit the plot along the vertical axis, ymin and ymax keywords can be provided to either this plot command or to the show command.

• This function does NOT simply sample equally spaced points between xmin and xmax. Instead it computes equally spaced points and adds small perturbations to them. This reduces the possibility of, e.g., sampling \( \sin \) only at multiples of \( 2\pi \), which would yield a very misleading graph.

• If there is a range of consecutive points where the function has no value, then those points will be excluded from the plot. See the example below on automatic exclusion of points.

• For the other keyword options that the plot function can take, refer to the method show() and the further options below.

COLOR OPTIONS:

• color - (Default: ‘blue’) One of:
  – an RGB tuple (r,g,b) with each of r,g,b between 0 and 1.
  – a color name as a string (e.g., ‘purple’).
  – an HTML color such as ‘#aaff0b’.
  – a list or dictionary of colors (valid only if \( X \) is a list): if a dictionary, keys are taken from \( \text{range}(\text{len}(X)) \); the entries/values of the list/dictionary may be any of the options above.
  – ‘automatic’ – maps to default (‘blue’) if \( X \) is a single Sage object; and maps to a fixed sequence of regularly spaced colors if \( X \) is a list.

• legend_color - the color of the text for \( X \) (or each item in \( X \)) in the legend. Default color is ‘black’. Options are as in color above, except that the choice ‘automatic’ maps to ‘black’ if \( X \) is a single Sage object.

• fillcolor - The color of the fill for the plot of \( X \) (or each item in \( X \)). Default color is ‘gray’ if \( X \) is a single Sage object or if color is a single color. Otherwise, options are as in color above.

APPEARANCE OPTIONS:
The following options affect the appearance of the line through the points on the graph of \( X \) (these are the same as for the line function):

INPUT:

• alpha – how transparent the line is
• thickness – how thick the line is
• rgbcolor – the color as an RGB tuple
• hue – the color given as a hue

LINE OPTIONS:
Any MATPLOTLIB line option may also be passed in. E.g.,

• linestyle - (default: “-”) The style of the line, which is one of
  – "-" or "solid"
  – "--" or "dashed"
  – "-." or "dash dot"
- "": or "dotted"
- "None" or " " or "" (nothing)
- a list or dictionary (see below)

The linestyle can also be prefixed with a drawing style (e.g., "steps--")
- "default" (connect the points with straight lines)
- "steps" or "steps-pre" (step function; horizontal line is to the left of point)
- "steps-mid" (step function; points are in the middle of horizontal lines)
- "steps-post" (step function; horizontal line is to the right of point)

If \( X \) is a list, then linestyle may be a list (with entries taken from the strings above) or a dictionary (with keys in range(len(X)) and values taken from the strings above).

- **marker** - The style of the markers, which is one of
  - "None" or " " or "" (nothing) – default
  - ",", (pixel), ".", (point)
  - "", (horizontal line), "|", (vertical line)
  - "o", "p", "s", "x", "+", "*", "D", "d", "H", "h", "<", ">", "^", "v", ";", "\$\ldots\$"
  - (numsides, style, angle) to create a custom, regular symbol
    - numsides – the number of sides
    - style – 0 (regular polygon), 1 (star shape), 2 (asterisk), 3 (circle)
    - angle – the angular rotation in degrees

- **markersize** - the size of the marker in points
- **markeredgecolor** – the color of the marker edge
- **markerfacecolor** – the color of the marker face
- **markeredgewidth** - the size of the marker edge in points
- **exclude** - (Default: None) values which are excluded from the plot range. Either a list of real numbers, or an equation in one variable.

**FILLING OPTIONS:**
- **fill** - (default: False) One of:
  - “axis” or True: Fill the area between the function and the x-axis.
  - “min”: Fill the area between the function and its minimal value.
  - “max”: Fill the area between the function and its maximal value.
- a number c: Fill the area between the function and the horizontal line $y = c$.
- a function g: Fill the area between the function that is plotted and g.
- a dictionary $d$ (only if a list of functions are plotted): The keys of the dictionary should be integers. The value of $d[i]$ specifies the fill options for the $i$-th function in the list. If $d[i] == [j]$: Fill the area between the $i$-th and the $j$-th function in the list. (But if $d[i] == j$: Fill the area between the $i$-th function in the list and the horizontal line $y = j$.)

- **fillalpha** - (default: 0.5) How transparent the fill is. A number between 0 and 1.

**MATPLOTLIB STYLE SHEET OPTION:**
- **stylesheet** - (Default: classic) Support for loading a full matplotlib style sheet. Any style sheet listed in `matplotlib.pyplot.style.available` is acceptable. If a non-existing style is provided the default classic is applied.

**EXAMPLES:**

We plot the sin function:

```python
sage: P = plot(sin, (0,10)); print(P)
Graphics object consisting of 1 graphics primitive
sage: len(P)  # number of graphics primitives
1
sage: len(P[0])  # how many points were computed (random)
225
sage: P  # render
Graphics object consisting of 1 graphics primitive
```

```python
sage: P = plot(sin, (0,10), plot_points=10); print(P)
Graphics object consisting of 1 graphics primitive
sage: len(P[0])  # random output
32
sage: P  # render
Graphics object consisting of 1 graphics primitive
```

We plot with `randomize=False`, which makes the initial sample points evenly spaced (hence always the same). Adaptive plotting might insert other points, however, unless `adaptive_recursion=0`.

```python
sage: p=plot(1, (x,0,3), plot_points=4, randomize=False, adaptive_recursion=0)
sage: list(p[0])
[(0.0, 1.0), (1.0, 1.0), (2.0, 1.0), (3.0, 1.0)]
```

Some colored functions:

```python
sage: plot(sin, 0, 10, color='purple')
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(sin, 0, 10, color='#ff00ff')
Graphics object consisting of 1 graphics primitive
```

We plot several functions together by passing a list of functions as input:

```python
sage: plot([x^exp(-n*x^2)/.4 for n in [1..5]], (0, 2), aspect_ratio=.8)
Graphics object consisting of 5 graphics primitives
```

By default, color will change from one primitive to the next. This may be controlled by modifying color option:
1.1. 2D Plotting
We can also build a plot step by step from an empty plot:

```
sage: a = plot([]); a  # passing an empty list returns an empty plot
    (Graphics() object)
    Graphics object consisting of 0 graphics primitives
sage: a += plot(x**2); a  # append another plot
    Graphics object consisting of 1 graphics primitive
sage: a += plot(x**3); a  # append yet another plot
    Graphics object consisting of 2 graphics primitives
```

The function $\sin(\frac{1}{x})$ wiggles wildly near 0. Sage adapts to this and plots extra points near the origin.

```
sage: plot(sin(1/x), (x, -1, 1))
    Graphics object consisting of 1 graphics primitive
```

1.1. 2D Plotting
1.1. 2D Plotting
Via the matplotlib library, Sage makes it easy to tell whether a graph is on both sides of both axes, as the axes only cross if the origin is actually part of the viewing area:

```
sage: plot(x^3,(x,0,2))  # this one has the origin
Graphics object consisting of 1 graphics primitive
```

```
sage: plot(x^3,(x,1,2))  # this one does not
Graphics object consisting of 1 graphics primitive
```

Another thing to be aware of with axis labeling is that when the labels have quite different orders of magnitude or are very large, scientific notation (the $e$ notation for powers of ten) is used:

```
sage: plot(x^2,(x,480,500))  # this one has no scientific notation
Graphics object consisting of 1 graphics primitive
```

```
sage: plot(x^2,(x,300,500))  # this one has scientific notation on y-axis
Graphics object consisting of 1 graphics primitive
```

You can put a legend with `legend_label` (the legend is only put once in the case of multiple functions):

```
sage: plot(exp(x), 0, 2, legend_label='$e^x$')
Graphics object consisting of 1 graphics primitive
```

Sage understands TeX, so these all are slightly different, and you can choose one based on your needs:
1.1. 2D Plotting
It is possible to use a different color for the text of each label:

```python
sage: p1 = plot(sin, legend_label='sin', legend_color='red')
sage: p2 = plot(cos, legend_label='cos', legend_color='green')
sage: p1 + p2
```

Prior to trac ticket #19485, legends by default had a shadowless gray background. This behavior can be recovered by setting the legend options on your plot object:

```python
sage: p = plot(sin(x), legend_label=r'$\sin(x)$')
sage: p.set_legend_options(back_color=(0.9,0.9,0.9), shadow=False)
```

If $X$ is a list of Sage objects and `legend_label` is ‘automatic’, then Sage will create labels for each function.
1.1. 2D Plotting
$\sin(x)$
according to their internal representation:

```
sage: plot([sin(x), tan(x), 1-x^2], legend_label='automatic')
Graphics object consisting of 3 graphics primitives
```

If `legend_label` is any single string other than 'automatic', then it is repeated for all members of \( X \):

```
sage: plot([sin(x), tan(x)], color='blue', legend_label='trig')
Graphics object consisting of 2 graphics primitives
```

Note that the independent variable may be omitted if there is no ambiguity:

```
sage: plot(sin(1.0/x), (-1, 1))
Graphics object consisting of 1 graphics primitive
```

Plotting in logarithmic scale is possible for 2D plots. There are two different syntaxes supported:

```
sage: plot(exp, (1, 10), scale='semilogy') # log axis on vertical
Graphics object consisting of 1 graphics primitive
```

```
sage: plot_semilogy(exp, (1, 10)) # same thing
Graphics object consisting of 1 graphics primitive
```
1.1. 2D Plotting
We can also change the scale of the axes in the graphics just before displaying:

```
sage: G = plot(exp, 1, 10)  # long time
sage: G.show(scale=('semilogy', 2))  # long time
```

The algorithm used to insert extra points is actually pretty simple. On the picture drawn by the lines below:

```
sage: p = plot(x^2, (-0.5, 1.4)) + line([(0,0), (1,1)], color='green')
sage: p += line([(0.5, 0.5), (0.5, 0.5^2)], color='purple')
sage: p += point(((0, 0), (0.5, 0.5), (0.5, 0.5^2), (1, 1)), color='red', pointsize=20)
sage: p += text('A', (-0.05, 0.1), color='red')
sage: p += text('B', (1.01, 1.1), color='red')
sage: p += text('C', (0.48, 0.57), color='red')
sage: p += text('D', (0.53, 0.18), color='red')
sage: p.show(axes=False, xmin=-0.5, xmax=1.4, ymin=0, ymax=2)
```
1.1. 2D Plotting
You have the function (in blue) and its approximation (in green) passing through the points A and B. The algorithm finds the midpoint C of AB and computes the distance between C and D. If that distance exceeds the adaptive_tolerance threshold (relative to the size of the initial plot subintervals), the point D is added to the curve. If D is added to the curve, then the algorithm is applied recursively to the points A and D, and D and B. It is repeated adaptive_recursion times (5, by default).

The actual sample points are slightly randomized, so the above plots may look slightly different each time you draw them.

We draw the graph of an elliptic curve as the union of graphs of 2 functions.

```
sage: def h1(x): return abs(sqrt(x^3 - 1))
sage: def h2(x): return -abs(sqrt(x^3 - 1))
sage: P = plot([h1, h2], 1, 4)
sage: P
# show the result
```

Graphics object consisting of 2 graphics primitives

It is important to mention that when we draw several graphs at the same time, parameters xmin, xmax, ymin and ymax are just passed directly to the show procedure. In fact, these parameters would be overwritten:

```
sage: p=plot(x^3, x, xmin=-1, xmax=1, ymin=-1, ymax=1)
sage: q=plot(exp(x), x, xmin=-2, xmax=2, ymin=0, ymax=4)
sage: (p+q).show()
```

As a workaround, we can perform the trick:
We can also directly plot the elliptic curve:

```python
sage: E = EllipticCurve([0,-1])
sage: plot(E, (1, 4), color=hue(0.6))
```

```
Graphics object consisting of 1 graphics primitive
```

We can change the line style as well:

```python
sage: plot(sin(x), (x, 0, 10), linestyle='--')
```

```
Graphics object consisting of 1 graphics primitive
```

If we have an empty linestyle and specify a marker, we can see the points that are actually being plotted:

```python
sage: plot(sin(x), (x,0,10), plot_points=20, linestyle='', marker='.')
```

```
Graphics object consisting of 1 graphics primitive
```

The marker can be a TeX symbol as well:

```python
sage: plot(sin(x), (x,0,10), plot_points=20, linestyle='', marker=r'$\checkmark$')
```

```
Graphics object consisting of 1 graphics primitive
```
Sage currently ignores points that cannot be evaluated

```
sage: from sage.misc.verbose import set_verbose
sage: set_verbose(-1)
sage: plot(-x*log(x), (x, 0, 1))  # this works fine since the failed endpoint is just skipped.
sage: set_verbose(0)
```

Graphics object consisting of 1 graphics primitive

```
sage: set_verbose(-1)
sage: plot(x^(1/3), (x, -1, 1))
sage: set_verbose(0)
```

Graphics object consisting of 1 graphics primitive

This prints out a warning and plots where it can (we turn off the warning by setting the verbose mode temporarily to -1.)

```
sage: set_verbose(-1)
sage: plot(real_nth_root(x, 3), (x, -1, 1))
sage: set_verbose(0)
```

Graphics object consisting of 1 graphics primitive

Plotting the real cube root function for negative input requires avoiding the complex numbers one would usually get. The easiest way is to use `real_nth_root(x, n)`

```
sage: plot(real_nth_root(x, 3), (x, -1, 1))
```

Graphics object consisting of 1 graphics primitive

We can also get the same plot in the following way:
A way to plot other functions without symbolic variants is to use lambda functions:

```sage
sage: plot(lambda x : RR(x).nth_root(3), (x, -1, 1))
Graphics object consisting of 1 graphics primitive
```

We can detect the poles of a function:

```sage
sage: plot(gamma, (-3, 4), detect_poles='show').show(ymin=-5, ymax=5)
```

We draw the Gamma-Function with its poles highlighted:

```sage
sage: plot(gamma, (-3, 4), detect_poles='show').show(ymin=-5, ymax=5)
```

The basic options for filling a plot:

```sage
sage: p1 = plot(sin(x), -pi, pi, fill='axis')
sage: p2 = plot(sin(x), -pi, pi, fill='min', fillalpha=1)
sage: p3 = plot(sin(x), -pi, pi, fill='max')
sage: p4 = plot(sin(x), -pi, pi, fill=(1-x)/3, fillcolor='blue', fillalpha=.2)
sage: graphics_array([[p1, p2], [p3, p4]]).show(frame=True, axes=False) # long time
```
1.1. 2D Plotting
1.1. 2D Plotting
The basic options for filling a list of plots:

```python
sage: (f1, f2) = x*exp(-1*x^2)/.35, x*exp(-2*x^2)/.35
sage: p1 = plot([f1, f2], -pi, pi, fill={1: [0]}, fillcolor='blue', fillalpha=.25,
           color='blue')
sage: p2 = plot([f1, f2], -pi, pi, fill={0: x/3, 1:[0]}, color=['blue'])
sage: p3 = plot([f1, f2], -pi, pi, fill=[0, [0]], fillcolor=['orange','red'],
           fillalpha=1, color={1: 'blue'})
sage: p4 = plot([f1, f2], (x,-pi, pi), fill=[x/3, 0], fillcolor=['grey'], color=['red', 'blue'])
sage: graphics_array([[p1, p2], [p3, p4]]).show(frame=True, axes=False) # long time
```

A example about the growth of prime numbers:

```python
sage: plot(1.13*log(x), 1, 100, fill=lambda x: nth_prime(x)/floor(x), fillcolor='red')
Graphics object consisting of 2 graphics primitives
```

Fill the area between a function and its asymptote:

```python
sage: f = (2*x^3+2*x-1)/((x-2)^2*(x+1))
sage: plot([f, 2*x+2], -7,7, fill={0: [1]}, fillcolor='#ccc').show(ymin=-20,
           ymax=20)
```

Fill the area between a list of functions and the x-axis:
sage: def b(n): return lambda x: bessel_J(n, x)
sage: plot([b(n) for n in [1..5]], 0, 20, fill='axis')
Graphics object consisting of 10 graphics primitives

Note that to fill between the ith and jth functions, you must use the dictionary key-value syntax i:j; using key-value pairs like i:j will fill between the ith function and the line y=j:

sage: def b(n): return lambda x: bessel_J(n, x) + 0.5*(n-1)
sage: plot([b(c) for c in [1..5]], 0, 20, fill={i:i-1 for i in [1..4]}, color={i:'blue' for i in [1..5]}, aspect_ratio=3, ymax=3)
Graphics object consisting of 9 graphics primitives
sage: plot([b(c) for c in [1..5]], 0, 20, fill={i:i-1 for i in [1..4]}, color='blue', aspect_ratio=3) # long time
Graphics object consisting of 9 graphics primitives

Extra options will get passed on to `show()`, as long as they are valid:

sage: plot(sin(x^2), (x, -3, 3), title=r'Plot of $\sin(x^2)$', axes_labels=['$x$', '$y$']) # These labels will be nicely typeset
Graphics object consisting of 1 graphics primitive
sage: plot(sin(x^2), (x, -3, 3), title='Plot of sin(x^2)', axes_labels=['x', 'y']) # These will not
1.1. 2D Plotting
Plot of $\sin(x^2)$
Graphics object consisting of 1 graphics primitive

```
sage: plot(sin(x^2), (x, -3, 3), axes_labels=['x', 'y'], axes_labels_size=2.5)  # Large axes labels (w.r.t. the tick marks)
```

```
sage: plot(sin(x^2), (x, -3, 3), figsize=[8,2])
```

```
sage: plot(sin(x^2), (x, -3, 3)).show(figsize=[8,2])  # These are equivalent
```

This includes options for custom ticks and formatting. See documentation for `show()` for more details.

```
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[-7,-3,0,3,7],[-1/2,0,1/2]])
```

```
sage: plot(2*x+1,(x,0,5),ticks=[[0,1,e,pi,sqrt(20)],[2]],tick_formatter="latex")
```

This is particularly useful when setting custom ticks in multiples of $\pi$.

```
sage: plot(sin(x),(x,0,2*pi),ticks=pi/3,tick_formatter=pi)
```

1.1. 2D Plotting

111
1.1. 2D Plotting
You can even have custom tick labels along with custom positioning.

\begin{Verbatim}
sage: plot(x^2, (x,0,3), ticks=[[1,2.5],[0.5,1,2]], tick_formatter=[["$x_1$","$x_2$"], ["$y_1$","$y_2$","$y_3$"]])
\end{Verbatim}

Graphics object consisting of 1 graphics primitive

You can force Type 1 fonts in your figures by providing the relevant option as shown below. This also requires that LaTeX, dvipng and Ghostscript be installed:

\begin{Verbatim}
sage: plot(x, typeset='type1') # optional - latex
\end{Verbatim}

Graphics object consisting of 1 graphics primitive

A example with excluded values:

\begin{Verbatim}
sage: plot(floor(x), (x, 1, 10), exclude=[1..10])
\end{Verbatim}

Graphics object consisting of 11 graphics primitives

We exclude all points where \texttt{PrimePi} makes a jump:

\begin{Verbatim}
sage: jumps = [n for n in [1..100] if prime_pi(n) != prime_pi(n-1)]
sage: plot(lambda x: prime_pi(x), (x, 1, 100), exclude=jumps)
\end{Verbatim}

Graphics object consisting of 26 graphics primitives

Excluded points can also be given by an equation:
```sage
sage: g(x) = x^2-2*x-2
sage: plot(1/g(x), (x, -3, 4), exclude=g(x)==0, ymin=-5, ymax=5) # long time
Graphics object consisting of 3 graphics primitives
```

```
# long time

```

```sage
sage: f(x) = (floor(x)+0.5) / (1-(x-0.5)^2)
sage: plot(f, (x, -3.5, 3.5), detect_poles='show', exclude=[-3..3], ymin=-5, ymax=5)
Graphics object consisting of 12 graphics primitives
```

Regions in which the plot has no values are automatically excluded. The regions thus excluded are in addition to the exclusion points present in the `exclude` keyword argument.

```sage
sage: set_verbose(-1)
sage: plot(arcsec, (x, -2, 2)) # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives
```

```sage
sage: plot(arcsec, (x, -2, 2), exclude=[1.5]) # x=1.5 is also excluded
Graphics object consisting of 3 graphics primitives
```

```sage
sage: plot(arcsec(x/2), -2, 2) # plot should be empty; no valid points
Graphics object consisting of 0 graphics primitives
```

(continues on next page)
sage: plot(sqrt(x^2-1), -2, 2)  # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives

sage: plot(arccsc, -2, 2)  # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives

sage: set_verbose(0)

sage.plot.plot.plot_loglog(funcs, base=10, *args, **kwds)

Plot graphics in 'loglog' scale, that is, both the horizontal and the vertical axes will be in logarithmic scale.

INPUT:

- base – (default: 10); the base of the logarithm. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.

- funcs – any Sage object which is acceptable to the plot().

For all other inputs, look at the documentation of plot().

EXAMPLES:

sage: plot_loglog(exp, (1, 10))  # plot in loglog scale with base 10
Graphics object consisting of 1 graphics primitive
1.1. 2D Plotting
sage: plot_loglog(exp, (1,10), base=2.1)  \# long time \# with base 2.1 on both axes
Graphics object consisting of 1 graphics primitive

sage.plot.plot.plot_semilogx(funcs, base=10, *args, **kwds)

Plot graphics in 'semilogx' scale, that is, the horizontal axis will be in logarithmic scale.

INPUT:

- base - (default: 10); the base of the logarithm. This must be greater than 1.
- funcs - any Sage object which is acceptable to the plot().

For all other inputs, look at the documentation of plot().

EXAMPLES:

sage: plot_semilogx(exp, (1,10))  \# long time \# plot in semilogx scale, base 10
Graphics object consisting of 1 graphics primitive

sage: plot_semilogx(exp, (1,10), base=2)  \# with base 2
Graphics object consisting of 1 graphics primitive
1.1. 2D Plotting
1.1. 2D Plotting
sage: s = var('s')  # Samples points logarithmically so graph is smooth
sage: f = 4000000/(4000000 + 4000*s*i - s*s)
sage: plot_semilogx(20*log(abs(f), 10), (s, 1, 1e6))
Graphics object consisting of 1 graphics primitive

sage.plot.plot.plot_semilogy(funcs, base=10, *args, **kwds)
Plot graphics in ‘semilogy’ scale, that is, the vertical axis will be in logarithmic scale.

INPUT:
- base – (default: 10); the base of the logarithm. This must be greater than 1.
- funcs – any Sage object which is acceptable to the plot().

For all other inputs, look at the documentation of plot().

EXAMPLES:

sage: plot_semilogy(exp, (1,10))  # long time  # plot in semilogy scale, base 10
Graphics object consisting of 1 graphics primitive

sage: plot_semilogy(exp, (1,10), base=2)  # long time  # with base 2
Graphics object consisting of 1 graphics primitive

sage.plot.plot.polar_plot(funcs, aspect_ratio=1.0, *args, **kwds)
polar_plot takes a single function or a list or tuple of functions and plots them with polar coordinates in the
given domain.

This function is equivalent to the `plot()` command with the options `polar=True` and `aspect_ratio=1`. For more help on options, see the documentation for `plot()`.

**INPUT:**

- `funcs` – a function
- `other options are passed to plot`

**EXAMPLES:**

Here is a blue 8-leaved petal:

```python
sage: polar_plot(sin(5*x)^2, (x, 0, 2*pi), color='blue')
Graphics object consisting of 1 graphics primitive
```

A red figure-8:

```python
sage: polar_plot(abs(sqrt(1 - sin(x)^2)), (x, 0, 2*pi), color='red')
Graphics object consisting of 1 graphics primitive
```

A green limacon of Pascal:

```python
sage: polar_plot(2 + 2*cos(x), (x, 0, 2*pi), color=hue(0.3))
Graphics object consisting of 1 graphics primitive
```
1.1. 2D Plotting
Several polar plots:

```python
sage: polar_plot([2*sin(x), 2*cos(x)], (x, 0, 2*pi))
Graphics object consisting of 2 graphics primitives
```

A filled spiral:

```python
sage: polar_plot(sqrt, 0, 2 * pi, fill=True)
Graphics object consisting of 2 graphics primitives
```

Fill the area between two functions:

```python
sage: polar_plot(cos(4*x) + 1.5, 0, 2*pi, fill=0.5 * cos(4*x) + 2.5, fillcolor='orange')
Graphics object consisting of 2 graphics primitives
```

Fill the area between several spirals:

```python
sage: polar_plot([(1.2+k*0.2)*log(x) for k in range(6)], 1, 3 * pi, fill={0: [1], 2: [3], 4: [5]})
Graphics object consisting of 9 graphics primitives
```

Exclude points at discontinuities:
1.1. 2D Plotting
1.1. 2D Plotting
sage: polar_plot(log(floor(x)), (x, 1, 4*pi), exclude=[1..12])
Graphics object consisting of 12 graphics primitives

sage.plot.plot.reshape(v, n, m)
Helper function for creating graphics arrays.

The input array is flattened and turned into an \textit{nimesm} array, with blank graphics object padded at the end, if necessary.

\textbf{INPUT:}

- \textit{v} – a list of lists or tuples
- \textit{n}, \textit{m} – integers

\textbf{OUTPUT:} a list of lists of graphics objects

\textbf{EXAMPLES:}

sage: L = [plot(sin(k*x),(x,-pi,pi)) for k in range(10)]
sage: graphics_array(L,3,4) # long time (up to 4s on sage.math, 2012)
Graphics Array of size 3 x 4

sage: M = [[plot(sin(k*x),(x,-pi,pi)) for k in range(3)],
       [plot(cos(j*x),(x,-pi,pi)) for j in [3..5]]]
sage: graphics_array(M,6,1)  # long time (up to 4s on sage.math, 2012)
Graphics Array of size 6 x 1

sage.plot.plot.to_float_list(v)
Given a list or tuple or iterable v, coerce each element of v to a float and make a list out of the result.

EXAMPLES:

sage: from sage.plot.plot import to_float_list
sage: to_float_list([1,1/2,3])
[1.0, 0.5, 3.0]

sage.plot.plot.xydata_from_point_list(points)
Return two lists (xdata, ydata), each coerced to a list of floats, which correspond to the x-coordinates and the y-coordinates of the points.
The points parameter can be a list of 2-tuples or some object that yields a list of one or two numbers.
This function can potentially be very slow for large point sets.

1.2 Text in plots

class sage.plot.text.Text(string, point, options)
Bases: sage.plot.primitive.GraphicPrimitive

Base class for Text graphics primitive.

get_minmax_data()
Return a dictionary with the bounding box data. Notice that, for text, the box is just the location itself.

EXAMPLES:

sage: T = text("Where am I?", (1,1))
sage: t=T[0]
sage: t.get_minmax_data()['ymin']
1.0
sage: t.get_minmax_data()['ymax']
1.0

plot3d(**kwds)
Plot 2D text in 3D.

EXAMPLES:

sage: T = text("ABC", (1, 1))
sage: t = T[0]
sage: s = t.plot3d()
sage: s.jmol_repr(s.testing_render_params())[0][2] 'label "ABC",
sage: s._trans
(1.0, 1.0, 0)

sage.plot.text.text(string, xy, fontsize=10, rgbcolor=(0, 0, 1), horizontal_alignment='center', vertical_alignment='center', axis_coords=False, clip=False, **options)

Return a 2D text graphics object at the point (x, y).
Type `text.options` for a dictionary of options for 2D text.

2D OPTIONS:

- **fontsize** - How big the text is. Either an integer that specifies the size in points or a string which specifies a size (one of ‘xx-small’, ‘x-small’, ‘small’, ‘medium’, ‘large’, ‘x-large’, ‘xx-large’)
- **fontstyle** - A string either ‘normal’, ‘italic’ or ‘oblique’
- **rgbcolor** - The color as an RGB tuple
- **hue** - The color given as a hue
- **alpha** - A float (0.0 transparent through 1.0 opaque)
- **background_color** - The background color
- **rotation** - How to rotate the text: angle in degrees, vertical, horizontal
- **vertical_alignment** - How to align vertically: top, center, bottom
- **horizontal_alignment** - How to align horizontally: left, center, right
- **zorder** - The layer level in which to draw
- **clip** - (default: False) Whether to clip or not
- **axis_coords** - (default: False) If True, use axis coordinates, so that (0,0) is the lower left and (1,1) upper right, regardless of the x and y range of plotted values.
- **bounding_box** - A dictionary specifying a bounding box. Currently the text location.

**EXAMPLES:**

```sage
text("Sage graphics are really neat because they use matplotlib!", (2,12))
Graphics object consisting of 1 graphics primitive
```
Larger font, bold, colored red and transparent text:

```sage
text("I had a dream!", (2,12), alpha=0.3, fontsize='large', fontweight='bold', color='red')
Graphics object consisting of 1 graphics primitive
```
By setting `horizontal_alignment` to ‘left’ the text is guaranteed to be in the lower left no matter what:

```sage
text("I got a horse and he lives in a tree", (0,0), axis_coords=True, horizontal_alignment='left')
Graphics object consisting of 1 graphics primitive
```
Various rotations:

```sage
text("noitator", (0,0), rotation=45.0, horizontal_alignment='left', vertical_alignment='bottom')
Graphics object consisting of 1 graphics primitive
```
```sage
text("Sage is really neat!!",(0,0), rotation="vertical")
Graphics object consisting of 1 graphics primitive
```
You can also align text differently:
Sage graphics are really neat because they use matplotlib!
I had a dream!
I got a horse and he lives in a tree
1.2. Text in plots
Sage is really neat!!
sage: t1 = text("Hello", (1,1), vertical_alignment="top")
sage: t2 = text("World", (1,0.5), horizontal_alignment="left")
sage: t1 + t2  # render the sum
Graphics object consisting of 2 graphics primitives

You can save text as part of PDF output:
sage: text("sage", (0,0), rgbcolor=(0,0,0)).save(os.path.join(SAGE_TMP, 'a.pdf'))

Some examples of bounding box:
sage: bbox = {'boxstyle':"rarrow,pad=0.3", 'fc':"cyan", 'ec':"b", 'lw':2}
sage: text("I feel good", (1,2), bounding_box=bbox)
Graphics object consisting of 1 graphics primitive
sage: text("So good", (0,0), bounding_box={'boxstyle':"round", 'fc':"w"})
Graphics object consisting of 1 graphics primitive


A text with a background color:
I feel good
1.2. Text in plots

```
So good
```

```
1.0
0.5
0.5
1.0
1.0
0.5
0.5
1.0
So good
```

```
-1.0
-0.5
 0.5
 0.5
0.5
0.5
-1.0
```

```
-1.0
-0.5
 0.5
 0.5
-1.0
```
Use dollar signs for LaTeX and raw strings to avoid having to escape backslash characters:

```
sage: A = arc((0, 0), 1, sector=(0.0, RDF.pi()))
sage: a = sqrt(1./2.)
sage: PQ = point2d([(-a, a), (a, a)])
sage: botleft = dict(horizontal_alignment='left', vertical_alignment='bottom')
sage: botright = dict(horizontal_alignment='right', vertical_alignment='bottom')
sage: tp = text(r'$z_P = e^{3i\pi/4}$', (-a, a), **botright)
sage: tq = text(r'$Q = (\sqrt{2}/2, \sqrt{2}/2)$', (a, a), **botleft)
sage: A + PQ + tp + tq
```

Text coordinates must be 2D, an error is raised if 3D coordinates are passed:

```
sage: t = text("hi", (1, 2, 3))
Traceback (most recent call last):
... ValueError: use text3d instead for text in 3d
```

Use the \texttt{text3d} function for 3D text:
\[ z_P = e^{3\pi/4} \quad Q = \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \]
Or produce 2D text with coordinates \((x, y)\) and plot it in 3D (at \(z = 0\)):

```
sage: t = text("hi", (1, 2, 3))
sage: t.plot3d() # text at position (1, 2, 0)
```

Extra options will get passed on to `show()`, as long as they are valid. Hence this

```
sage: text("MATH IS AWESOME", (0, 0), fontsize=40, axes=False)
```

is equivalent to

```
sage: text("MATH IS AWESOME", (0, 0), fontsize=40).show(axes=False)
```

### 1.3 Colors

This module defines a `Color` object and helper functions (see, e.g., `hue()`, `rainbow()`), as well as a set of colors and colormaps to use with `Graphics` objects in Sage.

For a list of pre-defined colors in Sage, evaluate:

```
sage: sorted(colors)
['aliceblue', 'antiquewhite', 'aqua', 'aquamarine', 'automatic', ...]
```

Apart from ‘automatic’ which just an alias for ‘lightblue’, this list comprises the “official” W3C CSS3 / SVG colors.

For a list of color maps in Sage, evaluate:

```
sage: sorted(colormaps)
['Accent', 'Blues', 'BrBG', ...]
```

These are imported from matplotlib’s cm module.

```
class sage.plot.colors.Color(r=#0000ff, g=None, b=None, space='rgb')
    Bases: object
    An Red-Green-Blue (RGB) color model color object. For most consumer-grade devices (e.g., CRTs, LCDs, and printers), as well as internet applications, this is a point in the sRGB absolute color space. The Hue-Saturation-Lightness (HSL), Hue-Lightness-Saturation (HLS), and Hue-Saturation-Value (HSV) spaces are useful alternate representations, or coordinate transformations, of this space. Coordinates in all of these spaces are floating point values in the interval \([0.0, 1.0]\).

    Note: All instantiations of `Color` are converted to an internal RGB floating point 3-tuple. This is likely to degrade precision.

    INPUT:
        * r, g, b - either a triple of floats between 0 and 1, OR r - a color name string or HTML color hex string
        * space - a string (default: ‘rgb’); the coordinate system (other choices are ‘hsl’, ‘hls’, and ‘hsv’) in which to interpret a triple of floats
```

In the above code, Sage is designed to support various methods for creating 2D graphics, including the ability to specify text with coordinates and plot it in 3D. The module also includes a set of predefined colors and colormaps which can be used with Graphics objects. The `Color` class is described with an input section that includes parameters for specifying colors in different spaces and coordinate transformations.
EXAMPLES:

```python
sage: Color('purple')
RGB color (0.5019607843137255, 0.0, 0.5019607843137255)
sage: Color('#8000ff')
RGB color (0.5019607843137255, 0.0, 1.0)
sage: Color(0.5,0,1)
RGB color (0.5, 0.0, 1.0)
sage: Color(0.5, 1.0, 1, space='hsv')
RGB color (0.0, 1.0, 1.0)
sage: Color(0.25, 0.5, 0.5, space='hls')
RGB color (0.5000000000000001, 0.75, 0.25)
sage: Color(1, 0, 1/3, space='hsl')
RGB color (0.3333333333333333, 0.3333333333333333, 0.3333333333333333)
sage: from sage.plot.colors import chocolate
sage: Color(chocolate)
RGB color (0.8235294117647058, 0.4117647058823529, 0.11764705882352941)
```

blend(color, fraction=0.5)

Return a color blended with the given color by a given fraction. The algorithm interpolates linearly between the colors' corresponding R, G, and B coordinates.

INPUT:

- **color** - a `Color` instance or float-convertible 3-tuple/list; the color with which to blend this color
- **fraction** - a float-convertible number; the fraction of color to blend with this color

OUTPUT:

- a new `Color` instance

EXAMPLES:

```python
sage: from sage.plot.colors import red, blue, lime
sage: red.blend(blue)
RGB color (0.5, 0.0, 0.5)
sage: red.blend(blue, fraction=0.0)
RGB color (1.0, 0.0, 0.0)
sage: red.blend(blue, fraction=1.0)
RGB color (0.0, 0.0, 1.0)
sage: lime.blend((0.3, 0.5, 0.7))
RGB color (0.15, 0.75, 0.35)
sage: blue.blend(blue)
RGB color (0.0, 0.0, 1.0)
sage: red.blend(lime, fraction=0.3)
RGB color (0.7, 0.3, 0.0)
sage: blue.blend((0.0, 0.9, 0.2), fraction=0.2)
RGB color (0.0, 0.18000000000000002, 0.8400000000000001)
sage: red.blend(0.2)
Traceback (most recent call last):
...TypeError: 0.2000000000000000 must be a Color or float-convertible 3-tuple/list
```

darker(fraction=0.3333333333333333)

Return a darker “shade” of this RGB color by `blend()`-ing it with black. This is not an inverse of `lighter()`.
INPUT:
- `fraction` - a float (default: 1/3); blending fraction to apply

OUTPUT:
- a new instance of `Color`

EXAMPLES:

```python
sage: from sage.plot.colors import black
sage: vector(black.darker().rgb()) == vector(black.rgb())
True
sage: Color(0.4, 0.6, 0.8).darker(0.1)
RGB color (0.36000000000000004, 0.54, 0.72000000000000001)
sage: Color(1.2, 1.3, space='hsl').darker()
RGB color (0.24000000000000002, 0.20800000000000002, 0.16)
```

`hls()`

Return the Hue-Lightness-Saturation (HLS) coordinates of this color.

OUTPUT:
- a 3-tuple of floats

EXAMPLES:

```python
sage: Color(0.3, 0.5, 0.7, space='hls').hls()
(0.30000000000000004, 0.5, 0.7)
sage: Color(0.3, 0.5, 0.7, space='hsl').hls() # abs tol 1e-15
(0.30000000000000004, 0.7, 0.50000000000000001)
sage: Color('#aabbcc').hls() # abs tol 1e-15
(0.5833333333333334, 0.7333333333333334, 0.250000000000000017)
sage: from sage.plot.colors import orchid
sage: orchid.hls() # abs tol 1e-15
(0.8396226415094339, 0.6470588235294117, 0.5888888888888889)
```

`hsl()`

Return the Hue-Saturation-Lightness (HSL) coordinates of this color.

OUTPUT:
- a 3-tuple of floats

EXAMPLES:

```python
sage: Color(1,0,0).hsl()
(0.0, 1.0, 0.5)
sage: from sage.plot.colors import orchid
sage: orchid.hsl() # abs tol 1e-15
(0.8396226415094339, 0.5888888888888889, 0.6470588235294117)
sage: Color('#aabbcc').hsl() # abs tol 1e-15
(0.5833333333333334, 0.250000000000000017, 0.7333333333333334)
```

`hsv()`

Return the Hue-Saturation-Value (HSV) coordinates of this color.

OUTPUT:
- a 3-tuple of floats

EXAMPLES:

```python
sage: Color(1,0,0).hsv()
(0.0, 1.0, 0.5)
sage: from sage.plot.colors import orchid
sage: orchid.hsv() # abs tol 1e-15
(0.8396226415094339, 0.5888888888888889, 0.6470588235294117)
sage: Color('#aabbcc').hsv() # abs tol 1e-15
(0.5833333333333334, 0.250000000000000017, 0.7333333333333334)
```
EXAMPLES:

```
sage: from sage.plot.colors import red
sage: red.hsv()
(0.0, 1.0, 1.0)
sage: Color(1,1,1).hsv()
(0.0, 0.0, 1.0)
sage: Color('gray').hsv()
(0.0, 0.0, 0.5019607843137255)
```

`html_color()`

Return a HTML hex representation for this color.

OUTPUT:

- a string of length 7.

EXAMPLES:

```
sage: Color('yellow').html_color()
'#ffff00'
sage: Color('#fedcba').html_color()
'#fedcba'
sage: Color(0.0, 1.0, 0.0).html_color()
'#00ff00'
sage: from sage.plot.colors import honeydew
sage: honeydew.html_color()
'#f0fff0'
```

`lighter(fraction=0.3333333333333333)`

Return a lighter “shade” of this RGB color by `blend()`-ing it with white. This is **not** an inverse of `darker()`.

INPUT:

- fraction - a float (default: 1/3); blending fraction to apply

OUTPUT:

- a new instance of `Color`

EXAMPLES:

```
sage: from sage.plot.colors import khaki
sage: khaki.lighter()
RGB color (0.9607843137254903, 0.934640522875817, 0.6993464052287582)
sage: Color('white').lighter().darker()
RGB color (0.6666666666666667, 0.6666666666666667, 0.6666666666666667)
sage: Color('#abcdef').lighter(1/4)
RGB color (0.7529411764705882, 0.8529411764705883, 0.9529411764705882)
sage: Color(1, 0, 8/9, space='hsv').lighter()
RGB color (0.925925925925926, 0.925925925925926, 0.925925925925926)
```

`rgb()`

Return the underlying Red-Green-Blue (RGB) coordinates of this color.

OUTPUT:

- a 3-tuple of floats
EXAMPLES:

```
sage: Color(0.3, 0.5, 0.7).rgb()
(0.3, 0.5, 0.7)
sage: Color('#8000ff').rgb()
(0.5019607843137255, 0.0, 1.0)
sage: from sage.plot.colors import orange
sage: orange.rgb()
(1.0, 0.6470588235294118, 0.0)
sage: Color('magenta').rgb()
(1.0, 0.0, 1.0)
sage: Color(1, 0.7, 0.9, space='hsv').rgb()
(0.9, 0.2700000000000001, 0.2700000000000001)
```

class sage.plot.colors.Colormaps
Bases: collections.abc.MutableMapping

A dict-like collection of lazily-loaded matplotlib color maps. For a list of map names, evaluate:

```
sage: sorted(colormaps)
['Accent', 'Blues', ...]
```

load_maps()

If it’s necessary, loads matplotlib’s color maps and adds them to the collection.

EXAMPLES:

```
sage: from sage.plot.colors import Colormaps
sage: maps = Colormaps()
sage: len(maps.maps)
0
sage: maps.load_maps()
sage: len(maps.maps)>60
True
```

class sage.plot.colors.ColorsDict
Bases: dict

A dict-like collection of colors, accessible via key or attribute. For a list of color names, evaluate:

```
sage: sorted(colors)
['aliceblue', 'antiquewhite', 'aqua', 'aquamarine', ...]
```

sage.plot.colors.check_color_data(cfcm)

Make sure that the arguments are in order (coloring function, colormap).

This will allow users to use both possible orders.

EXAMPLES:

```
sage: from sage.plot.colors import check_color_data
sage: cf = lambda x,y : (x+y) % 1
sage: cm = colormaps.autumn
sage: check_color_data((cf, cm)) == (cf, cm)
True
sage: check_color_data((cm, cf)) == (cf, cm)
True
```
sage.plot.colors.float_to_html(r, g, b)
Convert a Red-Green-Blue (RGB) color tuple to a HTML hex color.

Each input value should be in the interval [0.0, 1.0]; otherwise, the values are first reduced modulo one (see mod_one()).

INPUT:
• r – a real number; the RGB color’s “red” intensity
• g – a real number; the RGB color’s “green” intensity
• b – a real number; the RGB color’s “blue” intensity

OUTPUT:
• a string of length 7, starting with ‘#’

EXAMPLES:

```sage
def from sage.plot.colors import float_to_html
float_to_html(1., 1., 0.)
'#ffffff'
float_to_html(.03, .06, .02)
'#070f05'
float_to_html('Color('brown').rgb())
'#a52a2a'
```

sage.plot.colors.float_to_integer(r, g, b)
Convert a Red-Green-Blue (RGB) color tuple to an integer.

Each input value should be in the interval [0.0, 1.0]; otherwise, the values are first reduced modulo one (see mod_one()).

INPUT:
• r – a real number; the RGB color’s “red” intensity
• g – a real number; the RGB color’s “green” intensity
• b – a real number; the RGB color’s “blue” intensity

OUTPUT:
• the integer \(256^2 r + 256 g + b\), where \(r, g, b\) are obtained from \(r, g, b\) by converting from the real interval [0.0, 1.0] to the integer range 0, 1, ..., 255.

EXAMPLES:

```sage
def from sage.plot.colors import float_to_integer
float_to_integer(1., 1., 0.)
16776960
float_to_integer(.03, .06, .02)
462597
float_to_integer('Color('brown').rgb())
10824234
```

sage.plot.colors.get_cmap(cmap)
Returns a color map (actually, a matplotlib Colormap object), given its name or a [mixed] list/tuple of RGB list/tuples and color names. For a list of map names, evaluate:
sage: sorted(colormaps)
['Accent', 'Blues', ...]

See rgbcolor() for valid list/tuple element formats.

INPUT:
- `cmap` - a string, list, tuple, or matplotlib.colors.Colormap; a string must be a valid color map name

OUTPUT:
- a matplotlib.colors.Colormap instance

EXAMPLES:

```python
sage: from sage.plot.colors import get_cmap
sage: get_cmap('jet')
<matplotlib.colors.LinearSegmentedColormap object at 0x...>
sage: get_cmap([(0,0,0), (0.5,0.5,0.5), (1,1,1)])
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap(['green','lightblue','blue'])
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap(((0.5,0.3,0.2), [1.0,0.0,0.5], 'purple', Color(0.5,0.5,1, space='hsv')))
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap('jolies')
Traceback (most recent call last):
...
RuntimeError: Color map jolies not known (type import matplotlib.cm; matplotlib.cm.datad.keys() for valid names)
sage: get_cmap('mpl')
Traceback (most recent call last):
...
RuntimeError: Color map mpl not known (type import matplotlib.cm; matplotlib.cm.datad.keys() for valid names)
```

sage.plot.colors.html_to_float(c)
Convert a HTML hex color to a Red-Green-Blue (RGB) tuple.

INPUT:
- `c` - a string: a valid HTML hex color

OUTPUT:
- a RGB 3-tuple of floats in the interval [0.0, 1.0]

EXAMPLES:

```python
sage: from sage.plot.colors import html_to_float
sage: html_to_float('#fff')
(1.0, 1.0, 1.0)
sage: html_to_float('#abcdef')
(0.6705882352941176, 0.803921568627451, 0.9372549019607843)
sage: html_to_float('#123xyz')
Traceback (most recent call last):
...
ValueError: invalid literal for int() with base 16: '3x'
```
sage.plot.colors.hue(h, s=1, v=1)
Convert a Hue-Saturation-Value (HSV) color tuple to a valid Red-Green-Blue (RGB) tuple. All three inputs
should lie in the interval [0.0, 1.0]; otherwise, they are reduced modulo 1 (see mod_one()). In particular \(h=0\)
and \(h=1\) yield red, with the intermediate hues orange, yellow, green, cyan, blue, and violet as \(h\) increases.

This function makes it easy to sample a broad range of colors for graphics:

```python
sage: p = Graphics()
sage: for phi in xrange(0, 2 * pi, 1 / pi):
    ....:     p += plot(sin(x + phi), (x, -7, 7), rgbcolor = hue(phi))
sage: p
Graphics object consisting of 20 graphics primitives
```

**INPUT:**

- \(h\) - a number; the color’s hue
- \(s\) - a number (default: 1); the color’s saturation
- \(v\) - a number (default: 1); the color’s value

**OUTPUT:**

- a RGB 3-tuple of floats in the interval [0.0, 1.0]

**EXAMPLES:**

```python
sage: hue(0.6)
(0.0, 0.40000000000000036, 1.0)
sage: from sage.plot.colors import royalblue
sage: royalblue
RGB color (0.2549019607843137, 0.4117647058823529, 0.8823529411764706)
sage: hue(*royalblue.hsv())
(0.2549019607843137, 0.4117647058823529, 0.8823529411764706)
sage: hue(.5, .5, .5)
(0.25, 0.5, 0.5)
```

**Note:** The HSV to RGB coordinate transformation itself is given in the source code for the Python library’s
colorsys module:

```python
sage: from colorsys import hsv_to_rgb # not tested
sage: hsv_to_rgb?? # not tested
```

sage.plot.colors.mod_one(x)
Reduce a number modulo 1.

**INPUT:**

- \(x\) - an instance of Integer, int, RealNumber, etc.; the number to reduce

**OUTPUT:**

- a float

**EXAMPLES:**

```python
sage: from sage.plot.colors import mod_one
sage: mod_one(1)
```
sage.mod_one(7.0)
0.0
sage: mod_one(-11/7)
0.4285714285714286
sage: mod_one(pi) + mod_one(-pi)
1.0

sage.plot.colors.rainbow(n, format='hex')
Returns a list of colors sampled at equal intervals over the spectrum, from Hue-Saturation-Value (HSV) coordinates (0, 1, 1) to (1, 1, 1). This range is red at the extremes, but it covers orange, yellow, green, cyan, blue, violet, and many other hues in between. This function is particularly useful for representing vertex partitions on graphs.

INPUT:
• n - a number; the length of the list
• format - a string (default: 'hex'); the output format for each color in the list; the other choice is 'rgbtuple'

OUTPUT:
• a list of strings or RGB 3-tuples of floats in the interval [0.0, 1.0]

EXAMPLES:
sage: from sage.plot.colors import rainbow
dsage: rainbow(7)
['#ff0000', '#ffda00', '#48ff00', '#00ff91', '#0091ff', '#4800ff', '#ff00da']
sage: rainbow(int(7))
['#ff0000', '#ffda00', '#48ff00', '#00ff91', '#0091ff', '#4800ff', '#ff00da']
sage: rainbow(7, 'rgbtuple')
[(1.0, 0.0, 0.0), (1.0, 0.8571428571428571, 0.0), (0.2857142857142857, 1.0, 0.0), ...
 (0.0, 0.0, 0.0), (0.8571428571428571, 1.0, 0.0), (0.2857142857142857, 0.0, 1.0), (1.0, 0.0, 0.0), (0.8571428571428571, 0.0, 1.0), (1.0, 0.0, 0.0)]

AUTHORS:
• Robert L. Miller
• Karl-Dieter Crisman (directly use hsv_to_rgb() for hues)

sage.plot.colors.rgbcolor(c, space='rgb')
Convert a color (string, tuple, list, or Color) to a mod-one reduced (see mod_one()) valid Red-Green-Blue (RGB) tuple. The returned tuple is also a valid matplotlib RGB color.

INPUT:
• c - a Color instance, string (name or HTML hex), 3-tuple, or 3-list; the color to convert
• space - a string (default: 'rgb'); the color space coordinate system (other choices are ‘hsl’, ‘hls’, and ‘hsv’) in which to interpret a 3-tuple or 3-list

OUTPUT:
• a RGB 3-tuple of floats in the interval [0.0, 1.0]

EXAMPLES:
.. code-block:: python

    sage: from sage.plot.colors import rgbcolor
    sage: rgbcolor(Color(0.25, 0.4, 0.9))
    (0.25, 0.4, 0.9)
    sage: rgbcolor('purple')
    (0.5019607843137255, 0.0, 0.5019607843137255)
    sage: rgbcolor('#fa0')
    (1.0, 0.6666666666666666, 0.0)
    sage: rgbcolor('#ffffff')
    (1.0, 1.0, 1.0)
    sage: rgbcolor((1,1/2,1/3))
    (1.0, 0.5, 0.3333333333333333)
    sage: rgbcolor((0.5,0.75,1), space='hls')
    (0.5, 0.9999999999999999, 1.0)
    sage: rgbcolor((0.5,1.0,0.75), space='hsl')
    (0.5, 0.9999999999999999, 1.0)
    sage: rgbcolor([1,2,255]) # WARNING -- numbers are reduced mod 1!!
    (1.0, 0.0, 0.0)
    sage: rgbcolor('#abcd')
    Traceback (most recent call last):
    ...  ValueError: color hex string (= 'abcd') must have length 3 or 6
    sage: rgbcolor('fff')
    Traceback (most recent call last):
    ...  ValueError: unknown color 'fff'
    sage: rgbcolor(1)
    Traceback (most recent call last):
    ...  TypeError: '1' must be a Color, list, tuple, or string
    sage: rgbcolor((0.2,0.8,1), space='grassmann')
    Traceback (most recent call last):
    ...  ValueError: space must be one of 'rgb', 'hsv', 'hsl', 'hls'
    sage: rgbcolor((0.4, 0.1])
    Traceback (most recent call last):
    ...  ValueError: color list or tuple '[0.40000000000000000000000000, 0.10000000000000000000000000]' must have 3 entries, one for each RGB, HSV, HLS, or HSL channel

:sage: sage.plot.colors.to_mpl_color(c, space='rgb')

Convert a color (string, tuple, list, or Color) to a mod-one reduced (see :func:`mod_one()`) valid Red-Green-Blue (RGB) tuple. The returned tuple is also a valid matplotlib RGB color.

**INPUT:**

- c - a Color instance, string (name or HTML hex), 3-tuple, or 3-list; the color to convert
- space - a string (default: 'rgb'); the color space coordinate system (other choices are 'hsv', 'hls', and 'hsl') in which to interpret a 3-tuple or 3-list

**OUTPUT:**

1.3. Colors

- a RGB 3-tuple of floats in the interval [0.0, 1.0]

EXAMPLES:

```
sage: from sage.plot.colors import rgbcolor
sage: rgbcolor(Color(0.25, 0.4, 0.9))
(0.25, 0.4, 0.9)
sage: rgbcolor('purple')
(0.5019607843137255, 0.0, 0.5019607843137255)
sage: rgbcolor('#fa0')
(1.0, 0.6666666666666666, 0.0)
sage: rgbcolor('#ffffff')
(1.0, 1.0, 1.0)
sage: rgbcolor((1,1/2,1/3))
(1.0, 0.5, 0.3333333333333333)
sage: rgbcolor((1,1/2,1/3), space='hsv')
(1.0, 0.0, 0.0)
sage: rgbcolor((0.5,0.75,1), space='hls')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor((0.5,1.0,0.75), space='hsl')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor([1,2,255])  # WARNING -- numbers are reduced mod 1!!
(1.0, 0.0, 0.0)
sage: rgbcolor('#abcd')
Traceback (most recent call last):
  ... ValueError: color hex string (= 'abcd') must have length 3 or 6
sage: rgbcolor('fff')
Traceback (most recent call last):
  ... ValueError: unknown color 'fff'
sage: rgbcolor(1)
Traceback (most recent call last):
  ... TypeError: '1' must be a Color, list, tuple, or string
sage: rgbcolor((0.2,0.8,1), space='grassmann')
Traceback (most recent call last):
  ... ValueError: space must be one of 'rgb', 'hsv', 'hsl', 'hls'
sage: rgbcolor([0.4, 0.1])
Traceback (most recent call last):
  ... ValueError: color list or tuple '[0.400000000000000, 0.100000000000000]' must have
  →3 entries, one for each RGB, HSV, HLS, or HSL channel
```
Animations are generated from a list (or other iterable) of graphics objects. Images are produced by calling the `save_image` method on each input object, creating a sequence of PNG files. These are then assembled to various target formats using different tools. In particular, the convert program from ImageMagick can be used to generate an animated GIF file. FFmpeg (with the command line program ffmpeg) provides support for various video formats, but also an alternative method of generating animated GIFs. For browsers which support it, APNG can be used as another alternative which works without any extra dependencies.

**Warning:** Note that ImageMagick and FFmpeg are not included with Sage, and must be installed by the user. On unix systems, type `which convert` at a command prompt to see if `convert` (part of the ImageMagick suite) is installed. If it is, you will be given its location. Similarly, you can check for ffmpeg with `which ffmpeg`. See the websites of ImageMagick or FFmpeg for installation instructions.

EXEMPLARY:

The sine function:

```
sage: sines = [plot(c*sin(x), (-2*pi,2*pi), color=Color(c,0,0), ymin=-1, ymax=1) for c in sxrange(0,1,.2)]
sage: a = animate(sines)
sage: a  # optional -- ImageMagick
Animation with 5 frames
sage: a.show()  # optional -- ImageMagick
```

Animate using FFmpeg instead of ImageMagick:

```
sage: f = tmp_filename(ext='.gif')
sage: a.save(filename=f, use_ffmpeg=True)  # optional -- ffmpeg
```

Animate as an APNG:

```
sage: a.apng()  # long time
```

An animated `sage.plot.multigraphics.GraphicsArray` of rotating ellipses:

```
sage: E = animate((graphics_array([[ellipse((0,0),a,b,angle=t,xmin=-3,xmax=3)+circle((0,0),3,color='blue') for a in range(1,3)] for b in range(2,4)]) for t in sxrange(0,pi/4,.15))
sage: str(E)  # animations produced from a generator do not have a known length
'Animation with unknown number of frames'
sage: E.show()  # optional -- ImageMagick
```

A simple animation of a circle shooting up to the right:

```
sage: c = animate([circle((i,i), 1-1/(i+1), hue=i/10) for i in srange(0,2,.2)],
               xmin=0,ymin=0,xmax=2,ymax=2,figsize=[2,2])
sage: c.show()  # optional -- ImageMagick
```

Animations of 3d objects:

```
sage: var('s,t')
(s, t)
```
sage: def sphere_and_plane(x):
    ....:     return sphere((0,0,0),1,color='red',opacity=.5)+parametric_plot3d([t,x,s],(s,-
   ˓→1,1),(t,-1,1),color='green',opacity=.7)
sage: sp = animate([sphere_and_plane(x) for x in sxrange(-1,1,.3)])
sage: sp[0]  # first frame
Graphics3d Object
sage: sp[-1]  # last frame
Graphics3d Object
sage: sp.show()  # optional -- ImageMagick

sage: (x,y,z) = var('x,y,z')
sage: def frame(t):
    ....:     return implicit_plot3d((x^2 + y^2 + z^2), (x, -2, 2), (y, -2, 2), (z, -2, 2),
   ˓→plot_points=60, contour=[1,3,5], region=lambda x,y,z: x<=t or y>=t or z<=t)
sage: a = animate([frame(t) for t in srange(.01,1.5,.2)])
sage: a[0]  # long time
Graphics3d Object
sage: a.show()  # optional -- ImageMagick

If the input objects do not have a save_image method, then the animation object attempts to make an image by calling its internal method sage.plot.animate.Animation.make_image(). This is illustrated by the following example:

sage: t = var('t')
sage: a = animate((sin(c*pi*t) for c in sxrange(1,2,.2)))
sage: a.show()  # optional -- ImageMagick

AUTHORS:

• William Stein
• John Palmieri
• Niles Johnson (2013-12): Expand to animate more graphics objects
• Martin von Gagern (2014-12): Added APNG support
• Joshua Campbell (2020): interactive animation via Three.js viewer

REFERENCES:

• ImageMagick
• FFmpeg
• APNG
• browsers which support it

class sage.plot.animate.APngAssembler(out, num_frames, num_plays=0, delay=200, delay_denominator=100)

Bases: object

Builds an APNG (Animated PNG) from a sequence of PNG files. This is used by the sage.plot.animate.Animation.apng() method.

This code is quite simple; it does little more than copying chunks from input PNG files to the output file. There is no optimization involved. This does not depend on external programs or libraries.

INPUT:
• out – a file opened for binary writing to which the data will be written
• num_frames – the number of frames in the animation
• num_plays – how often to iterate, 0 means infinitely
• delay – numerator of the delay fraction in seconds
• delay_denominator – denominator of the delay in seconds

EXAMPLES:

```python
sage: from sage.plot.animate import APngAssembler
sage: def assembleAPNG():
....: a = animate([sin(x + float(k)) for k in srange(0,2*pi,0.7)],
....: xmin=0, xmax=2*pi, figsize=[2,1])
....: pngdir = a.png()
....: outfile = sage.misc.temporary_file.tmp_filename(ext='\.png')
....: with open(outfile, "wb") as f:
....:     apng = APngAssembler(f, len(a))
....:     for i in range(len(a)):
....:         png = os.path.join(pngdir, "{:08d}.png".format(i))
....:         apng.add_frame(png, delay=10*i + 10)
....: return outfile
sage: assembleAPNG()  # long time
'...png'
```

**add_frame**(pngfile, delay=None, delay_denominator=None)

Adds a single frame to the APNG file.

**set_default**(pngfile)

Adds a default image for the APNG file.

This image is used as a fallback in case some application does not understand the APNG format. This method must be called prior to any calls to the **add_frame** method, if it is called at all. If it is not called, then the first frame of the animation will be the default.

**class sage.plot.animate.Animation**(v=None, **kwds)

Bases: sage.misc.fast_methods.WithEqualityById, sage.structure.sage_object.SageObject

Return an animation of a sequence of plots of objects.

**INPUT:**

• v - iterable of Sage objects. These should preferably be graphics objects, but if they aren’t then **make_image()** is called on them.

• xmin, xmax, ymin, ymax - the ranges of the x and y axes.
**kwds** - all additional inputs are passed onto the rendering command. E.g., use figsize to adjust the resolution and aspect ratio.

**EXAMPLES:**

```python
sage: a = animate([(sin(x + float(k))) for k in srange(0,2*pi,0.3)],
       xmin=0, xmax=2*pi, figsize=[2,1])
```

The `show()` method takes arguments to specify the delay between frames (measured in hundredths of a second, default value 20) and the number of iterations (default value 0, which means to iterate forever). To iterate 4 times with half a second between each frame:

```python
sage: a.show(delay=50, iterations=4) # optional -- ImageMagick
```

An animation of drawing a parabola:

```python
sage: step = 0.1
sage: L = Graphics()

sage: for i in srange(0,1,step):
   \[ L += line([(i,i^2),(i+step,(i+step)^2)], rgbcolor=(1,0,0), thickness=2) \]

sage: a = animate(L, xmin=0, ymin=0)
```

`apng()` creates an animated PNG composed from rendering the graphics objects in self. Return the absolute path to that file.

Notice that not all web browsers are capable of displaying APNG files, though they should still present the first frame of the animation as a fallback.

The generated file is not optimized, so it may be quite large.

**Input:**

- `delay` - (default: 20) delay in hundredths of a second between frames
- `savefile` - file that the animated gif gets saved to
- `iterations` - integer (default: 0); number of iterations of animation. If 0, loop forever.
- `show_path` - boolean (default: False); if True, print the path to the saved file

**EXAMPLES:**

```python
sage: a = animate([(sin(x + float(k))) for k in srange(0,2*pi,0.7)],
       xmin=0, xmax=2*pi, figsize=[2,1])
```

(continues on next page)
If the individual frames have different sizes, an error will be raised:

```
sage: a = animate([plot(sin(x), (x, 0, k)) for k in range(1,4)],
.....:       ymin=-1, ymax=1, aspect_ratio=1, figsize=[2,1])
sage: a.apng()  # long time
Traceback (most recent call last):
...    ValueError: Chunk IHDR mismatch
```

**ffmpeg**

```
(savefile=None, show_path=False, output_format=None, ffmpeg_options='', delay=None,
iterations=0, pix_fmt='rgb24')
```

Return a movie showing an animation composed from rendering the frames in `self`.

This method will only work if ffmpeg is installed. See https://www.ffmpeg.org for information about ffmpeg.

**INPUT:**

- `savefile` – file that the mpeg gets saved to.

**Warning:** This will overwrite `savefile` if it already exists.

- `show_path` – boolean (default: False); if True, print the path to the saved file
- `output_format` - string (default: None); format and suffix to use for the video. This may be 'mpg', 'mpeg', 'avi', 'gif', or any other format that ffmpeg can handle. If this is None and the user specifies `savefile` with a suffix, say `savefile='animation.avi'`, try to determine the format ('avi' in this case) from that file name. If no file is specified or if the suffix cannot be determined, 'mpg' is used.
- `ffmpeg_options` - string (default: ''); this string is passed directly to ffmpeg.
- `delay` - integer (default: None); delay in hundredths of a second between frames. The framerate is 100/delay. This is not supported for mpeg files: for mpegs, the frame rate is always 25 fps.
- `iterations` - integer (default: 0); number of iterations of animation. If 0, loop forever. This is only supported for animated gif output and requires ffmpeg version 0.9 or later. For older versions, set `iterations=None`.
- `pix_fmt` - string (default: 'rgb24'); used only for gif output. Different values such as 'rgb8' or 'pal8' may be necessary depending on how ffmpeg was installed. Set `pix_fmt=None` to disable this option.

If `savefile` is not specified: in notebook mode, display the animation; otherwise, save it to a default file name. Use `sage.misc.verbose.set_verbose()` with `level=1` to see additional output.

**EXAMPLES:**

```
sage: a = animate([sin(x + float(k)) for k in srange(0,2*pi,0.7)],
.....:      xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2,1])
sage: td = tmp_dir()
sage: a.ffmpeg(savefile=td + 'new.mpg')  # optional -- ffmpeg
sage: a.ffmpeg(savefile=td + 'new.avi')  # optional -- ffmpeg
```

(continues on next page)
sage: a.ffmpeg(savefile=td + 'new.gif') # optional -- ffmpeg
sage: a.ffmpeg(savefile=td + 'new.mpg', show_path=True) # optional -- ffmpeg

Animation saved to .../new.mpg.

Note: If ffmpeg is not installed, you will get an error message like this:

```python
FeatureNotPresentError: ffmpeg is not available.
Executable 'ffmpeg' not found on PATH.
Further installation instructions might be available at https://www.ffmpeg.org/.
```

gif(delay=20, savefile=None, iterations=0, show_path=False, use_ffmpeg=False)

Returns an animated gif composed from rendering the graphics objects in self.

This method will only work if either (a) the ImageMagick software suite is installed, i.e., you have the convert command or (b) ffmpeg is installed. See the web sites of ImageMagick and FFmpeg for more details. By default, this produces the gif using convert if it is present. If this can't find convert or if use_ffmpeg is True, then it uses ffmpeg instead.

INPUT:

- delay - (default: 20) delay in hundredths of a second between frames
- savefile - file that the animated gif gets saved to
- iterations - integer (default: 0); number of iterations of animation. If 0, loop forever.
- show_path - boolean (default: False); if True, print the path to the saved file
- use_ffmpeg - boolean (default: False); if True, use 'ffmpeg' by default instead of 'convert'.

If savefile is not specified: in notebook mode, display the animation; otherwise, save it to a default file name.

EXAMPLES:

```python
sage: a = animate([sin(x + float(k)) for k in srange(0,2*pi,0.7)],
....: xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2,1])
sage: td = tmp_dir()
sage: a.gif() # not tested
sage: a.gif(savefile=td + 'my_animation.gif', delay=35, iterations=3) #
    # optional -- ImageMagick
sage: with open(td + 'my_animation.gif', 'rb') as f: print(b'GIF8' in f.read())
True
sage: a.gif(savefile=td + 'my_animation.gif', show_path=True) # optional --
    # ImageMagick
    Animation saved to .../my_animation.gif.
sage: a.gif(savefile=td + 'my_animation_2.gif', show_path=True, use_
    # ffmpeg=True) # optional -- ffmpeg
    Animation saved to .../my_animation_2.gif.
```

Note: If neither ffmpeg nor ImageMagick is installed, you will get an error message like this:
Error: Neither ImageMagick nor ffmpeg appears to be installed. Saving an animation to a GIF file or displaying an animation requires one of these packages, so please install one of them and try again.


**graphics_array**(ncols=3)

Return a `sage.plot.multigraphics.GraphicsArray` with plots of the frames of this animation, using the given number of columns. The frames must be acceptable inputs for `sage.plot.multigraphics.GraphicsArray`.

EXAMPLES:

```python
sage: E = EllipticCurve('37a')
sage: v = [E.change_ring(GF(p)).plot(pointsize=30) for p in [97, 101, 103, 107]]
sage: a = animate(v, xmin=0, ymin=0, axes=False)  
   # optional -- ImageMagick
sage: a  
Animation with 4 frames
sage: a.show()  
# optional -- ImageMagick
```

Modify the default arrangement of array:

```python
sage: g = a.graphics_array(); print(g)
Graphics Array of size 2 x 3
sage: g.show(figsize=[6,3])  
# not tested
```

Specify different arrangement of array and save it with a given file name:

```python
sage: g = a.graphics_array(ncols=2); print(g)
Graphics Array of size 2 x 2
sage: f = tmp_filename(ext='.png')
sage: g.save(f)
```

Frames can be specified as a generator too; it is internally converted to a list:

```python
sage: t = var('t')
sage: b = animate((plot(sin(c*pi*t)) for c in sxrange(1,2,.2)))
sage: g = b.graphics_array()
sage: g
Graphics Array of size 2 x 3
```

**interactive**(**kwds**)

Create an interactive depiction of the animation.

INPUT:

- **kwds** – any of the viewing options accepted by `show()` are valid as keyword arguments to this function and they will behave in the same way. Those that are animation-related and recognized by the Three.js viewer are: `animate`, `animation_controls`, `auto_play`, `delay`, and `loop`.

OUTPUT:

A 3D graphics object which, by default, will use the Three.js viewer.

EXAMPLES:
sage: frames = [point3d((sin(x), cos(x), x)) for x in (0, pi/16, ..., 2*pi)]
sage: animate(frames).interactive(online=True)
Graphics3d Object

Works with frames that are 2D or 3D graphics objects or convertible to 2D or 3D graphics objects via a
plot or plot3d method:

sage: frames = [dodecahedron(), circle(center=(0,0), radius=1), x^2]
sage: animate(frames).interactive(online=True, delay=100)
Graphics3d Object

See also:

Three.js JavaScript WebGL Renderer

make_image(frame, filename, **kwds)
Given a frame which has no save_image() method, make a graphics object and save it as an image with
the given filename. By default, this is sage.plot.plot.plot(). To make animations of other objects,
override this method in a subclass.

EXAMPLES:

sage: from sage.plot.animate import Animation
sage: class MyAnimation(Animation):
....:     def make_image(self, frame, filename, **kwds):
....:         P = parametric_plot(frame[0], frame[1], **frame[2])
....:         P.save_image(filename, **kwds)

sage: t = var('t')
sage: x = lambda t: cos(t)
sage: y = lambda n,t: sin(t)/n
sage: B = MyAnimation([[x(t), y(i+1,t)],(t,0,1), {'color':Color((1,0,i/4)),
    # 'aspect_ratio':1, 'ymax':1}) for i in range(4)]

sage: d = B.png(); v = os.listdir(d); v.sort(); v
# long time
['00000000.png', '00000001.png', '00000002.png', '00000003.png']
sage: B.show()  # not tested

sage: class MyAnimation(Animation):
....:     def make_image(self, frame, filename, **kwds):
....:         G = frame.plot()
....:         G.set_axes_range(floor(G.xmin()),ceil(G.xmax()),floor(G.ymin()),
            # ceil(G.ymax()))
....:         G.save_image(filename, **kwds)

sage: B = MyAnimation([graphs.CompleteGraph(n) for n in range(7,11)], figsize=5)
sage: d = B.png()
sage: v = os.listdir(d); v.sort(); v
['00000000.png', '00000001.png', '00000002.png', '00000003.png']
sage: B.show()  # not tested

png(dir=None)
Render PNG images of the frames in this animation, saving them in dir. Return the absolute path to that
directory. If the frames have been previously rendered and dir is None, just return the directory in which
ey are stored.
When \texttt{dir} is other than \texttt{None}, force re-rendering of frames.

**INPUT:**
- \texttt{dir} – Directory in which to store frames. Default \texttt{None}; in this case, a temporary directory will be created for storing the frames.

**OUTPUT:**
Absolute path to the directory containing the PNG images

**EXAMPLES:**

```sage
sage: a = animate([plot(x^2 + n) for n in range(4)], ymin=0, ymax=4)
sage: d = a.png(); v = os.listdir(d); v.sort(); v
# long time
['00000000.png', '00000001.png', '00000002.png', '00000003.png']
```

**save**(*filename=None, show\_path=False, use\_ffmpeg=False, **kwds*)
Save this animation.

**INPUT:**
- \texttt{filename} - (default: \texttt{None}) name of save file
- \texttt{show\_path} - boolean (default: \texttt{False}); if True, print the path to the saved file
- \texttt{use\_ffmpeg} - boolean (default: \texttt{False}); if True, use ‘ffmpeg’ by default instead of ‘convert’ when creating GIF files.

If \texttt{filename} is \texttt{None}, then in notebook mode, display the animation; otherwise, save the animation to a GIF file. If \texttt{filename} ends in \texttt{.html}, save an \texttt{interactive()} version of the animation to an HTML file that uses the Three.js viewer. If \texttt{filename} ends in \texttt{.sobj}, save to an sobj file. Otherwise, try to determine the format from the filename extension (\texttt{.mpg}, \texttt{.gif}, \texttt{.avi}, etc.). If the format cannot be determined, default to GIF.

For GIF files, either \texttt{ffmpeg} or the ImageMagick suite must be installed. For other movie formats, \texttt{ffmpeg} must be installed. \texttt{sobj} and HTML files can be saved with no extra software installed.

**EXAMPLES:**

```sage
sage: a = animate([sin(x + float(k)) for k in srange(0,2*pi,0.7)],
                 xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2,1])
sage: td = tmp\_dir()
sage: a.save()  # not tested
sage: a.save(td + 'wave\_gif.png')  # optional -- ImageMagick
sage: a.save(td + 'wave.gif', show\_path=True)  # optional -- ImageMagick
Animation saved to file \texttt{.../wave.gif}.
sage: a.save(td + 'wave\_avi.png', show\_path=True)  # optional -- ffmpeg
Animation saved to file \texttt{.../wave.\_avi}.
sage: a.save(td + 'wave0.sobj')
sage: a.save(td + 'wave1.sobj', show\_path=True)
Animation saved to file \texttt{.../wave1.sobj}.
sage: a.save(td + 'wave0.html', online=True)
sage: a.save(td + 'wave1.html', show\_path=True, online=True)
Animation saved to file \texttt{.../wave1.html}.
```

**show**(*delay=None, iterations=None, **kwds*)
Show this animation immediately.

1.4. Animated plots 173
This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

INPUT:

- delay – (default: 20) delay in hundredths of a second between frames.
- iterations – integer (default: 0); number of iterations of animation. If 0, loop forever.
- format - (default: gif) format to use for output. Currently supported formats are: gif, ogg, webm, mp4, flash, matroska, avi, wmv, quicktime.

OUTPUT:

This method does not return anything. Use `save()` if you want to save the figure as an image.

**Note:** Currently this is done using an animated gif, though this could change in the future. This requires that either ffmpeg or the ImageMagick suite (in particular, the `convert` command) is installed.

See also the `ffmpeg()` method.

**EXAMPLES:**

```sage
a = animate([sin(x + float(k)) for k in srange(0, 2*pi, 0.7)],
....:   xmin=0, xmax=2*pi, figsize=[2,1])
sage: a.show()  # optional -- ImageMagick
```

The preceding will loop the animation forever. If you want to show only three iterations instead:

```sage
sage: a.show(iterations=3)  # optional -- ImageMagick
```

To put a half-second delay between frames:

```sage
sage: a.show(delay=50)  # optional -- ImageMagick
```

You can also make use of the HTML5 video element in the Sage Notebook:

```sage
sage: a.show(format="ogg")  # optional -- ffmpeg
sage: a.show(format="webm")  # optional -- ffmpeg
sage: a.show(format="mp4")  # optional -- ffmpeg
sage: a.show(format="webm", iterations=1)  # optional -- ffmpeg
```

Other backends may support other file formats as well:

```sage
sage: a.show(format="flash")  # optional -- ffmpeg
sage: a.show(format="matroska")  # optional -- ffmpeg
sage: a.show(format="avi")  # optional -- ffmpeg
sage: a.show(format="wmv")  # optional -- ffmpeg
sage: a.show(format="quicktime")  # optional -- ffmpeg
```

**Note:** If you don’t have ffmpeg or ImageMagick installed, you will get an error message like this:

```sage
Error: Neither ImageMagick nor ffmpeg appears to be installed. Saving an animation to a GIF file or displaying an animation requires one of these packages, so please install one of them and try again.
```
sage.plot.animate.animate(frames, **kwds)

Animate a list of frames by creating a sage.plot.animate.Animation object.

EXAMPLES:

```
sage: t = var('t')
sage: a = animate((cos(c*pi*t) for c in sxrange(1,2,.2)))
sage: a.show()  # optional -- ImageMagick
```

See also sage.plot.animate for more examples.
CHAPTER TWO

FUNCTION AND DATA PLOTS

2.1 Complex Plots

class sage.plot.complex_plot.ComplexPlot(rgb_data, x_range, y_range, options)
Bases: sage.plot.primitive.GraphicPrimitive

The GraphicsPrimitive to display complex functions in using the domain coloring method

INPUT:
• rgb_data – An array of colored points to be plotted.
• x_range – A minimum and maximum x value for the plot.
• y_range – A minimum and maximum y value for the plot.

get_minmax_data()
Return a dictionary with the bounding box data.

EXAMPLES:

sage: p = complex_plot(lambda z: z, (-1, 2), (-3, 4))
sage: sorted(p.get_minmax_data().items())
[('xmax', 2.0), ('xmin', -1.0), ('ymax', 4.0), ('ymin', -3.0)]

sage: p = complex_plot(lambda z: z, (1, 2), (3, 4))
sage: sorted(p.get_minmax_data().items())
[('xmax', 2.0), ('xmin', 1.0), ('ymax', 4.0), ('ymin', 3.0)]

sage.plot.complex_plot.complex_plot(f, x_range, y_range, plot_points=100, interpolation='catrom', **options)

complex_plot takes a complex function of one variable, \( f(z) \) and plots output of the function over the specified x_range and y_range as demonstrated below. The magnitude of the output is indicated by the brightness (with zero being black and infinity being white) while the argument is represented by the hue (with red being positive real, and increasing through orange, yellow,... as the argument increases).

complex_plot(f, (xmin, xmax), (ymin, ymax), ...)

INPUT:
• f – a function of a single complex value \( x + iy \)
• (xmin, xmax) – 2-tuple, the range of x values
• (ymin, ymax) – 2-tuple, the range of y values

The following inputs must all be passed in as named parameters:
• plot_points – integer (default: 100); number of points to plot in each direction of the grid
• **interpolation** – string (default: 'catrom'), the interpolation method to use: 'bilinear', 'bicubic', 'spline16', 'spline36', 'quadric', 'gaussian', 'sinc', 'bessel', 'mitchell', 'lanczos', 'catrom', 'hermite', 'hanning', 'hamming', 'kaiser'

**EXAMPLES:**

Here we plot a couple of simple functions:

```python
sage: complex_plot(sqrt(x), (-5, 5), (-5, 5))
Graphics object consisting of 1 graphics primitive
```

```

```

```

```

```

```

A function with some nice zeros and a pole:
2.1. Complex Plots
sage: f(z) = z^5 + z - 1 + 1/z
sage: complex_plot(f, (-3, 3), (-3, 3))
Graphics object consisting of 1 graphics primitive

Here is the identity, useful for seeing what values map to what colors:

sage: complex_plot(lambda z: z, (-3, 3), (-3, 3))
Graphics object consisting of 1 graphics primitive

The Riemann Zeta function:

sage: complex_plot(zeta, (-30,30), (-30,30))
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to show(), as long as they are valid:

sage: complex_plot(lambda z: z, (-3, 3), (-3, 3), figsize=[1,1])
Graphics object consisting of 1 graphics primitive

sage: complex_plot(lambda z: z, (-3, 3), (-3, 3)).show(figsize=[1,1]) # These are equivalent

sage.plot.complex_plot.complex_to_rgb(z_values)

INPUT:
2.1. Complex Plots
• \texttt{z_values} – A grid of complex numbers, as a list of lists

\textbf{OUTPUT:}

An $N \times M \times 3$ floating point Numpy array $X$, where $X[i,j]$ is an (r,g,b) tuple.

\textbf{EXAMPLES:}

```python
sage: from sage.plot.complex_plot import complex_to_rgb
sage: complex_to_rgb([[0, 1, 1000]])
array([[0.0, 0.0, 0.0],
       [0.77172568, 0.0, 0.0],
       [1.0, 0.64421177, 0.64421177]])
```

```
2.2 Contour Plots

class sage.plot.contour_plot.ContourPlot(xy_data_array, xrange, yrange, options)
Bases: sage.plot.primitive.GraphicPrimitive

Primitive class for the contour plot graphics type.

See \texttt{contour_plot?} for help actually doing contour plots.

\textbf{INPUT:}

• \texttt{xy_data_array} - list of lists giving evaluated values of the function on the grid
• \texttt{xrange} - tuple of 2 floats indicating range for horizontal direction
• \texttt{yrange} - tuple of 2 floats indicating range for vertical direction
• \texttt{options} - dict of valid plot options to pass to constructor

\textbf{EXAMPLES:}

Note this should normally be used indirectly via \texttt{contour_plot:}

```python
sage: from sage.plot.contour_plot import ContourPlot
sage: C = ContourPlot([[1,3],[2,4]], (1,2), (2,3), options={})
sage: C
ContourPlot defined by a 2 x 2 data grid
sage: C.xrange
(1, 2)
```

\textbf{get\_minmax\_data()}

Return a dictionary with the bounding box data.

\textbf{EXAMPLES:}

```python
sage: x,y = var('x,y')
sage: f(x,y) = x^2 + y^2
sage: d = contour_plot(f, (3,6), (3,6))[0].get_minmax_data()
sage: d['xmin']
3.0
```
sage.plot.contour_plot\(\text{contour_plot}(f, \text{xrange}, \text{yrange}, \text{plot_points}=100, \text{fill}=\text{True}, \text{contours}=\text{None}, \text{linewidths}=\text{None}, \text{linestyles}=\text{None}, \text{labels}=\text{False}, \text{frame}=\text{True}, \text{axes}=\text{False}, \text{colorbar}=\text{False}, \text{legend_label}=\text{None}, \text{aspect_ratio}=1, \text{region}=\text{None}, \text{label_fontsize}=9, \text{label_colors}=\text{'blue'}, \text{label_inline}=\text{None}, \text{label_inline_spacing}=3, \text{label_fmt}=\text{%1.2f'}, \text{colorbar_orientation}=\text{'vertical'}, \text{colorbar_format}=\text{None}, \text{colorbar_spacing}=\text{'uniform'}, **\text{options})\)

\text{contour_plot} takes a function of two variables, \(f(x, y)\) and plots contour lines of the function over the specified \text{xrange} and \text{yrange} as demonstrated below.

\text{contour_plot}(f, (\text{xmin, xmax}), (\text{ymin, ymax}), ...) \]

**INPUT:**
- \(f\) – a function of two variables
- (\text{xmin, xmax}) – 2-tuple, the range of \text{x} values OR 3-tuple (\text{x, xmin, xmax})
- (\text{ymin, ymax}) – 2-tuple, the range of \text{y} values OR 3-tuple (\text{y, ymin, ymax})

The following inputs must all be passed in as named parameters:
- \text{plot_points} – integer (default: 100); number of points to plot in each direction of the grid. For old computers, 25 is fine, but should not be used to verify specific intersection points.
- \text{fill} – bool (default: True), whether to color in the area between contour lines
- \text{cmap} – a colormap (default: 'gray'), the name of a predefined colormap, a list of colors or an instance of a matplotlib Colormap. Type: \text{import matplotlib.cm; matplotlib.cm.datad.keys()} for available colormap names.
- \text{contours} – integer or list of numbers (default: None): If a list of numbers is given, then this specifies the contour levels to use. If an integer is given, then this many contour lines are used, but the exact levels are determined automatically. If None is passed (or the option is not given), then the number of contour lines is determined automatically, and is usually about 5.
- \text{linewidths} – integer or list of integer (default: None), if a single integer all levels will be of the width given, otherwise the levels will be plotted with the width in the order given. If the list is shorter than the number of contours, then the widths will be repeated cyclically.
- \text{linestyles} – string or list of strings (default: None), the style of the lines to be plotted, one of: "solid", "dashed", "dashdot", "dotted", respectively "-", "--", "-.", ":" If the list is shorter than the number of contours, then the styles will be repeated cyclically.
- \text{labels} – boolean (default: False) Show level labels or not.

The following options are to adjust the style and placement of labels, they have no effect if no labels are shown.
- \text{label_fontsize} – integer (default: 9), the font size of the labels.
- \text{label_colors} – string or sequence of colors (default: None) If a string, gives the name of a single color with which to draw all labels. If a sequence, gives the colors of the labels. A color is a string giving the name of one or a 3-tuple of floats.
- \text{label_inline} – boolean (default: False if fill is True, otherwise True), controls whether the underlying contour is removed or not.
- **label_inline_spacing** – integer (default: 3), When inline, this is the amount of contour that is removed from each side, in pixels.

- **label_fmt** – a format string (default: “%1.2f”), this is used to get the label text from the level. This can also be a dictionary with the contour levels as keys and corresponding text string labels as values. It can also be any callable which returns a string when called with a numeric contour level.

- **colorbar** – boolean (default: False) Show a colorbar or not.

The following options are to adjust the style and placement of colorbars. They have no effect if a colorbar is not shown.

- **colorbar_orientation** – string (default: ‘vertical’), controls placement of the colorbar, can be either ‘vertical’ or ‘horizontal’

- **colorbar_format** – a format string, this is used to format the colorbar labels.

- **colorbar_spacing** – string (default: ‘proportional’). If ‘proportional’, make the contour divisions proportional to values. If ‘uniform’, space the colorbar divisions uniformly, without regard for numeric values.

- **legend_label** – the label for this item in the legend

- **region** – (default: None) If region is given, it must be a function of two variables. Only segments of the surface where region(x,y) returns a number >0 will be included in the plot.

**Warning:** Due to an implementation detail in matplotlib, single-contour plots whose data all lie on one side of the sole contour may not be plotted correctly. We attempt to detect this situation and to produce something better than an empty plot when it happens; a **UserWarning** is emitted in that case.

**EXAMPLES:**

Here we plot a simple function of two variables. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

```python
sage: x,y = var('x,y')
sage: contour_plot(cos(x^2 + y^2), (x,-4,4), (y,-4,4))
Graphics object consisting of 1 graphics primitive
```

Here we change the ranges and add some options:

```python
sage: x,y = var('x,y')
sage: contour_plot((x^2) * cos(x*y), (x,-10,5), (y,-5,5), fill=False, plot_points=150)
Graphics object consisting of 1 graphics primitive
```

An even more complicated plot:

```python
sage: x,y = var('x,y')
sage: contour_plot(sin(x^2+y^2) * cos(x) * sin(y), (x,-4,4), (y,-4,4), plot_points=150)
Graphics object consisting of 1 graphics primitive
```

Some elliptic curves, but with symbolic endpoints. In the first example, the plot is rotated 90 degrees because we switch the variables x, y:
Chapter 2. Function and Data Plots
We can play with the contour levels:

```python
sage: x, y = var('x, y')
sage: f(x, y) = x^2 + y^2
sage: contour_plot(f, (-2, 2), (-2, 2))
Graphics object consisting of 1 graphics primitive
```

```python
contour_plot(f, (-2, 2), (-2, 2), contours=(0.1, 1.0, 1.2, 1.4), cmap='hsv')
Graphics object consisting of 1 graphics primitive
```
2.2. Contour Plots
2.2. Contour Plots
sage: contour_plot(f, (-2,2), (-2,2), contours=(1.0,), fill=False)
Graphics object consisting of 1 graphics primitive

We can change the style of the lines:

sage: contour_plot(f, (-2,2), (-2,2), fill=False, linewidths=10)
Graphics object consisting of 1 graphics primitive

sage: contour_plot(f, (-2,2), (-2,2), fill=False, linestyles='dashdot')
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(x^2 - y^2, (x,-3,3), (y,-3,3),
   ....: contours=[0,1,2,3,4], linewidths=[1,5],
   ....: linestyles=['solid','dashed'], fill=False)
sage: P
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(x^2 - y^2, (x,-3,3), (y,-3,3),
   ....: contours=[0,1,2,3,4], linewidths=[1,5],
(continues on next page)
2.2. Contour Plots
2.2. Contour Plots
linestyles=['solid', 'dashed'])

We can add labels and play with them:

```
sage: contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
....:                 fill=False, cmap='hsv', labels=True)
Graphics object consisting of 1 graphics primitive
```

```
sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
....:                 fill=False, cmap='hsv', labels=True, label_fmt="%1.0f",
....:                 label_colors='black')
sage: P
Graphics object consisting of 1 graphics primitive
```
sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
....: fill=False, cmap='hsv', labels=True,
....: contours=[-4,0,4],
....: label_fmt={-4:"low", 0:"medium", 4: "hi"},
....: label_colors='black')
sage: P
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
....: fill=False, cmap='hsv', labels=True,
....: contours=[-4,0,4], label_fmt=lambda x: "z=\%s\"%x,
....: label_colors='black', label_inline=True,
....: label_fontsize=12)
sage: P
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
....: fill=False, cmap='hsv', labels=True,
....: label_fontsize=18)
sage: P
Graphics object consisting of 1 graphics primitive
$z = -4.0$

$z = 0.0$

$z = 4.0$
2.2. Contour Plots
We can change the color of the labels if so desired:

```
sage: contour_plot(f, (-2,2), (-2,2), labels=True, label_colors='red')
```

Graphics object consisting of 1 graphics primitive

We can add a colorbar as well:

```
sage: f(x, y)=x^2-y^2
sage: contour_plot(f, (x,-3,3), (y,-3,3), colorbar=True)
```

Graphics object consisting of 1 graphics primitive
2.2. Contour Plots
2.2. Contour Plots
The image contains a screenshot of a page from the Sage 9.5 Reference Manual, focusing on 2D Graphics. It includes code examples and their corresponding graphics outputs. The code examples demonstrate the use of the `contour_plot` function with various parameters to customize the appearance of contour plots. The parameters include `colorbar`, `colorbar_orientation`, `contours`, `colorbar_spacing`, and `label_colors`. The page is part of Chapter 2, Function and Data Plots.
2.2. Contour Plots
This should plot concentric circles centered at the origin:

```python
sage: x, y = var('x, y')
sage: contour_plot(x^2 + y^2 - 2, (x, -1, 1), (y, -1, 1))
```

Extra options will get passed on to `show()`, as long as they are valid:

```python
sage: f(x, y) = cos(x) + sin(y)
sage: contour_plot(f, (0, pi), (0, pi), axes=True)
sage: contour_plot(f, (0, pi), (0, pi)).show(axes=True) # These are equivalent
```

One can also plot over a reduced region:

```python
sage: contour_plot(x^2 - y^2, (x, -2, 2), (y, -2, 2), region=x - y, plot_points=300)
```

### 2.2. Contour Plots
2.2. Contour Plots
Note that with `fill=False` and grayscale contours, there is the possibility of confusion between the contours and the axes, so use `fill=False` together with `axes=True` with caution:

```
sage: contour_plot(f, (-pi,pi), (-pi,pi), fill=False, axes=True)
```

If you are plotting a sole contour and if all of your data lie on one side of it, then (as part of trac ticket #21042) a heuristic may be used to improve the result; in that case, a warning is emitted:

```
sage: contour_plot(lambda x,y: abs(x^2-y^2), (-1,1), (-1,1), ...
....: contours=[0], fill=False, cmap=['blue'])
```

UserWarning: pathological contour plot of a function whose values all lie on one side of the sole contour; we are adding more plot points and perturbing your function values.

```
sage: contour_plot(lambda x,y: 0, (-1,1), (-1,1), ...
....: contours=[0], fill=False, cmap=['blue'])
```

UserWarning: No contour levels were found within the data range.

Constant functions (with a single contour) can be plotted as well; this was not possible before trac ticket #21042:
Chapter 2. Function and Data Plots
Return the equation rewritten as a symbolic function to give negative values when True, positive when False.

EXAMPLES:

```python
sage: from sage.plot.contour_plot import equify
sage: var('x, y')
(x, y)
sage: equify(x^2 < 2)
x^2 - 2
sage: equify(x^2 > 2)
-x^2 + 2
sage: equify(x*y > 1)
-x*y + 1
sage: equify(y > 0)
-y
sage: f = equify(lambda x, y: x > y)
sage: f(1, 2)
1
sage: f(2, 1)
-1
```

*sage.plot.contour_plot.implicit_plot*(f, xrange, yrange, plot_points=150, contours=(0,), fill=False, cmap=['blue'], **options)

implicit_plot takes a function of two variables, \(f(x, y)\) and plots the curve \(f(x, y) = 0\) over the specified xrange and yrange as demonstrated below.

```
implicit_plot(f, (xmin,xmax), (ymin,ymax), ...)
implicit_plot(f, (x,xmin,xmax), (y,ymin,ymax), ...)
```

The following inputs must all be passed in as named parameters:

- \(f\) – a function of two variables or equation in two variables
- \((xmin,xmax)\) – 2-tuple, the range of \(x\) values or \((x,xmin,xmax)\)
- \((ymin,ymax)\) – 2-tuple, the range of \(y\) values or \((y,ymin,ymax)\)

The following inputs must all be passed in as named parameters:

- `plot_points` – integer (default: 150); number of points to plot in each direction of the grid
- `fill` – boolean (default: False); if True, fill the region \(f(x, y) < 0\).
- `fillcolor` – string (default: 'blue'), the color of the region where \(f(x, y) < 0\) if fill = True. Colors are defined in `sage.plot.colors`; try colors? to see them all.
- `linewidth` – integer (default: None), if a single integer all levels will be of the width given, otherwise the levels will be plotted with the widths in the order given.
- `linestyle` – string (default: None), the style of the line to be plotted, one of: "solid", "dashed", "dashdot" or "dotted", respectively "-", "--", "-.", or ":".
- `color` – string (default: 'blue'), the color of the plot. Colors are defined in `sage.plot.colors`; try colors? to see them all. If fill = True, then this sets only the color of the border of the plot. See fillcolor for setting the color of the fill region.
- `legend_label` – the label for this item in the legend
• **base** – (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can also be given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.

• **scale** – (default: "linear") string. The scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy".

  The scale can also be given as a single argument that is a list or tuple (scale, base) or (scale, basex, basey).

  The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when Graphics is initialized.

**Warning:** Due to an implementation detail in matplotlib, implicit plots whose data are all nonpositive or nonnegative may not be plotted correctly. We attempt to detect this situation and to produce something better than an empty plot when it happens; a UserWarning is emitted in that case.

**EXAMPLES:**

A simple circle with a radius of 2. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

```sage```
var("x y")
(x, y)
sage: implicit_plot(x^2 + y^2 - 2, (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```

We can do the same thing, but using a callable function so we do not need to explicitly define the variables in the ranges. We also fill the inside:

```sage```
f(x,y) = x^2 + y^2 - 2
sage: implicit_plot(f, (-3,3), (-3,3), fill=True, plot_points=500) # long time
Graphics object consisting of 2 graphics primitives
```

The same circle but with a different line width:

```sage```
sage: implicit_plot(f, (-3,3), (-3,3), linewidth=6)
Graphics object consisting of 1 graphics primitive
```

Again the same circle but this time with a dashdot border:

```sage```
sage: implicit_plot(f, (-3,3), (-3,3), linestyle='dashdot')
Graphics object consisting of 1 graphics primitive
```

The same circle with different line and fill colors:

```sage```
sage: implicit_plot(f, (-3,3), (-3,3), color='red', fill=True, fillcolor='green',
....: plot_points=500) # long time
Graphics object consisting of 2 graphics primitives
```

You can also plot an equation:

```sage```
var("x y")
(x, y)
```
2.2. Contour Plots
You can even change the color of the plot:

```python
sage: implicit_plot(x^2 + y^2 == 2, (x,-3,3), (y,-3,3), color="red")
```

Graphics object consisting of 1 graphics primitive

The color of the fill region can be changed:

```python
sage: implicit_plot(x^2 + y^2 == 2, (x,-3,3), (y,-3,3), fill=True, fillcolor='red')
```

Graphics object consisting of 2 graphics primitives

Here is a beautiful (and long) example which also tests that all colors work with this:

```python
sage: G = Graphics()
sage: counter = 0
sage: for col in colors.keys():  # long time
    ....:     G += implicit_plot(x^2 + y^2 == 1 + counter*.1, (x,-4,4),(y,-4,4),
                      -->color=col)
    ....:     counter += 1
```

(continues on next page)
We can define a level-$n$ approximation of the boundary of the Mandelbrot set:

```
sage: def mandel(n):
    c = polygen(CDF, 'c')
    z = 0
    for i in range(n):
        z = z*z + c
    def f(x,y):
        val = z(CDF(x, y))
        return val.norm() - 4
    return f
```

The first-level approximation is just a circle:

```
sage: implicit_plot(mandel(1), (-3,3), (-3,3))
```

A third-level approximation starts to get interesting:
2.2. Contour Plots
The seventh-level approximation is a degree 64 polynomial, and \texttt{implicit_plot} does a pretty good job on this part of the curve. (\texttt{plot_points=200} looks even better, but it takes over a second.)

When making a filled implicit plot using a python function rather than a symbolic expression the user should increase the number of plot points to avoid artifacts:

An example of an implicit plot on ‘loglog’ scale:

```python
sage: implicit_plot(x^2 + y^2 == 200, (x,1,200), (y,1,200), scale='loglog')
Graphics object consisting of 1 graphics primitive
```
2.2. Contour Plots
2.2. Contour Plots
region_plot(f, xrangexyrange, plot_points, incol, outcol, bordercol, borderstyle, borderwidth, alpha, plot_points=100, incol='blue', outcol=None, bordercol=None, borderstyle=None, borderwidth=None, frame=False, axes=True, legend_label=None, aspect_ratio=1, alpha=1, **options)

region_plot takes a boolean function of two variables, \( f(x, y) \) and plots the region where \( f \) is True over the specified xrange and yrange as demonstrated below.

\[
\text{region_plot}(f, (x_{\text{min}}, x_{\text{max}}), (y_{\text{min}}, y_{\text{max}}), \ldots)
\]

INPUT:

- \( f \) – a boolean function or a list of boolean functions of two variables
- \((x_{\text{min}}, x_{\text{max}})\) – 2-tuple, the range of \( x \) values OR 3-tuple \((x, x_{\text{min}}, x_{\text{max}})\)
- \((y_{\text{min}}, y_{\text{max}})\) – 2-tuple, the range of \( y \) values OR 3-tuple \((y, y_{\text{min}}, y_{\text{max}})\)
- plot_points – integer (default: 100); number of points to plot in each direction of the grid
- incol – a color (default: 'blue'), the color inside the region
- outcol – a color (default: None), the color of the outside of the region
- bordercol – a color (default: None), the color of the border ('black' if borderwidth or borderstyle is specified but not bordercol)
- borderstyle – string (default: 'solid'), one of 'solid', 'dashed', 'dotted', 'dashdot', respectively '-', '--', ':', '-.',
- borderwidth – integer (default: None), the width of the border in pixels
- alpha – (default: 1) how transparent the fill is; a number between 0 and 1
- legend_label – the label for this item in the legend
- base – (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple \((\text{basex}, \text{basey})\). \text{basex} sets the base of the logarithm along the horizontal axis and \text{basey} sets the base along the vertical axis.
- scale – (default: "linear") string. The scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy".

The scale can be also be given as single argument that is a list or tuple \((\text{scale}, \text{base})\) or \((\text{scale}, \text{basex}, \text{basey})\).

The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when Graphics is initialized.

EXAMPLES:

Here we plot a simple function of two variables:

```
sage: x,y = var('x,y')
sage: region_plot(cos(x^2 + y^2) <= 0, (x,-3,3), (y,-3,3))
```

Graphics object consisting of 1 graphics primitive

Here we play with the colors:
2.2. Contour Plots
An even more complicated plot, with dashed borders:

```python
sage: region_plot(sin(x) * sin(y) >= 1/4, (x,-10,10), (y,-10,10),
    incol='yellow', bordercol='black',
    borderstyle='dashed', plot_points=250)
```

A disk centered at the origin:

```python
sage: region_plot(x^2 + y^2 < 1, (x,-1,1), (y,-1,1))
```

A plot with more than one condition (all conditions must be true for the statement to be true):

```python
sage: region_plot([x^2 + y^2 < 1, x < y], (x,-2,2), (y,-2,2))
```

Since it does not look very good, let us increase `plot_points`:
2.2. Contour Plots

To get plots where only one condition needs to be true, use a function. Using lambda functions, we definitely need the extra plot_points:

```python
sage: region_plot(lambda x, y: x^2 + y^2 < 1 or x < y, (x,-2,2), (y,-2,2), plot_points=400)
```

The first quadrant of the unit circle:

```python
sage: region_plot([y > 0, x > 0, x^2 + y^2 < 1], (x,-1.1,1.1), (y,-1.1,1.1), plot_points=400)
```

Here is another plot, with a huge border:

```python
sage: region_plot(x*(x-1)*(x+1) + y^2 < 0, (x,-3,2), (y,-3,3),
               incol='lightblue', bordercol='gray', borderwidth=10,
               plot_points=50)
```

If we want to keep only the region where x is positive:
2.2. Contour Plots

247
2.2. Contour Plots
region_plot([x*(x-1)*(x+1) + y^2 < 0, x > -1], (x,-3,2), (y,-3,3),
....: incol='lightblue', plot_points=50)
Graphics object consisting of 1 graphics primitive

Here we have a cut circle:

region_plot([x^2 + y^2 < 4, x > -1], (x,-2,2), (y,-2,2),
....: incol='lightblue', bordercol='gray', plot_points=200)
Graphics object consisting of 2 graphics primitives

The first variable range corresponds to the horizontal axis and the second variable range corresponds to the vertical axis:

sage: s, t = var('s, t')
sage: region_plot(s > 0, (t,-2,2), (s,-2,2))
Graphics object consisting of 1 graphics primitive

sage: region_plot(s>0,(s,-2,2),(t,-2,2))
Graphics object consisting of 1 graphics primitive

An example of a region plot in ‘loglog’ scale:
2.2. Contour Plots
2.2. Contour Plots
2.3 Density Plots

```
class sage.plot.density_plot.DensityPlot(xy_data_array, xrange, yrange, options)
   Bases: sage.plot.primitive.GraphicPrimitive

   Primitive class for the density plot graphics type. See density_plot? for help actually doing density plots.

   INPUT:
   • xy_data_array - list of lists giving evaluated values of the function on the grid
   • xrange - tuple of 2 floats indicating range for horizontal direction
   • yrange - tuple of 2 floats indicating range for vertical direction
   • options - dict of valid plot options to pass to constructor

   EXAMPLES:
   Note this should normally be used indirectly via density_plot:
```
```python
sage: from sage.plot.density_plot import DensityPlot
sage: D = DensityPlot([[1,3],[2,4]], (1,2), (2,3), options={})
sage: D
DensityPlot defined by a 2 x 2 data grid
sage: D.yrange
(2, 3)
sage: D.options()
{}
```

**get_minmax_data()**

Returns a dictionary with the bounding box data.

**EXAMPLES:**

```python
sage: x, y = var('x, y')
sage: f(x, y) = x^2 + y^2
sage: d = density_plot(f, (3,6), (3,6))[0].get_minmax_data()
sage: d['xmin']
3.0
sage: d['ymin']
3.0
```

`sage.plot.density_plot.density_plot(f, xrange, yrange, plot_points=25, cmap='gray', interpolation='catrom', **options)`

density_plot takes a function of two variables, \( f(x, y) \) and plots the height of the function over the specified xrange and yrange as demonstrated below.

density_plot\( f, (x_{\text{min}}, x_{\text{max}}), (y_{\text{min}}, y_{\text{max}}), \ldots \)

**INPUT:**

- \( f \) – a function of two variables
- \( (x_{\text{min}}, x_{\text{max}}) \) – 2-tuple, the range of \( x \) values OR 3-tuple \( (x, x_{\text{min}}, x_{\text{max}}) \)
- \( (y_{\text{min}}, y_{\text{max}}) \) – 2-tuple, the range of \( y \) values OR 3-tuple \( (y, y_{\text{min}}, y_{\text{max}}) \)

The following inputs must all be passed in as named parameters:

- **plot_points** – integer (default: 25); number of points to plot in each direction of the grid
- **cmap** – a colormap (default: 'gray'), the name of a predefined colormap, a list of colors or an instance of a matplotlib Colormap. Type: \import{matplotlib.cm}; matplotlib.cm.datad.keys() for available colormap names.
- **interpolation** – string (default: 'catrom'), the interpolation method to use: 'bilinear', 'bicubic', 'spline16', 'spline36', 'quadric', 'gaussian', 'sinc', 'bessel', 'mitchell', 'lanczos', 'catrom', 'hermite', 'hanning', 'hamming', 'kaiser'

**EXAMPLES:**

Here we plot a simple function of two variables. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

```python
sage: x, y = var('x, y')
sage: density_plot(sin(x) * sin(y), (x,-2,2), (y,-2,2))
Graphics object consisting of 1 graphics primitive
```

Here we change the ranges and add some options; note that here \( f \) is callable (has variables declared), so we can use 2-tuple ranges:
```
sage: x, y = var('x, y')
sage: f(x, y) = x^2 * cos(x*y)
sage: density_plot(f, (x,-10,5), (y,-5,5), interpolation='sinc', plot_points=100)
Graphics object consisting of 1 graphics primitive
```

An even more complicated plot:
```
sage: x, y = var('x, y')
sage: density_plot(sin(x^2+y^2) * cos(x) * sin(y), (x,-4,4), (y,-4,4), cmap='jet', plot_points=100)
Graphics object consisting of 1 graphics primitive
```

This should show a “spotlight” right on the origin:
```
sage: x, y = var('x, y')
sage: density_plot(1/(x^10 + y^10), (x,-10,10), (y,-10,10))
Graphics object consisting of 1 graphics primitive
```

Some elliptic curves, but with symbolic endpoints. In the first example, the plot is rotated 90 degrees because we switch the variables $x, y$:
```
sage: density_plot(y^2 + 1 - x^3 - x, (y,-pi,pi), (x,-pi,pi))
Graphics object consisting of 1 graphics primitive
```
Extra options will get passed on to show(), as long as they are valid:

```python
sage: density_plot(log(x) + log(y), (x,1,10), (y,1,10), dpi=20)
```
Graphics object consisting of 1 graphics primitive

```python
sage: density_plot(log(x) + log(y), (x,1,10), (y,1,10)).show(dpi=20) # These are...
```

---

### 2.4 Plotting fields

```python
class sage.plot.plot_field.PlotField(xpos_array, ypos_array, xvec_array, yvec_array, options)
```
Bases: `sage.plot.primitive.GraphicPrimitive`

Primitive class that initializes the PlotField graphics type

```python
def get_minmax_data()
    Returns a dictionary with the bounding box data.
```

**EXAMPLES:**
Chapter 2. Function and Data Plots
sage: x,y = var('x,y')
sage: d = plot_vector_field((.01*x, x+y), (x,10,20), (y,10,20))[0].get_minmax_data()
sage: d['xmin']
10.0
sage: d['ymin']
10.0

sage.plot.plot_field.plot_slope_field(f, xrange, yrange, **kwds)

plot_slope_field takes a function of two variables xvar and yvar (for instance, if the variables are x and y, take \( f(x, y) \)), and at representative points \((x_i, y_i)\) between xmin, xmax, and ymin, ymax respectively, plots a line with slope \( f(x_i, y_i) \) (see below).

\[
\text{plot_slope_field}(f, (xvar, xmin, xmax), (yvar, ymin, ymax))
\]

EXAMPLES:

A logistic function modeling population growth:

sage: x,y = var('x y')
sage: capacity = 3 # thousand
sage: growth_rate = 0.7 # population increases by 70% per unit of time
sage: plot_slope_field(growth_rate * (1-y/capacity) * y, (x,0,5), (y,0,capacity*2))
Graphics object consisting of 1 graphics primitive
Plot a slope field involving sin and cos:

```
sage: x,y = var('x y')
sage: plot_slope_field(sin(x+y) + cos(x+y), (x,-3,3), (y,-3,3))
```

Graphics object consisting of 1 graphics primitive

Plot a slope field using a lambda function:

```
sage: plot_slope_field(lambda x,y: x + y, (-2,2), (-2,2))
```

Graphics object consisting of 1 graphics primitive

`sage.plot.plot_field.plot_vector_field(f_g, xrange, yrange, plot_points=20, frame=True, **options)`

`plot_vector_field` takes two functions of two variables xvar and yvar (for instance, if the variables are $x$ and $y$, take $(f(x, y), g(x, y))$) and plots vector arrows of the function over the specified ranges, with xrange being of xvar between xmin and xmax, and yrange similarly (see below).

```
plot_vector_field((f,g), (xvar,xmin,xmax), (yvar,ymin,ymax))
```

EXAMPLES:

Plot some vector fields involving sin and cos:

```
sage: x,y = var('x y')
sage: plot_vector_field((sin(x),cos(y)), (x,-3,3), (y,-3,3))
```

Graphics object consisting of 1 graphics primitive
2.4. Plotting fields
Plot a gradient field:

```python
sage: u, v = var('u v')
sage: f = exp(-(u^2 + v^2))
sage: plot_vector_field(f.gradient(), (u,-2,2), (v,-2,2), color='blue')
```

Plot two orthogonal vector fields:

```python
sage: x,y = var('x,y')
sage: a = plot_vector_field((x,y), (x,-3,3), (y,-3,3), color='blue')
sage: b = plot_vector_field((y,-x), (x,-3,3), (y,-3,3), color='red')
sage: show(a + b)
```

We ignore function values that are infinite or NaN:

```python
sage: x,y = var('x,y')
sage: plot_vector_field((-x/sqrt(x^2+y^2),-y/sqrt(x^2+y^2)), (x,-10,10), (y,-10,10))
```
2.4. Plotting fields
2.5 Streamline Plots

```python
class sage.plot.streamline_plot.StreamlinePlot(xpos_array, ypos_array, xvec_array, yvec_array, options)
```
Bases: `sage.plot.primitive.GraphicPrimitive`

Primitive class that initializes the StreamlinePlot graphics type

```python
get_minmax_data()
```
Returns a dictionary with the bounding box data.

```python
EXAMPLES:
```
sage: x, y = var('x y')
sage: d = streamline_plot((.01*x, x+y), (x,10,20), (y,10,20))[0].get_minmax_data()
sage: d['xmin']
10.0
sage: d['ymin']
10.0

sage.plot.streamline_plot.streamline_plot(f_g, xrange, yrange, plot_points=20, density=1.0, frame=True, **options)

Return a streamline plot in a vector field.

streamline_plot can take either one or two functions. Consider two variables $x$ and $y$.

If given two functions $(f(x, y), g(x, y))$, then this function plots streamlines in the vector field over the specified ranges with $xrange$ being of $x$, denoted by $xvar$ below, between $xmin$ and $xmax$, and $yrange$ similarly (see below).

streamline_plot((f, g), (xvar, xmin, xmax), (yvar, ymin, ymax))

Similarly, if given one function $f(x, y)$, then this function plots streamlines in the slope field $dy/dx = f(x, y)$ over the specified ranges as given above.

PLOT OPTIONS:
- plot_points – (default: 200) the minimal number of plot points
- density – float (default: 1.); controls the closeness of streamlines
- start_points – (optional) list of coordinates of starting points for the streamlines; coordinate pairs can be tuples or lists

EXAMPLES:
Plot some vector fields involving $\sin$ and $\cos$:

sage: x, y = var('x y')
sage: streamline_plot((sin(x), cos(y)), (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive

sage: streamline_plot((y, (cos(x)-2) * sin(x)), (x,-pi,pi), (y,-pi,pi))
Graphics object consisting of 1 graphics primitive

We increase the density of the plot:

sage: streamline_plot((y, (cos(x)-2) * sin(x)), (x,-pi,pi), (y,-pi,pi), density=2)
Graphics object consisting of 1 graphics primitive

We ignore function values that are infinite or NaN:

sage: x, y = var('x y')
sage: streamline_plot((-x/sqrt(x^2+y^2), -y/sqrt(x^2+y^2)), (x,-10,10), (y,-10,10))
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to show(), as long as they are valid:
2.5. Streamline Plots
Chapter 2. Function and Data Plots
2.5. Streamline Plots
We can also construct streamlines in a slope field:

```
sage: x, y = var('x y')
sage: streamline_plot((x + y) / sqrt(x^2 + y^2), (x,-3,3), (y,-3,3))
```

We choose some particular points the streamlines pass through:

```
sage: pts = [[1, 1], [-2, 2], [1, -3/2]]
sage: g = streamline_plot((x + y) / sqrt(x^2 + y^2), (x,-3,3), (y,-3,3), start_points=pts)
sage: g += point(pts, color='red')
sage: g
```

**Note:** Streamlines currently pass close to start_points but do not necessarily pass directly through them. That is part of the behavior of matplotlib, not an error on your part.
2.5. Streamline Plots
2.6 Scatter Plots

**class** `sage.plot.scatter_plot.ScatterPlot(xdata, ydata, options)`

Bases: `sage.plot.primitive.GraphicPrimitive`

Scatter plot graphics primitive.

Input consists of two lists/arrays of the same length, whose values give the horizontal and vertical coordinates of each point in the scatter plot. Options may be passed in dictionary format.

**EXAMPLES:**

```python
sage: from sage.plot.scatter_plot import ScatterPlot
sage: ScatterPlot([0,1,2], [3.5,2,5.1], {'facecolor': 'white', 'marker': 's'})
```

Scatter plot graphics primitive on 3 data points

**get_minmax_data()**

Returns a dictionary with the bounding box data.

**EXAMPLES:**

```python
sage: s = scatter_plot([[0,1],[2,4],[3.2,6]])
sage: d = s.get_minmax_data()
sage: d['xmin']
0.0
sage: d['ymin']
1.0
```

```python
sage.plot.scatter_plot.scatter_plot(datalist, alpha=1, markersize=50, marker='o', zorder=5,
                                    facecolor='#fec7b8', edgecolor='black', clip=True,
                                    aspect_ratio='automatic', **options)
```

Returns a Graphics object of a scatter plot containing all points in the datalist. Type `scatter_plot.options` to see all available plotting options.

**INPUT:**

- **datalist** – a list of tuples `(x, y)`
- **alpha** – default: 1
- **markersize** – default: 50
- **marker** - The style of the markers (default "o"). See the documentation of `plot()` for the full list of markers.
- **facecolor** – default: '#fec7b8'
- **edgecolor** – default: 'black'
- **zorder** – default: 5

**EXAMPLES:**

```python
sage: scatter_plot([[0,1],[2,2],[4.3,1.1]], marker='s')
```

Graphics object consisting of 1 graphics primitive

Extra options will get passed on to `show()`, as long as they are valid:
2.7 Step function plots

`sage.plot.step.plot_step_function(v, vertical_lines=True, **kwds)`

Return the line graphics object that gives the plot of the step function \( f \) defined by the list \( v \) of pairs \((a, b)\). Here if \((a, b)\) is in \( v \), then \( f(a) = b \). The user does not have to worry about sorting the input list \( v \).

**INPUT:**

- \( v \) – list of pairs (a,b)
- `vertical_lines` – bool (default: True) if True, draw vertical risers at each step of this step function. Technically these vertical lines are not part of the graph of this function, but they look very nice in the plot so we include them by default

**EXAMPLES:**

We plot the prime counting function:

```python
sage: plot_step_function([(i,prime_pi(i)) for i in range(20)])
```

Graphics object consisting of 1 graphics primitive

```python
sage: plot_step_function([(i,sin(i)) for i in range(5,20)])
```

Graphics object consisting of 1 graphics primitive

We pass in many options and get something that looks like “Space Invaders”:

```python
sage: v = [(i,sin(i)) for i in range(5,20)]
sage: plot_step_function(v, vertical_lines=False, thickness=30, rgbcolor='purple', axes=False)
```

Graphics object consisting of 14 graphics primitives

2.8 Histograms

`class sage.plot.histogram.Histogram(datalist, options)`

Bases: `sage.plot.primitive.GraphicPrimitive`

Graphics primitive that represents a histogram. This takes quite a few options as well.

**EXAMPLES:**

```python
sage: from sage.plot.histogram import Histogram
sage: g = Histogram([1,3,2,0], {}); g
Histogram defined by a data list of size 4
sage: type(g)
<class 'sage.plot.histogram.Histogram'>
sage: opts = { 'bins':20, 'label':'mydata'}
sage: g = Histogram([random() for _ in range(500)], opts); g
Histogram defined by a data list of size 500
```
We can accept multiple sets of the same length:

```
sage: g = Histogram([[1,3,2,0], [4,4,3,3]], {}); g
Histogram defined by 2 data lists
```

```
get_minmax_data()
Get minimum and maximum horizontal and vertical ranges for the Histogram object.

EXAMPLES:
```
sage: H = histogram([10,3,5], density=True); h = H[0]
sage: h.get_minmax_data()
{'xmax': 10.0, 'xmin': 3.0, 'ymax': 0.4761904761904765, 'ymin': 0}
sage: G = histogram([random() for _ in range(500)]); g = G[0]
sage: g.get_minmax_data()
# random output
{'xmax': 0.99729312925213209, 'xmin': 0.00013024562219410285, 'ymax': 61, 'ymin': 0}
sage: Y = histogram([random()*10 for _ in range(500)], range=[2,8]); y = Y[0]
sage: ymm = y.get_minmax_data(); ymm['xmax'], ymm['xmin']
(8.0, 2.0)
sage: Z = histogram([[1,3,2,0], [4,4,3,3]]); z = Z[0]
sage: z.get_minmax_data()
{'xmax': 4.0, 'xmin': 0, 'ymax': 2, 'ymin': 0}
```

```
sage.plot.histogram.histogram(datalist, aspect_ratio='automatic', align='mid', weights=None, range=None, bins=10, edgecolor='black', **options)
Computes and draws the histogram for list(s) of numerical data. See examples for the many options; even more customization is available using matplotlib directly.

INPUT:
• datalist – A list, or a list of lists, of numerical data
• align – (default: “mid”) How the bars align inside of each bin. Acceptable values are “left”, “right” or “mid”
• alpha – (float in [0,1], default: 1) The transparency of the plot
• bins – The number of sections in which to divide the range. Also can be a sequence of points within the range that create the partition
• color – The color of the face of the bars or list of colors if multiple data sets are given
• cumulative – (boolean - default: False) If True, then a histogram is computed in which each bin gives the counts in that bin plus all bins for smaller values. Negative values give a reversed direction of accumulation
• edgecolor – The color of the border of each bar
• fill – (boolean - default: True) Whether to fill the bars
• hatch – (default: None) symbol to fill the bars with - one of “\", “\n”, “\n\n", “+”, “x”, “o”, “O”, “.”, “*”, “\n” (or None)
• hue – The color of the bars given as a hue. See hue for more information on the hue
• label – A string label for each data list given
• linewidth – (float) width of the lines defining the bars
• linestyle – (default: ‘solid’) Style of the line. One of ‘solid’ or ‘-‘, ‘dashed’ or ‘--’, ‘dotted’ or ‘:’, ‘dashdot’ or ‘-.’

2.8. Histograms 283
• **density** – (boolean - default: False) If True, the result is the value of the probability density function at the bin, normalized such that the integral over the range is 1.

• **range** – A list [min, max] which define the range of the histogram. Values outside of this range are treated as outliers and omitted from counts

• **rwidth** – (float in [0,1], default: 1) The relative width of the bars as a fraction of the bin width

• **stacked** – (boolean - default: False) If True, multiple data are stacked on top of each other

• **weights** – (list) A sequence of weights the same length as the data list. If supplied, then each value contributes its associated weight to the bin count

• **zorder** – (integer) the layer level at which to draw the histogram

**Note:** The **weights** option works only with a single list. List of lists representing multiple data are not supported.

**EXAMPLES:**

A very basic histogram for four data points:

```
sage: histogram([1,2,3,4], bins=2)
Graphics object consisting of 1 graphics primitive
```

We can see how the histogram compares to various distributions. Note the use of the **density** keyword to guarantee the plot looks like the probability density function:

```sage
nv = normalvariate
sage: H = histogram([nv(0,1) for _ in range(1000)], bins=20, density=True, range=[-5,5])
sage: P = plot( 1/sqrt(2*pi)*e^(-x^2/2), (x,-5,5), color='red', linestyle='--')
sage: H+P
Graphics object consisting of 2 graphics primitives
```

There are many options one can use with histograms. Some of these control the presentation of the data, even if it is boring:

```
sage: T = RealDistribution('lognormal', [0,1])
sage: histogram( [T.get_random_element() for _ in range(100)], alpha=0.3, edgecolor='red', fill=False, linestyle='dashed', hatch='O', linewidth=5)
sage: histogram( [T.get_random_element() for _ in range(100)], linestyle='-.')
```

We can do several data sets at once if desired:

```
sage: histogram([srange(0,1,.1)*10, [nv(0, 1) for _ in range(100)]], color=['red', 'green'], bins=5)
```

---

Chapter 2. Function and Data Plots
We have the option of stacking the data sets too:

```
sage: histogram([ [1,1,1,2,2,2,3,3,3], [4,4,4,4,3,3,3,2,2,2] ], stacked=True, ...
             color=['blue', 'red'])
Graphics object consisting of 1 graphics primitive
```

It is possible to use weights with the histogram as well:

```
sage: histogram(list(range(10)), bins=3, weights=[1,2,3,4,5,5,4,3,2,1])
Graphics object consisting of 1 graphics primitive
```

## 2.9 Bar Charts

The `sage.plot.bar_chart.BarChart` class is defined as:

```
class sage.plot.bar_chart.BarChart(ind, datalist, options)
```

This class represents a bar chart and is a graphics primitive.

**Examples:**

```
sage: from sage.plot.bar_chart import BarChart
sage: g = BarChart(list(range(4)), [1,3,2,0], {}); g
BarChart defined by a 4 datalist
sage: type(g)
<class 'sage.plot.bar_chart.BarChart'>
```

**get_minmax_data()**

This method returns a dictionary with the bounding box data.

**Examples:**

```
sage: b = bar_chart([-2.3,5,-6,12])
sage: d = b.get_minmax_data()
sage: d['xmin']
0
sage: d['xmax']
4
```

The `sage.plot.bar_chart.bar_chart` function is defined as:

```
sage.plot.bar_chart.bar_chart(datalist, width=0.5, rgbcolor=(0, 0, 1), legend_label=None, aspect_ratio='automatic', **options)
```

This function creates a bar chart of (currently) one list of numerical data. Support for more data lists is in progress.

**Examples:**

A bar chart with blue bars:

```
sage: bar_chart([1,2,3,4])
```

A bar chart with thinner bars:

```
sage: bar_chart([x^2 for x in range(1,20)], width=0.2)
```

A bar chart with negative values and red bars:
A bar chart with a legend (it’s possible, not necessarily useful):

```python
sage: bar_chart([-1,1,-1,1], legend_label='wave')
```

Extra options will get passed on to `show()`, as long as they are valid:

```python
sage: bar_chart([-2,8,-7,3], rgbcolor=(1,0,0), axes=False)  # These are equivalent
```
3.1 Graph Plotting

(For \LaTeX\ drawings of graphs, see the graph_latex module.)

All graphs have an associated Sage graphics object, which you can display:

```
sage: G = graphs.WheelGraph(15)
sage: P = G.plot()
sage: P.show()  # long time
```

When plotting a graph created using Sage’s Graph command, node positions are determined using the spring-layout
algorithm. Special graphs available from graphs.* have preset positions. For example, compare the two plots of the Petersen graph, as obtained using Graph or as obtained from that database:

```python
sage: petersen_spring = Graph('I\`ES@obGkqegW~')
sage: petersen_spring.show()  # long time
```

```python
sage: petersen_database = graphs.PetersenGraph()
sage: petersen_database.show()  # long time
```

All constructors in this database (except some random graphs) prefill the position dictionary, bypassing the spring-layout positioning algorithm.

**Plot options**

Here is the list of options accepted by plot() and the constructor of GraphPlot. Those two functions also accept all options of `sage.plot.graphics.Graphics.show()`.

---

288 Chapter 3. Plots of Other Mathematical Objects
| **layout** | A layout algorithm – one of: “acyclic”, “circular” (plots the graph with vertices evenly distributed on a circle), “ranked”, “graphviz”, “planar”, “spring” (traditional spring layout, using the graph’s current positions as initial positions), or “tree” (the tree will be plotted in levels, depending on minimum distance for the root). |
| **iterations** | The number of times to execute the spring layout algorithm. |
| **heights** | A dictionary mapping heights to the list of vertices at this height. |
| **spring** | Use spring layout to finalize the current layout. |
| **tree_root** | A vertex designation for drawing trees. A vertex of the tree to be used as the root for the `layout='tree'` option. If no root is specified, then one is chosen close to the center of the tree. Ignored unless `layout='tree'`. |
| **forest_roots** | An iterable specifying which vertices to use as roots for the `layout='forest'` option. If no root is specified for a tree, then one is chosen close to the center of the tree. Ignored unless `layout='forest'`. |
| **tree_orientation** | The direction of tree branches – ‘up’, ‘down’, ‘left’ or ‘right’. |
| **save_pos** | Whether or not to save the computed position for the graph. |
| **dim** | The dimension of the layout – 2 or 3. |
| **prog** | Which graphviz layout program to use – one of “circo”, “dot”, “fdp”, “neato”, or “twopi”. |
| **by_component** | Whether to do the spring layout by connected component – a boolean. |
| **pos** | The position dictionary of vertices. |
| **vertex_labels** | Vertex labels to draw. This can be True/False to indicate whether to print the vertex string representation of not, a dictionary keyed by vertices and associating to each vertex a label string, or a function taking as input a vertex and returning a label string. |
| **vertex_color** | Default color for vertices not listed in vertex_colors dictionary. |
| **vertex_colors** | A dictionary specifying vertex colors: each key is a color recognizable by matplotlib, and each corresponding value is a list of vertices. |
| **vertex_size** | The size to draw the vertices. |
| **vertex_shape** | The shape to draw the vertices. Currently unavailable for Multi-edged DiGraphs. |
| **edge_labels** | Whether or not to draw edge labels. |
| **edge_style** | The linestyle of the edges. It should be one of “solid”, “dashed”, “dotted”, dash-dot”, or “\"", “\"", “\"", “\"", respectively. |
| **edge_thickness** | The thickness of the edges. |
| **edge_color** | The default color for edges not listed in edge_colors. |
| **edge_colors** | A dictionary specifying edge colors: each key is a color recognized by matplotlib, and each corresponding value is a list of edges. |
| **color_by_label** | Whether to color the edges according to their labels. This also accepts a function or dictionary mapping labels to colors. |
| **partition** | A partition of the vertex set. If specified, plot will show each cell in a different color; vertex_colors takes precedence. |
| **loop_size** | The radius of the smallest loop. |
| **dist** | The distance between multiedges. |
| **max_dist** | The max distance range to allow multiedges. |
| **talk** | Whether to display the vertices in talk mode (larger and white). |
| **graph_border** | Whether or not to draw a frame around the graph. |
| **edge_labels_background** | The color of the background of the edge labels. |

**Default options**

This module defines two dictionaries containing default options for the `plot()` and `show()` methods. These two dictionaries are `sage.graphs.graph_plot.DEFAULT_PLOT_OPTIONS` and `sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS`, respectively.
Obviously, these values are overruled when arguments are given explicitly.

Here is how to define the default size of a graph drawing to be $(6, 6)$. The first two calls to `show()` use this option, while the third does not (a value for `figsize` is explicitly given):

```python
sage: import sage.graphs.graph_plot
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (6, 6)
sage: graphs.PetersenGraph().show()  # long time
sage: graphs.ChvatalGraph().show()  # long time
sage: graphs.PetersenGraph().show(figsize=(4, 4))  # long time
```

We can now reset the default to its initial value, and now display graphs as previously:

```python
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (4, 4)
sage: graphs.PetersenGraph().show()  # long time
sage: graphs.ChvatalGraph().show()  # long time
```

Note:

- While `DEFAULT_PLOT_OPTIONS` affects both `G.show()` and `G.plot()`, settings from `DEFAULT_SHOW_OPTIONS` only affects `G.show()`.
- In order to define a default value permanently, you can add a couple of lines to Sage's startup scripts. Example:

```python
sage: import sage.graphs.graph_plot
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (4, 4)
```

Index of methods and functions

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>GraphPlot.set_pos()</code></td>
<td>Set the position plotting parameters for this GraphPlot.</td>
</tr>
<tr>
<td><code>GraphPlot.set_vertices()</code></td>
<td>Set the vertex plotting parameters for this GraphPlot.</td>
</tr>
<tr>
<td><code>GraphPlot.set_edges()</code></td>
<td>Set the edge (or arrow) plotting parameters for the GraphPlot object.</td>
</tr>
<tr>
<td><code>GraphPlot.show()</code></td>
<td>Show the (Di)Graph associated with this GraphPlot object.</td>
</tr>
<tr>
<td><code>GraphPlot.plot()</code></td>
<td>Return a graphics object representing the (di)graph.</td>
</tr>
<tr>
<td><code>GraphPlot.layout_tree()</code></td>
<td>Compute a nice layout of a tree.</td>
</tr>
</tbody>
</table>

**class** `sage.graphs.graph_plot.GraphPlot(graph, options)`

Bases: `sage.structure.sage_object.SageObject`

Return a GraphPlot object, which stores all the parameters needed for plotting (Di)Graphs.

A GraphPlot has a plot and show function, as well as some functions to set parameters for vertices and edges. This constructor assumes default options are set. Defaults are shown in the example below.

**EXAMPLES:**

```python
sage: from sage.graphs.graph_plot import GraphPlot
sage: options = {
    'vertex_size': 200,
    'vertex_labels': True,
    'layout': None,
    'edge_style': 'solid',
    'edge_color': 'black',
    'edge_colors': None,
}
```

(continues on next page)
..:  'edge_labels': False,
..:  'iterations': 50,
..:  'tree_orientation': 'down',
..:  'heights': None,
..:  'graph_border': False,
..:  'talk': False,
..:  'color_by_label': False,
..:  'partition': None,
..:  'dist': .075,
..:  'max_dist': 1.5,
..:  'loop_size': .075,
..:  'edge_labels_background': 'transparent'}
sage: g = Graph({0: [1, 2], 2: [3], 4: [0, 1]})
sage: GP = GraphPlot(g, options)

layout_tree(root, orientation)
Compute a nice layout of a tree.

INPUT:

- root – the root vertex.
- orientation – whether to place the root at the top or at the bottom:
  - orientation="down" – children are placed below their parent
  - orientation="top" – children are placed above their parent

EXAMPLES:

sage: from sage.graphs.graph_plot import GraphPlot
sage: G = graphs.HoffmanSingletonGraph()
sage: T = Graph()
sage: T.add_edges(G.min_spanning_tree(starting_vertex=0))
sage: T.show(layout='tree', tree_root=0) # indirect doctest

plot(**kwds)
Return a graphics object representing the (di)graph.

INPUT:

The options accepted by this method are to be found in the documentation of the sage.graphs.
graph_plot module, and the show() method.

Note: See the module's documentation for information on default values of this method.

We can specify some pretty precise plotting of familiar graphs:

sage: from math import sin, cos, pi
sage: P = graphs.PetersenGraph()
sage: d = {'#FF0000': [0, 5], '#FF9900': [1, 6], '#FFFF00': [2, 7],
.....: '#00FF00': [3, 8], '#0000FF': [4, 9]}
sage: pos_dict = {}
sage: for i in range(5):
.....:  x = float(cos(pi/2 + ((2*pi)/5)*i))
Here are some more common graphs with typical options:

```python
sage: C = graphs.CubeGraph(8)
sage: P = C.graphplot(vertex_labels=False, vertex_size=0,
....:                  graph_border=True)
sage: P.show()
```

```python
sage: G = graphs.HeawoodGraph().copy(sparse=True)
sage: for u, v, l in G.edges():
....:  G.set_edge_label(u, v, f'({u},{v})')
sage: G.graphplot(edge_labels=True).show()
```
3.1. Graph Plotting
The options for plotting also work with directed graphs:

```python
sage: D = DiGraph({
    ...: 0: [1, 10, 19], 1: [8, 2], 2: [3, 6], 3: [19, 4],
    ...: 4: [17, 5], 5: [6, 15], 6: [7], 7: [8, 14], 8: [9],
    ...: 9: [10, 13], 10: [11], 11: [12, 18], 12: [16, 13],
    ...: 13: [14], 14: [15], 15: [16], 16: [17], 17: [18],
    ...: 18: [19], 19: []})
sage: for u, v, l in D.edges():
    ...:     D.set_edge_label(u, v, f'({u}, {v})')
sage: D.graphplot(edge_labels=True, layout='circular').show()
```

This example shows off the coloring of edges:

```python
sage: from sage.plot.colors import rainbow
sage: C = graphs.CubeGraph(5)
sage: R = rainbow(5)
sage: edge_colors = {}
sage: for i in range(5):
    ...:     edge_colors[R[i]] = []
sage: for u, v, l in C.edges():
    ...:     for i in range(5):
    ...:         if u[i] != v[i]:
```

(continues on next page)
With the partition option, we can separate out same-color groups of vertices:

```
sage: D = graphs.DodecahedralGraph()
sage: Pi = [[6, 5, 15, 14, 7], [16, 13, 8, 2, 4],
        [12, 17, 9, 3, 1], [0, 19, 18, 10, 11]]
sage: D.show(partition=Pi)
```

Loops are also plotted correctly:

```
sage: G = graphs.PetersenGraph()
sage: G.allow_loops(True)
sage: G.add_edge(0,0)
sage: G.show()
```

```
sage: D = DiGraph({0:[0,1], 1:[2], 2:[3]}, loops=True)
sage: D.show()
sage: D.show(edge_colors={(0, 1, 0): [(0, 1, None), (1, 2, None)],
                      (0, 0, 0): [(2, 3, None)]})
```
Chapter 3. Plots of Other Mathematical Objects
More options:

```python
sage: pos = {0: [0.0, 1.5], 1: [-0.8, 0.3], 2: [-0.6, -0.8],
.....: 3: [0.6, -0.8], 4: [0.8, 0.3]}
```

```python
sage: g = Graph({0: [1], 1: [2], 2: [3], 3: [4], 4: [0]})
sage: g.graphplot(pos=pos, layout='spring', iterations=0).plot()
```

Graphics object consisting of 11 graphics primitives

We can plot multiple graphs:

```python
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.graphplot(heights={0: [0], 1: [4, 5, 1],
.....: 2: [2], 3: [3, 6]})
```

(continues on next page)
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.graphplot(heights={0: [0], 1: [4, 5, 1],
2: [2], 3: [3, 6]})

Graphics object consisting of 14 graphics primitives

sage: t.set_edge_label(0, 1, -7)
sage: t.set_edge_label(0, 5, 3)
sage: t.set_edge_label(0, 5, 99)
sage: t.set_edge_label(1, 2, 1000)
sage: t.set_edge_label(3, 2, 'spam')
sage: t.set_edge_label(2, 6, 3/2)
sage: t.set_edge_label(0, 4, 66)

sage: t.graphplot(heights={0: [0], 1: [4, 5, 1],
2: [2], 3: [3, 6]},
edge_labels=True)

Graphics object consisting of 14 graphics primitives
Graphics object consisting of 20 graphics primitives

```
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.graphplot(layout='tree').show()
```

The tree layout is also useful:
```
sage: t = DiGraph('JCC???@A??G0??C0??G0??')
sage: t.graphplot(layout='tree', tree_root=0, tree_orientation='up').show()
```

More examples:
```
sage: D = DiGraph({0:[1,2,3], 2:[1,4], 3:[0]})
sage: D.graphplot().show()
```
```
sage: D = DiGraph(multiedges=True, sparse=True)
sage: for i in range(5):
...:    D.add_edge((i, i + 1, 'a'))
...:    D.add_edge((i, i - 1, 'b'))
```

(continues on next page)
3.1. Graph Plotting
sage: D.graphplot(edge_labels=True,
...:                edge_colors=D._color_by_label())
...:                ).plot()
Graphics object consisting of 34 graphics primitives

sage: g = Graph({}, loops=True, multiedges=True, sparse=True)

sage: g.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
...:                (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
...:                (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h')])

sage: g.graphplot(edge_labels=True,
...:                color_by_label=True,
...:                edge_style='dashed'
...:                ).plot()
Graphics object consisting of 22 graphics primitives

The `edge_style` option may be provided in the short format too:

sage: g.graphplot(edge_labels=True,
...:                color_by_label=True,
...:                edge_style='--'
...:                ).plot()
Graphics object consisting of 22 graphics primitives
**set_edges(**edge_options**)

Set edge plotting parameters for the GraphPlot object.

This function is called by the constructor but can also be called to update the vertex options of an existing GraphPlot object. Note that the changes are cumulative.

**EXAMPLES:**

```python
sage: g = Graph(loops=True, multiedges=True, sparse=True)
sage: g.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
                (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
                (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h'))]

sage: GP = g.graphplot(vertex_size=100, edge_labels=True,
                    color_by_label=True, edge_style='dashed')

sage: GP.set_edges(edge_style='solid')

sage: GP.plot()

sage: d = DiGraph(loops=True, multiedges=True, sparse=True)

sage: d.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
                (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
                (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h'))]

sage: GP.set_edges(edge_color='black')

sage: GP.plot()

sage: d = Digraph(loops=True, multiedges=True, sparse=True)

sage: d.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
                (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
                (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h'))]
```
3.1. Graph Plotting
....: (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
....: (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h'))
sage: GP = d.graphplot(vertex_size=100, edge_labels=True,
....: color_by_label=True, edge_style='dashed')
sage: GP.set_edges(edge_style='solid')
sage: GP.plot()
Graphics object consisting of 24 graphics primitives

sage: GP.set_edges(edge_color='black')
sage: GP.plot()
Graphics object consisting of 24 graphics primitives

```
set_pos()
```
Set the position plotting parameters for this GraphPlot.

EXAMPLES:

This function is called implicitly by the code below:

```
sage: g = Graph({0: [1, 2], 2: [3], 4: [0, 1]})
sage: g.graphplot(save_pos=True, layout='circular') # indirect doctest
GraphPlot object for Graph on 5 vertices
```
The following illustrates the format of a position dictionary, but due to numerical noise we do not check the values themselves:

```
sage: g.get_pos()
{0: (0.0, 1.0),
  1: (-0.951..., 0.309...),
  2: (-0.587..., -0.809...),
  3: (0.587..., -0.809...),
  4: (0.951..., 0.309...)}
```

```
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.plot(heights={0: [0], 1: [4, 5, 1], 2: [2], 3: [3, 6]})
Graphics object consisting of 14 graphics primitives
```

**set_vertices(**vertex_options**)**

Set the vertex plotting parameters for this GraphPlot.

This function is called by the constructor but can also be called to make updates to the vertex options of an existing GraphPlot object. Note that the changes are cumulative.

EXAMPLES:
sage: g = Graph({}, loops=True, multiedges=True, sparse=True)
sage: g.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
               ....:            (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
               ....:            (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h')])
sage: GP = g.graphplot(vertex_size=100, edge_labels=True,
               ....:            color_by_label=True, edge_style='dashed')
sage: GP.set_vertices(talk=True)
sage: GP.plot()
Graphics object consisting of 22 graphics primitives
sage: GP.set_vertices(vertex_color='green', vertex_shape='^')
sage: GP.plot()
Graphics object consisting of 22 graphics primitives

Vertex labels are flexible:

sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=False)
Graphics object consisting of 4 graphics primitives
sage: g.plot(vertex_labels=True)
Graphics object consisting of 8 graphics primitives

3.1. Graph Plotting  315
Chapter 3. Plots of Other Mathematical Objects
```
sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=dict(zip(g, ['+', '-', '/', '*']))))
Graphics object consisting of 8 graphics primitives
```

```
sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=lambda x: str(x % 2))
Graphics object consisting of 8 graphics primitives
```

### show(**kwds)
Show the (di)graph associated with this GraphPlot object.

**INPUT:**
This method accepts all parameters of `sage.plot.graphics.Graphics.show()`.

**Note:**
- See the module's documentation for information on default values of this method.
- Any options not used by plot will be passed on to the `show()` method.

**EXAMPLES:**
3.2 Matrix Plots

class sage.plot.matrix_plot.MatrixPlot(xy_data_array, xrange, yrange, options)
   Bases: sage.plot.primitive.GraphicsPrimitive

Primitive class for the matrix plot graphics type. See matrix_plot? for help actually doing matrix plots.

INPUT:

- xy_data_array - list of lists giving matrix values corresponding to the grid
- xrange - tuple of 2 floats indicating range for horizontal direction (number of columns in the matrix). If None, the defaults are used as indicated in matrix_plot().
- yrange - tuple of 2 floats indicating range for vertical direction (number of rows in the matrix). If None, the defaults are used as indicated in matrix_plot().
- options - dict of valid plot options to pass to constructor
EXAMPLES:

Note this should normally be used indirectly via \texttt{matrix_plot()}: 

\begin{verbatim}
sage: from sage.plot.matrix_plot import MatrixPlot
sage: M = MatrixPlot([[1,3],[2,4]],(1,2),(2,3),options={'cmap':'winter'})
sage: M
MatrixPlot defined by a 2 x 2 data grid
sage: M.yrange
(2, 3)
sage: M.xy_data_array
[[1, 3], [2, 4]]
sage: M.options()
{'cmap': 'winter'}
\end{verbatim}

Extra options will get passed on to \texttt{show()}, as long as they are valid:

\begin{verbatim}
sage: matrix_plot([[1, 0], [0, 1]], fontsize=10)
Graphics object consisting of 1 graphics primitive
sage: matrix_plot([[1, 0], [0, 1]]).show(fontsize=10) # These are equivalent
\end{verbatim}

\texttt{get\_minmax\_data()}

Returns a dictionary with the bounding box data.

EXAMPLES:

\begin{verbatim}
sage: m = matrix_plot(matrix([[1,3,5,1],[2,4,5,6],[1,3,5,7]]))[0]
sage: list(sorted(m.get_minmax_data().items()))
[('xmax', 3.5), ('xmin', -0.5), ('ymax', 2.5), ('ymin', -0.5)]
\end{verbatim}

\texttt{sage.plot.matrix\_plot.matrix\_plot(mat, xrange=None, yrange=None, aspect\_ratio=1, axes=False, cmap='Greys', colorbar=False, frame=True, marker='', norm=None, flip_y=True, subdivisions=False, ticks_integer=True, vmin=None, vmax=None, subdivision_boundaries=None, subdivision_style=None, colorbar\_orientation='vertical', colorbar\_format=None, **options)}

A plot of a given matrix or 2D array.

If the matrix is sparse, colors only indicate whether an element is nonzero or zero, so the plot represents the sparsity pattern of the matrix.

If the matrix is dense, each matrix element is given a different color value depending on its relative size compared to the other elements in the matrix.

The default is for the lowest number to be black and the highest number to be white in a greyscale pattern; see the information about normalizing below. To reverse this, use \texttt{cmap='Greys'}.

The tick marks drawn on the frame axes denote the row numbers (vertical ticks) and the column numbers (horizontal ticks) of the matrix.

INPUT:

* \texttt{mat} - a 2D matrix or array
* \texttt{xrange} - (default: None) tuple of the horizontal extent (\texttt{xmin}, \texttt{xmax}) of the bounding box in which to draw the matrix. The image is stretched individually along x and y to fill the box.

If None, the extent is determined by the following conditions. Matrix entries have unit size in data coordinates. Their centers are on integer coordinates, and their center coordinates range from 0 to columns-1 horizontally and from 0 to rows-1 vertically.
If the matrix is sparse, this keyword is ignored.

- **yrange** - (default: None) tuple of the vertical extent \((y_{\text{min}}, y_{\text{max}})\) of the bounding box in which to draw the matrix. See **xrange** for details.

The following input must all be passed in as named parameters, if default not used:

- **cmap** - a colormap (default: ‘Greys’), the name of a predefined colormap, a list of colors, or an instance of a matplotlib Colormap.

  The list of predefined color maps can be visualized in matplotlib’s documentation. You can also type `import matplotlib.cm; matplotlib.cm.datad.keys()` to list their names.

- **colorbar** – boolean (default: False) Show a colorbar or not (dense matrices only).

  The following options are used to adjust the style and placement of colorbars. They have no effect if a colorbar is not shown.

  - **colorbar_orientation** – string (default: ‘vertical’), controls placement of the colorbar, can be either ‘vertical’ or ‘horizontal’

    - **colorbar_format** – a format string, this is used to format the colorbar labels.

    - **colorbar_options** – a dictionary of options for the matplotlib.colorbar API. Documentation for the matplotlib.colorbar module has details.

- **norm** - If None (default), the value range is scaled to the interval \([0,1]\). If ‘value’, then the actual value is used with no scaling. A matplotlib.colors.Normalize instance may also passed.

- **vmin** - The minimum value (values below this are set to this value)

- **vmax** - The maximum value (values above this are set to this value)

- **flip_y** - (default: True) boolean. If False, the first row of the matrix is on the bottom of the graph. Otherwise, the first row is on the top of the graph.

- **subdivisions** - If True, plot the subdivisions of the matrix as lines.

- **subdivision_boundaries** - a list of lists in the form \([\text{row_subdivisions}, \text{column_subdivisions}]\), which specifies the row and column subdivisions to use. If not specified, defaults to the matrix subdivisions

- **subdivision_style** - a dictionary of properties passed on to the `line2d()` command for plotting subdivisions. If this is a two-element list or tuple, then it specifies the styles of row and column divisions, respectively.

**EXAMPLES:**

A matrix over \(\mathbb{Z}\) colored with different grey levels:

```python
sage: matrix_plot(matrix([[1,3,5,1],[2,4,5,6],[1,3,5,7]]))
Graphics object consisting of 1 graphics primitive
```

Here we make a random matrix over \(\mathbb{R}\) and use `cmap=`’hsv’ to color the matrix elements different RGB colors:

```python
sage: matrix_plot(random_matrix(RDF, 50), cmap='hsv')
Graphics object consisting of 1 graphics primitive
```

By default, entries are scaled to the interval \([0,1]\) before determining colors from the color map. That means the two plots below are the same:

```python
sage: P = matrix_plot(matrix(2,[1,1,3,3]))
sage: Q = matrix_plot(matrix(2,[2,2,3,3]))
sage: P; Q
```

(continues on next page)
Graphics object consisting of 1 graphics primitive
Graphics object consisting of 1 graphics primitive

However, we can specify which values scale to 0 or 1 with the \texttt{vmin} and \texttt{vmax} parameters (values outside the range are clipped). The two plots below are now distinguished:

\begin{verbatim}
sage: P = matrix_plot(matrix(2,[[1,1,3,3]]), vmin=0, vmax=3, colorbar=True)
sage: Q = matrix_plot(matrix(2,[[2,2,3,3]]), vmin=0, vmax=3, colorbar=True)
sage: P; Q
Graphics object consisting of 1 graphics primitive
Graphics object consisting of 1 graphics primitive
\end{verbatim}

We can also specify a norm function of ‘value’, which means that there is no scaling performed:

\begin{verbatim}
sage: matrix_plot(random_matrix(ZZ,10)*.05, norm='value', colorbar=True)
Graphics object consisting of 1 graphics primitive
\end{verbatim}

Matrix subdivisions can be plotted as well:

\begin{verbatim}
sage: m=random_matrix(RR,10)
sage: m.subdivide([2,4],[6,8])
sage: matrix_plot(m, subdivisions=True, subdivision_style=dict(color='red', thickness=3))
Graphics object consisting of 1 graphics primitive
\end{verbatim}

You can also specify your own subdivisions and separate styles for row or column subdivisions:

\begin{verbatim}
sage: m=random_matrix(RR,10)
sage: matrix_plot(m, subdivisions=True, subdivision_boundaries=[[2,4],[6,8]], subdivision_style=[dict(color='red', thickness=3),dict(linestyle='--', thickness=6)])
Graphics object consisting of 1 graphics primitive
\end{verbatim}

Generally matrices are plotted with the (0,0) entry in the upper left. However, sometimes if we are plotting an image, we’d like the (0,0) entry to be in the lower left. We can do that with the \texttt{flip_y} argument:

\begin{verbatim}
sage: matrix_plot(identity_matrix(100), flip_y=False)
Graphics object consisting of 1 graphics primitive
\end{verbatim}

A custom bounding box in which to draw the matrix can be specified using the \texttt{xrange} and \texttt{yrange} arguments:

\begin{verbatim}
sage: P = matrix_plot(identity_matrix(10), xrange=(0, pi), yrange=(-pi, 0))
sage: P
Graphics object consisting of 1 graphics primitive
sage: P.get_minmax_data()
\{'xmax': 3.14159..., 'xmin': 0.0, 'ymax': 0.0, 'ymin': -3.14159...\}
\end{verbatim}

If the horizontal and vertical dimension of the image are very different, the default \texttt{aspect_ratio=1} may be unsuitable and can be changed to \texttt{automatic}:

\begin{verbatim}
sage: matrix_plot(random_matrix(RDF, 2, 2), (-100, 100), (0, 1), ...
\quad aspect_ratio='automatic')
Graphics object consisting of 1 graphics primitive
\end{verbatim}
Another random plot, but over $\mathbb{F}_{389}$:

```
sage: m = random_matrix(GF(389), 10)
sage: matrix_plot(m, cmap='Oranges')
Graphics object consisting of 1 graphics primitive
```

It also works if you lift it to the polynomial ring:

```
sage: matrix_plot(m.change_ring(GF(389)['x']), cmap='Oranges')
Graphics object consisting of 1 graphics primitive
```

We have several options for colorbars:

```
sage: matrix_plot(random_matrix(RDF, 50), colorbar=True, colorbar_orientation='horizontal')
Graphics object consisting of 1 graphics primitive
```

```
sage: matrix_plot(random_matrix(RDF, 50), colorbar=True, colorbar_format='%.3f')
Graphics object consisting of 1 graphics primitive
```

The length of a color bar and the length of the adjacent matrix plot dimension may be quite different. This example shows how to adjust the length of the colorbar by passing a dictionary of options to the matplotlib colorbar routines.

```
sage: m = random_matrix(ZZ, 40, 80, x=-10, y=10)
sage: m.plot(colorbar=True, colorbar_orientation='vertical', colorbar_options={'shrink':0.50})
Graphics object consisting of 1 graphics primitive
```

Here we plot a random sparse matrix:

```
sage: sparse = matrix(dict([(randint(0, 10), randint(0, 10)), 1] for i in range(100)))
sage: matrix_plot(sparse)
Graphics object consisting of 1 graphics primitive
```

```
sage: A=random_matrix(ZZ,100000,density=.00001,sparse=True)
sage: matrix_plot(A,marker=',')
Graphics object consisting of 1 graphics primitive
```

As with dense matrices, sparse matrix entries are automatically converted to floating point numbers before plotting. Thus the following works:

```
sage: b=random_matrix(GF(2),200,sparse=True,density=0.01)
sage: matrix_plot(b)
Graphics object consisting of 1 graphics primitive
```

While this returns an error:

```
sage: b=random_matrix(CDF,200,sparse=True,density=0.01)
sage: matrix_plot(b)
Traceback (most recent call last):
...
ValueError: cannot convert entries to floating point numbers
```
To plot the absolute value of a complex matrix, use the `apply_map` method:

```python
sage: b=random_matrix(CDF,200,sparse=True,density=0.01)
sage: matrix_plot(b.apply_map(abs))
```

Graphics object consisting of 1 graphics primitive

Plotting lists of lists also works:

```python
sage: matrix_plot([[1,3,5,1],[2,4,5,6],[1,3,5,7]])
```

Graphics object consisting of 1 graphics primitive

As does plotting of NumPy arrays:

```python
sage: import numpy
sage: matrix_plot(numpy.random.rand(10, 10))
```

Graphics object consisting of 1 graphics primitive

A plot title can be added to the matrix plot:

```python
sage: matrix_plot(identity_matrix(50), flip_y=False, title='not identity')
```

Graphics object consisting of 1 graphics primitive

The title position is adjusted upwards if the `flip_y` keyword is set to True (this is the default):

```python
sage: matrix_plot(identity_matrix(50), title='identity')
```

Graphics object consisting of 1 graphics primitive
4.1 Arcs of circles and ellipses

```python
class sage.plot.arc.Arc(x, y, r1, r2, angle, s1, s2, options):
    Bases: sage.plot.primitive.GraphicPrimitive

    Primitive class for the Arc graphics type. See arc? for information about actually plotting an arc of a circle or an ellipse.

    INPUT:
    • x, y - coordinates of the center of the arc
    • r1, r2 - lengths of the two radii
    • angle - angle of the horizontal with width
    • sector - sector of angle
    • options - dict of valid plot options to pass to constructor

    EXAMPLES:
    Note that the construction should be done using arc:

    sage: from sage.plot.arc import Arc
    sage: print(Arc(0,0,1,1,pi/4,pi/4,pi/2,{}))
    Arc with center (0.0,0.0) radii (1.0,1.0) angle 0.78539816339... inside the sector→(0.78539816339...,1.5707963267...)
```

`bezier_path()`

Return self as a Bezier path.

This is needed to concatenate arcs, in order to create hyperbolic polygons.

EXAMPLES:

```python
sage: from sage.plot.arc import Arc
sage: op = {'alpha':1,'thickness':1,'rgbcolor':'blue','zorder':0,.....: 'linestyle': '--'}
sage: Arc(2,3,2.2,2.2,0,2,3,op).bezier_path()
Graphics object consisting of 1 graphics primitive
```

(continues on next page)
get_minmax_data()

Return a dictionary with the bounding box data.

The bounding box is computed as minimal as possible.

EXAMPLES:

An example without angle:

```
sage: p = arc((-2, 3), 1, 2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-3.0
sage: d['xmax']
-1.0
sage: d['ymin']
1.0
sage: d['ymax']
5.0
```

The same example with a rotation of angle $\pi/2$:

```
sage: p = arc((-2, 3), 1, 2, pi/2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-4.0
sage: d['xmax']
0.0
sage: d['ymin']
2.0
sage: d['ymax']
4.0
```

plot3d()

sage.plot.arc.arc(center, r1, r2=None, angle=0.0, sector=(0.0, 6.283185307179586), alpha=1, thickness=1, linestyle='solid', zorder=5, rgbcolor='blue', aspect_ratio=1.0, **options)

An arc (that is a portion of a circle or an ellipse)

Type arc.options to see all options.

INPUT:

- center - 2-tuple of real numbers - position of the center.
- r1, r2 - positive real numbers - radii of the ellipse. If only $r1$ is set, then the two radii are supposed to be equal and this function returns an arc of circle.
- angle - real number - angle between the horizontal and the axis that corresponds to $r1$.
- sector - 2-tuple (default: (0,2*pi))- angles sector in which the arc will be drawn.

OPTIONS:

- alpha - float (default: 1) - transparency
- thickness - float (default: 1) - thickness of the arc
• **color**, **rgbcolor** - string or 2-tuple (default: 'blue') - the color of the arc

• **linestyle** - string (default: 'solid') - The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', '-', ':', ' -. ', respectively.

**EXAMPLES:**

Plot an arc of circle centered at (0,0) with radius 1 in the sector \((\pi/4, 3\pi/4)\):

```python
sage: arc((0,0), 1, sector=(pi/4,3*pi/4))
Graphics object consisting of 1 graphics primitive
```

Plot an arc of an ellipse between the angles 0 and \(\pi/2\):

```python
sage: arc((2,3), 2, 1, sector=(0,pi/2))
Graphics object consisting of 1 graphics primitive
```

Plot an arc of a rotated ellipse between the angles 0 and \(\pi/2\):

```python
sage: arc((2,3), 2, 1, angle=pi/5, sector=(0,pi/2))
Graphics object consisting of 1 graphics primitive
```

Plot an arc of an ellipse in red with a dashed linestyle:

```python
sage: arc((0,0), 2, 1, 0, (0,pi/2), linestyle="dashed", color="red")
Graphics object consisting of 1 graphics primitive
sage: arc((0,0), 2, 1, 0, (0,pi/2), linestyle="--", color="red")
Graphics object consisting of 1 graphics primitive
```

The default aspect ratio for arcs is 1.0:

```python
sage: arc((0,0), 1, sector=(pi/4,3*pi/4)).aspect_ratio()
1.0
```

It is not possible to draw arcs in 3D:

```python
sage: A = arc((0,0,0), 1)
Traceback (most recent call last):
... NotImplementedError
```

### 4.2 Arrows

**class** `sage.plot.arrow.Arrow(xtail, ytail, xhead, yhead, options)`

**Bases:** `sage.plot.primitive.GraphicPrimitive`

Primitive class that initializes the (line) arrow graphics type

**EXAMPLES:**

We create an arrow graphics object, then take the 0th entry in it to get the actual Arrow graphics primitive:

```python
sage: P = arrow((0,1), (2,3))[0]
sage: type(P)
<class 'sage.plot.arrow.Arrow'>
```
sage: P
Arrow from (0.0,1.0) to (2.0,3.0)

get_minmax_data()
Returns a bounding box for this arrow.

EXAMPLES:

sage: d = arrow((1,1), (5,5)).get_minmax_data()
sage: d['xmin']
1.0
sage: d['xmax']
5.0

plot3d(ztail=0, zhead=0, **kwds)
Takes 2D plot and places it in 3D.

EXAMPLES:

sage: A = arrow((0,0),(1,1))[0].plot3d()
sage: A.jmol_repr(A.testing_render_params())[0]
'draw line_1 diameter 2 arrow {0.0 0.0 0.0} {1.0 1.0 0.0} '

Note that we had to index the arrow to get the Arrow graphics primitive. We can also change the height via the Graphics.plot3d() method, but only as a whole:

sage: A = arrow((0,0),(1,1)).plot3d(3)
sage: A.jmol_repr(A.testing_render_params())[0][0]
'draw line_1 diameter 2 arrow {0.0 0.0 3.0} {1.0 1.0 3.0} '

Optional arguments place both the head and tail outside the xy-plane, but at different heights. This must be done on the graphics primitive obtained by indexing:

sage: A=arrow((0,0),(1,1))[0].plot3d(3,4)
sage: A.jmol_repr(A.testing_render_params())[0]
'draw line_1 diameter 2 arrow {0.0 0.0 3.0} {1.0 1.0 4.0} '

class sage.plot.arrow.CurveArrow(path, options)
Bases: sage.plot.primitive.GraphicPrimitive

Returns an arrow graphics primitive along the provided path (bezier curve).

EXAMPLES:

sage: from sage.plot.arrow import CurveArrow
sage: b = CurveArrow(path=[[0,0],[.5,.5],[1,0],[.5,1],[0,0]],
.....: options={})
sage: b
CurveArrow from (0, 0) to (0, 0)

get_minmax_data()
Returns a dictionary with the bounding box data.

EXAMPLES:
sage: from sage.plot.arrow import CurveArrow
sage: b = CurveArrow(path=[[0,0], [.5,.5], [1,0], [.5,1], [0,0]], options={})
sage: d = b.get_minmax_data()
sage: d['xmin']
0.0
sage: d['xmax']
1.0

sage.plot.arrow.arrow(tailpoint=None, headpoint=None, **kwds)
Returns either a 2-dimensional or 3-dimensional arrow depending on value of points.

For information regarding additional arguments, see either arrow2d? or arrow3d?.

EXAMPLES:

sage: arrow((0,0), (1,1))
Graphics object consisting of 1 graphics primitive

sage: arrow((0,0,1), (1,1,1))
Graphics3d Object

4.2. Arrows
sage.plot.arrow.arrow2d(tailpoint=None, headpoint=None, path=None, width=2, rgbcolor=(0, 0, 1),
zorder=2, head=1, linestyle='solid', legend_label=None, **options)

If tailpoint and headpoint are provided, returns an arrow from (xtail, ytail) to (xhead, yhead). If tailpoint or headpoint is None and path is not None, returns an arrow along the path. (See further info on paths in bezier_path).

INPUT:

- **tailpoint** - the starting point of the arrow
- **headpoint** - where the arrow is pointing to
- **path** - the list of points and control points (see bezier_path for detail) that the arrow will follow from source to destination
- **head** - 0, 1 or 2, whether to draw the head at the start (0), end (1) or both (2) of the path (using 0 will swap headpoint and tailpoint). This is ignored in 3D plotting.
- **linestyle** - (default: 'solid') The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '---', ':', '-', '.-', respectively.
- **width** - (default: 2) the width of the arrow shaft, in points
- **color** - (default: (0,0,1)) the color of the arrow (as an RGB tuple or a string)
- **hue** - the color of the arrow (as a number)
- **arrowsize** - the size of the arrowhead
- **arrowshorten** - the length in points to shorten the arrow (ignored if using path parameter)
- **legend_label** - the label for this item in the legend
- **legend_color** - the color for the legend label
- **zorder** - the layer level to draw the arrow– note that this is ignored in 3D plotting.

EXAMPLES:

A straight, blue arrow:

```sage
sage: arrow2d((1,1), (3,3))
Graphics object consisting of 1 graphics primitive
```

Make a red arrow:

```sage
sage: arrow2d((-1,-1), (2,3), color=(1,0,0))
Graphics object consisting of 1 graphics primitive
```

```sage
sage: arrow2d((-1,-1), (2,3), color='red')
Graphics object consisting of 1 graphics primitive
```

You can change the width of an arrow:

```sage
sage: arrow2d((1,1), (3,3), width=5, arrowsize=15)
Graphics object consisting of 1 graphics primitive
```

Use a dashed line instead of a solid one for the arrow:

```sage
sage: arrow2d((1,1), (3,3), linestyle='dashed')
Graphics object consisting of 1 graphics primitive
```

(continues on next page)
Chapter 4. Basic Shapes
4.2. Arrows
sage: arrow2d((1,1), (3,3), linestyle='--')
Graphics object consisting of 1 graphics primitive

A pretty circle of arrows:

sage: sum([arrow2d((0,0), (cos(x),sin(x)), hue=x/(2*pi)) for x in [0..2*pi,step=0.˓→1]])
Graphics object consisting of 63 graphics primitives

If we want to draw the arrow between objects, for example, the boundaries of two lines, we can use the arrowshorten option to make the arrow shorter by a certain number of points:

sage: L1 = line([(0,0), (1,0)], thickness=10)
sage: L2 = line([(0,1), (1,1)], thickness=10)
sage: A = arrow2d((0.5,0), (0.5,1), arrowshorten=10, rgbcolor=(1,0,0))
sage: L1 + L2 + A
Graphics object consisting of 3 graphics primitives

If BOTH headpoint and tailpoint are None, then an empty plot is returned:

sage: arrow2d(headpoint=None, tailpoint=None)
Graphics object consisting of 0 graphics primitives
4.2. Arrows
We can also draw an arrow with a legend:

```
sage: arrow((0,0), (0,2), legend_label='up', legend_color='purple')
```
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to `Graphics.show()`, as long as they are valid:

```
sage: arrow2d((-2,2), (7,1), frame=True)
```
Graphics object consisting of 1 graphics primitive

```
sage: arrow2d((-2,2), (7,1)).show(frame=True)
```

### 4.3 Bezier Paths

**class** `sage.plot.bezier_path.BezierPath(path, options)`

Bases: `sage.plot.primitive.GraphicPrimitive_xydata`

Path of Bezier Curves graphics primitive.

The input to this constructor is a list of curves, each a list of points, along which to create the curves, along with a dict of any options passed.

**EXAMPLES:**
We use `bezier_path()` to actually plot Bezier curves:

```python
sage: bezier_path([[0,0],(0.5,0.5),(1,0)],[(0.5,1),(0,0)], linestyle="dashed")
Graphics object consisting of 1 graphics primitive
```

```
plot3d(z=0, **kwds)
```
and nonidentical) \( z \) coordinates in the path and control points, use the function `bezier3d()` instead of `bezier_path()`.

**EXAMPLES:**

```python
sage: b = bezier_path([[0,0],[0,1],[1,0]])
sage: A = b.plot3d()
sage: B = b.plot3d(z=2)
sage: A + B
Graphics3d Object
```

```python
sage: bezier3d([[0,0,0],[1,0,0],[0,1,0],[0,1,1]])
Graphics3d Object
```

Sage's `bezier3d()` function takes a list of points, where each point is a tuple `(x, y, z)`, and returns a 3D Bezier path. The first curve contains the endpoints as the first and last point in the list, and all other curves assume a starting point given by the last entry in the preceding list, and take the last point in the list as their opposite endpoint. A curve can have 0, 1 or 2 control points listed between the endpoints. In the input example for `path` below, the first and second curves have 2 control points, the third has one, and the fourth has no control points.

```python
sage.plot.bezier_path.bezier_path(path, alpha=1, fill=False, thickness=1, rgbcolor=(0, 0, 0), zorder=2, linestyle='solid', **options)
```

Returns a Graphics object of a Bezier path corresponding to the path parameter. The path is a list of curves, and each curve is a list of points. Each point is a tuple \((x, y)\).
In the case of no control points, a straight line will be drawn between the two endpoints. If one control point is supplied, then the curve at each of the endpoints will be tangent to the line from that endpoint to the control point. Similarly, in the case of two control points, at each endpoint the curve will be tangent to the line connecting that endpoint with the control point immediately after or immediately preceding it in the list.

So in our example above, the curve between \( p_1 \) and \( p_2 \) is tangent to the line through \( p_1 \) and \( c_1 \) at \( p_1 \), and tangent to the line through \( p_2 \) and \( c_2 \) at \( p_2 \). Similarly, the curve between \( p_2 \) and \( p_3 \) is tangent to line(\( p_2, c_3 \)) at \( p_2 \) and tangent to line(\( p_3, c_4 \)) at \( p_3 \). Curve(\( p_3, p_4 \)) is tangent to line(\( p_3, c_5 \)) at \( p_3 \) and tangent to line(\( p_4, c_5 \)) at \( p_4 \). Curve(\( p_4, p_5 \)) is a straight line.

**INPUT:**

- path – a list of lists of tuples (see above)
- alpha – default: 1
- fill – default: False
- thickness – default: 1
- linestyle – default: 'solid', The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '-' ', ':' ':' '-' ' - ', respectively.
- rgbcolor – default: (0,0,0)
- zorder – the layer in which to draw

**EXAMPLES:**
To construct a simple curve, create a list containing a single list:

```sage
path = [[(0,0),(.5,1),(1,0)]]
sage: curve = bezier_path(path, linestyle='dashed', rgbcolor='green')
sage: curve
Graphics object consisting of 1 graphics primitive
```

Extra options will get passed on to `show()`, as long as they are valid:

```sage
bezier_path([[0,1],(.5,0),(1,1)], fontsize=50)
Graphics object consisting of 1 graphics primitive
sage: bezier_path([[0,1],(.5,0),(1,1)]).show(fontsize=50) # These are equivalent
```

4.3. Bezier Paths 349
4.3. Bezier Paths
4.4 Circles

class sage.plot.circle.Circle(x, y, r, options)
Bases: sage.plot.primitive.GraphicPrimitive

Primitive class for the Circle graphics type. See circle? for information about actually plotting circles.

INPUT:

• x – x-coordinate of center of Circle
• y – y-coordinate of center of Circle
• r – radius of Circle object
• options – dict of valid plot options to pass to constructor

EXAMPLES:

Note this should normally be used indirectly via circle:

```
sage: from sage.plot.circle import Circle
sage: C = Circle(2,3,5,{'zorder':2})
sage: C
Circle defined by (2.0,3.0) with r=5.0
sage: C.options()['zorder']
2
sage: C.r
5.0
```

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

```
sage: p = circle((3, 3), 1)
sage: d = p.get_minmax_data()
sage: d['xmin']
2.0
sage: d['ymin']
2.0
```

plot3d(z=0, **kwds)

Plots a 2D circle (actually a 50-gon) in 3D, with default height zero.

INPUT:

• z - optional 3D height above xy-plane.

EXAMPLES:

```
sage: circle((0,0), 1).plot3d()
Graphics3d Object
```

This example uses this method implicitly, but does not pass the optional parameter z to this method:

```
sage: sum([circle((random(),random()), random()).plot3d(z=random()) for _ in range(20)])
Graphics3d Object
```
These examples are explicit, and pass \( z \) to this method:

```python
sage: C = circle((2,pi), 2, hue=.8, alpha=.3, fill=True)
sage: c = C[0]
sage: d = c.plot3d(z=2)
sage: d.texture.opacity
0.3
```

```python
sage: C = circle((2,pi), 2, hue=.8, alpha=.3, linestyle='dotted')
sage: c = C[0]
sage: d = c.plot3d(z=2)
sage: d.jmol_repr(d.testing_render_params())[0][-1]
'color $line_1 translucent 0.7 [204,0,255]'
```

```python
circle(center, radius, alpha=1, fill=False, thickness=1, edgecolor='blue', facecolor='blue', linestyle='solid', zorder=5, legend_label=None, legend_color=None, clip=True, aspect_ratio=1.0, **options)
```

Return a circle at a point \((x, y)\) (or \((x, y, z)\) and parallel to the \(xy\)-plane) with radius \(r\). Type `circle`.

**OPTIONS:**

- **alpha** - default: 1
- **fill** - default: False
- **thickness** - default: 1
- **linestyle** - default: 'solid' (2D plotting only) The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '.', respectively.
- **edgecolor** - default: 'blue' (2D plotting only)
- **facecolor** - default: 'blue' (2D plotting only, useful only if fill=True)
- **rgbcolor** - 2D or 3D plotting. This option overrides edgecolor and facecolor for 2D plotting.
- **legend_label** - the label for this item in the legend
- **legend_color** - the color for the legend label

**EXAMPLES:**

The default color is blue, the default linestyle is solid, but this is easy to change:

```python
sage: c = circle((1,1), 1)
sage: c
Graphics object consisting of 1 graphics primitive
```

```python
sage: c = circle((1,1), 1, rgbcolor=(1,0,0), linestyle='--')
sage: c
Graphics object consisting of 1 graphics primitive
```

We can also use this command to plot three-dimensional circles parallel to the \(xy\)-plane:

```python
sage: c = circle((1,1,3), 1, rgbcolor=(1,0,0))
sage: c
Graphics3d Object
```

```python
sage: type(c)
<class 'sage.plot.plot3d.base.TransformGroup'>
```
4.4. Circles
4.4. Circles
To correct the aspect ratio of certain graphics, it is necessary to show with a \texttt{figsize} of square dimensions:

\begin{verbatim}
\texttt{sage: c.show(figsize=[5,5],xmin=-1,xmax=3,ymin=-1,ymax=3)}
\end{verbatim}

Here we make a more complicated plot, with many circles of different colors:

\begin{verbatim}
sage: g = Graphics()
sage: step=6; ocur=1/5; paths=16
sage: PI = math.pi # numerical for speed -- fine for graphics
sage: for r in range(1,paths+1):
    ....:     for x,y in [((r+ocur)*math.cos(n), (r+ocur)*math.sin(n)) for n in srange(0, 2*PI+PI/step, PI/step)]:
    ....:         g += circle((x,y), ocur, rgbcolor=hue(r/paths))
    ....:     rnext = (r+1)^2
    ....:     ocur = (rnext-r)-ocur
sage: g.show(xmin=-(paths+1)^2, xmax=(paths+1)^2, ymin=-(paths+1)^2, ymax=(paths+1)^2, figsize=[6,6])
\end{verbatim}

Note that the \texttt{rgbcolor} option overrides the other coloring options. This produces red fill in a blue circle:

\begin{verbatim}
sage: circle((2,3), 1, fill=True, edgecolor='blue', facecolor='red')
Graphics object consisting of 1 graphics primitive
\end{verbatim}

This produces an all-green filled circle:
4.4. Circles
The option `hue` overrides all other options, so be careful with its use. This produces a purplish filled circle:

```python
sage: circle((2,3), 1, fill=True, edgecolor='blue', rgbcolor='green', hue=.8)
```

Graphics object consisting of 1 graphics primitive

And circles with legends:

```python
sage: circle((4,5), 1, rgbcolor='yellow', fill=True, legend_label='the sun').show(xmin=0, ymin=0)
```

Extra options will get passed on to `show()`, as long as they are valid:

```python
sage: circle((0, 0), 2, figsize=[10,10]) # That circle is huge!
```

Graphics object consisting of 1 graphics primitive

```python
sage: circle((0, 0), 2).show(figsize=[10,10]) # These are equivalent
```
4.4. Circles
4.5 Disks

class sage.plot.disk.Disk(point, r, angle, options)
   Bases: sage.plot.primitive.GraphicPrimitive

Primitive class for the Disk graphics type. See disk? for information about actually plotting a disk (the Sage term for a sector or wedge of a circle).

INPUT:

   • point - coordinates of center of disk
   • r - radius of disk
   • angle - beginning and ending angles of disk (i.e. angle extent of sector/wedge)
   • options - dict of valid plot options to pass to constructor

EXAMPLES:

Note this should normally be used indirectly via disk:

```python
sage: from sage.plot.disk import Disk
sage: D = Disk((1,2), 2, (pi/2,pi), {'zorder':3})
sage: D
Disk defined by (1.0,2.0) with r=2.0 spanning (1.5707963267..., 3.1415926535...) → radians
sage: D.options()['zorder']
3
sage: D.x
1.0
```

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

```python
sage: D = disk((5,4), 1, (pi/2, pi))
sage: d = D.get_minmax_data()
sage: d['xmin']
4.0
sage: d['ymin']
3.0
sage: d['xmax']
6.0
sage: d['ymax']
5.0
```

plot3d(z=0, **kwds)

Plots a 2D disk (actually a 52-gon) in 3D, with default height zero.

INPUT:

   • z - optional 3D height above xy-plane.

AUTHORS:

   • Karl-Dieter Crisman (05-09)

EXAMPLES:
A disk (that is, a sector or wedge of a circle) with center at a point \((x, y)\) (or \((x, y, z)\) and parallel to the \(xy\)-plane) with radius \(r\) spanning (in radians) \(\theta\) spanned \((\theta_1, \theta_2)\).

Type `disk.options` to see all options.

**EXAMPLES:**

Make some dangerous disks:

```
sage: bl = disk((0.0,0.0), 1, (pi, 3*pi/2), color='yellow')
sage: tr = disk((0.0,0.0), 1, (0, pi/2), color='yellow')
sage: tl = disk((0.0,0.0), 1, (pi/2, pi), color='black')
sage: br = disk((0.0,0.0), 1, (3*pi/2, 2*pi), color='black')
sage: P = tl+tr+bl+br
sage: P.show(xmin=-2,xmax=2,ymin=-2,ymax=2)
```

The default aspect ratio is 1.0:

```
sage: disk((0.0,0.0), 1, (pi, 3*pi/2)).aspect_ratio() 1.0
```

Another example of a disk:

```
sage: bl = disk((0.0,0.0), 1, (pi, 3*pi/2), rgbcolor=(1,1,0))
sage: bl.show(figsize=[5,5])
```

Note that since `thickness` defaults to zero, it is best to change that option when using `fill=False`:

```
sage: disk((2,3), 1, (pi/4,pi/3), hue=.8, alpha=.3, fill=False, thickness=2)
```

The previous two examples also illustrate using `hue` and `rgbcolor` as ways of specifying the color of the graphic. We can also use this command to plot three-dimensional disks parallel to the \(xy\)-plane:
sage: d = disk((1,1,3), 1, (pi,3*pi/2), rgbcolor=(1,0,0))
sage: d
Graphics3d Object
sage: type(d)
<... 'sage.plot.plot3d.index_face_set.IndexFaceSet'>

Extra options will get passed on to show(), as long as they are valid:

sage: disk((0, 0), 5, (0, pi/2), xmin=0, xmax=5, ymin=0, ymax=5, figsize=(2,2),
→rgbcolor=(1, 0, 1))
Graphics object consisting of 1 graphics primitive
sage: disk((0, 0), 5, (0, pi/2), rgbcolor=(1, 0, 1)).show(xmin=0, xmax=5, ymin=0,␣
→ymax=5, figsize=(2,2))  # These are equivalent

4.6 Ellipses

class sage.plot.ellipse.Ellipse(x, y, r1, r2, angle, options)
Bases: sage.plot.primitive.GraphicPrimitive

Primitive class for the Ellipse graphics type. See ellipse? for information about actually plotting ellipses.

INPUT:
• x,y - coordinates of the center of the ellipse
• r1, r2 - radii of the ellipse
• angle - angle
• options - dictionary of options

EXAMPLES:

Note that this construction should be done using ellipse:

sage: from sage.plot.ellipse import Ellipse
sage: Ellipse(0, 0, 2, 1, pi/4, {})
Ellipse centered at (0.0, 0.0) with radii (2.0, 1.0) and angle 0.78539816339...

get_minmax_data()
Return a dictionary with the bounding box data.

The bounding box is computed to be as minimal as possible.

EXAMPLES:

An example without an angle:

sage: p = ellipse((-2, 3), 1, 2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-3.0
sage: d['xmax']
-1.0
sage: d['ymin']
1.0
(continues on next page)
The same example with a rotation of angle $\pi/2$:

```python
sage: p = ellipse((-2, 3), 1, 2, pi/2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-4.0
sage: d['xmax']
0.0
sage: d['ymin']
2.0
sage: d['ymax']
4.0
```

plot3d()

Plotting in 3D is not implemented.

```python
sage.plot.ellipse.ellipse(center, r1, r2, angle=0, alpha=1, fill=False, thickness=1, edgecolor='blue', facecolor='blue', linestyle='solid', zorder=5, aspect_ratio=1.0, legend_label=None, legend_color=None, **options)
```

Return an ellipse centered at a point center = (x,y) with radii = r1,r2 and angle angle. Type `ellipse.options` to see all options.

**INPUT:**

- `center` - 2-tuple of real numbers - coordinates of the center
- `r1, r2` - positive real numbers - the radii of the ellipse
- `angle` - real number (default: 0) - the angle between the first axis and the horizontal

**OPTIONS:**

- `alpha` - default: 1 - transparency
- `fill` - default: False - whether to fill the ellipse or not
- `thickness` - default: 1 - thickness of the line
- `linestyle` - default: 'solid' - The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '---', ':', '-', ' ', respectively.
- `edgecolor` - default: 'black' - color of the contour
- `facecolor` - default: 'red' - color of the filling
- `rgbcolor` - 2D or 3D plotting. This option overrides `edgecolor` and `facecolor` for 2D plotting.
- `legend_label` - the label for this item in the legend
- `legend_color` - the color for the legend label

**EXAMPLES:**

An ellipse centered at (0,0) with major and minor axes of lengths 2 and 1. Note that the default color is blue:

```python
sage: ellipse((0,0),2,1)
```

Graphics object consisting of 1 graphics primitive

More complicated examples with tilted axes and drawing options:
other way to indicate dashed linestyle:

```python
sage: ellipse((0,0),3,1,pi/6,fill=True,alpha=0.3,linestyle="dashed")
```

Graphics object consisting of 1 graphics primitive

with colors

```python
sage: ellipse((0,0),3,1,pi/6,fill=True,edgecolor='black',facecolor='red')
```

Graphics object consisting of 1 graphics primitive

We see that rgbcolor overrides these other options, as this plot is green:

```python
sage: ellipse((0,0),3,1,pi/6,fill=True,edgecolor='black',facecolor='red',rgbcolor=\rightarrow 'green')
```

Graphics object consisting of 1 graphics primitive

The default aspect ratio for ellipses is 1.0:

```python
sage: ellipse((0,0),2,1).aspect_ratio()
1.0
```
One cannot yet plot ellipses in 3D:

```python
sage: ellipse((0,0,0),2,1)
Traceback (most recent call last):
...  
NotImplementedError: plotting ellipse in 3D is not implemented
```

We can also give ellipses a legend:

```python
sage: ellipse((0,0),2,1,legend_label="My ellipse", legend_color='green')
Graphics object consisting of 1 graphics primitive
```

4.7 Line Plots

```python
class sage.plot.line.Line(xdata, ydata, options)
    Bases: sage.plot.primitive.GraphicPrimitive_xydata

    Primitive class that initializes the line graphics type.

    EXAMPLES:
```
```python
sage: from sage.plot.line import Line
sage: Line([1,2,7], [1,5,-1], {})  
Line defined by 3 points
```

```python
plot3d(z=0, **kwds)
Plots a 2D line in 3D, with default height zero.
EXAMPLES:

```python
sage: E = EllipticCurve('37a').plot(thickness=5).plot3d()
sage: F = EllipticCurve('37a').plot(thickness=5).plot3d(z=2)
sage: E + F  # long time (5s on sage.math, 2012)
Graphics3d Object
```

```python
sage.plot.line.line(points, **kwds)
Returns either a 2-dimensional or 3-dimensional line depending on value of points.

INPUT:
  * points - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either line2d? or line3d?.

EXAMPLES:

```python
sage: line([(0,0), (1,1)])
Graphics object consisting of 1 graphics primitive
```

```python
sage: line([(0,0,1), (1,1,1)])
Graphics3d Object
```

```python
sage.plot.line.line2d(points, alpha=1, rgbcolor=(0, 0, 1), thickness=1, legend_label=None,
legend_color=None, aspect_ratio='automatic', **options)
Create the line through the given list of points.

INPUT:
  * points - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

Type line2d.options for a dictionary of the default options for lines. You can change this to change the defaults for all future lines. Use line2d.reset() to reset to the default options.

INPUT:
  * alpha – How transparent the line is
  * thickness – How thick the line is
  * rgbcolor – The color as an RGB tuple
  * hue – The color given as a hue
  * legend_color – The color of the text in the legend
  * legend_label – the label for this item in the legend

Any MATPLOTLIB line option may also be passed in. E.g.,
  * linestyle - (default: “-”) The style of the line, which is one of
```

- "-" or "solid"
- "--" or "dashed"
- ".-" or "dash dot"
- ":" or "dotted"
- "None" or " " or "" (nothing)

The linestyle can also be prefixed with a drawing style (e.g., "steps--")

- "default" (connect the points with straight lines)
- "steps" or "steps-pre" (step function; horizontal line is to the left of point)
- "steps-mid" (step function; points are in the middle of horizontal lines)
- "steps-post" (step function; horizontal line is to the right of point)

• **marker** - The style of the markers, which is one of
  - "None" or " " or "" (nothing) – default
  - "," (pixel), "." (point)
  - "," (horizontal line), "|" (vertical line)
  - "o" (circle), "p" (pentagon), "s" (square), "x" (x), "+" (plus), "*" (star)
  - "D" (diamond), "d" (thin diamond)
  - "H" (hexagon), "h" (alternative hexagon)
  - ";" (triangle left), ";" (triangle right), "^" (triangle up), "v" (triangle down)
  - "1" (tri down), "2" (tri up), "3" (tri left), "4" (tri right)
  - ø (tick left), 1 (tick right), 2 (tick up), 3 (tick down)
  - 4 (caret left), 5 (caret right), 6 (caret up), 7 (caret down)
  - "$...$" (math TeX string)

• **markersize** – the size of the marker in points

• **markeredgecolor** – the color of the marker edge

• **markerfacecolor** – the color of the marker face

• **markeredgewidth** – the size of the marker edge in points

**EXAMPLES:**

A line with no points or one point:

```
sage: line([])       #returns an empty plot
Graphics object consisting of 0 graphics primitives
```

```
sage: import numpy; line(numpy.array([]))
Graphics object consisting of 0 graphics primitives
```

```
sage: line([[1,1]])
Graphics object consisting of 1 graphics primitive
```

A line with numpy arrays:

```
sage: line(numpy.array([[1,2], [3,4]]))
Graphics object consisting of 1 graphics primitive
```
A line with a legend:

```python
sage: line([(0,0),(1,1)], legend_label='line')
```

Graphics object consisting of 1 graphics primitive

Lines with different colors in the legend text:

```python
sage: p1 = line([(0,0),(1,1)], legend_label='line')
sage: p2 = line([(1,1),(2,4)], legend_label='squared', legend_color='red')
sage: p1 + p2
```

Graphics object consisting of 2 graphics primitives

Extra options will get passed on to show(), as long as they are valid:

```python
sage: line([(0,1), (3,4)], figsize=[10, 2])
```

Graphics object consisting of 1 graphics primitive

```python
sage: line([(0,1), (3,4)]).show(figsize=[10, 2])  # These are equivalent
```

We can also use a logarithmic scale if the data will support it:

```python
sage: line([(1,2),(2,4),(3,4),(4,8),(4.5,32)],scale='loglog',base=2)
```

Graphics object consisting of 1 graphics primitive

Many more examples below!

A blue conchoid of Nicomedes:

```python
sage: L = [[1+5*cos(pi/2+pi*i/100), tan(pi/2+pi*i/100)*(1+5*cos(pi/2+pi*i/100))] for i in range(1,100)]
sage: line(L, rgbcolor=(1/4,1/8,3/4))
```

Graphics object consisting of 1 graphics primitive

A line with 2 complex points:

```python
sage: i = CC.0
sage: line([1+i, 2+3*i])
```

Graphics object consisting of 1 graphics primitive

A blue hypotrochoid (3 leaves):

```python
sage: n = 4; h = 3; b = 2
sage: L = [[n*cos(pi*i/100)+h*cos((n/b)*pi*i/100),n*sin(pi*i/100)-h*sin((n/b)*pi*i/100)] for i in range(200)]
sage: line(L, rgbcolor=(1/4,1/4,3/4))
```

Graphics object consisting of 1 graphics primitive

A blue hypotrochoid (4 leaves):

```python
sage: n = 6; h = 5; b = 2
sage: L = [[n*cos(pi*i/100)+h*cos((n/b)*pi*i/100),n*sin(pi*i/100)-h*sin((n/b)*pi*i/100)] for i in range(200)]
sage: line(L, rgbcolor=(1/4,1/4,3/4))
```

Graphics object consisting of 1 graphics primitive

A red limacon of Pascal:
A light green trisectrix of Maclaurin:

```
sage: L = [(2*(1-4*cos(-pi/2+pi*i/100)^2), 10*tan(-pi/2+pi*i/100)*(1-4*cos(-pi/2+pi*i/100)^2)) for i in range(1,100)]
sage: line(L, rgbcolor=(1/4,1,1/8))
```

Graphics object consisting of 1 graphics primitive

A green lemniscate of Bernoulli:

```
sage: cosines = [cos(-pi/2+pi*i/100) for i in range(201)]
sage: v = [(1/c, tan(-pi/2+pi*i/100)) for i,c in enumerate(cosines) if c != 0]
sage: L = [(a/(a^2+b^2), b/(a^2+b^2)) for a,b in v]
sage: line(L, rgbcolor=(1/4,3/4,1/8))
```

Graphics object consisting of 1 graphics primitive

A red plot of the Jacobi elliptic function \( sn(x,2) \), \(-3 < x < 3 \):

```
sage: L = [(i/100.0, real_part(jacobi('sn', i/100.0, 2.0))) for i in range(-300, 300, 30)]
sage: line(L, rgbcolor=(3/4, 1/4, 1/8))
```

Graphics object consisting of 1 graphics primitive

A red plot of \( J_2(x) \), \(0 < x < 10\):

```
sage: L = [(i/10.0, bessel_J(2,i/10.0)) for i in range(100)]
sage: line(L, rgbcolor=(3/4,1/4,5/8))
```

Graphics object consisting of 1 graphics primitive

A purple plot of the Riemann zeta function \( \zeta(1/2 + it) \), \(0 < t < 30\):

```
sage: i = CDF.gen()
sage: v = [zeta(0.5 + n/10 * i) for n in range(300)]
sage: L = [(z[1].real(), z[1].imag()) for z in v]
sage: line(L, rgbcolor=(3/4,1/2,5/8))
```

Graphics object consisting of 1 graphics primitive

A purple plot of the Hasse-Weil \( L \)-function \( L(E,1+it) \), \(-1 < t < 10\):

```
sage: E = EllipticCurve('37a')
sage: vals = E.lseries().values_along_line(1-I, 1+10*I, 100) # critical line
sage: L = [(z[1].real(), z[1].imag()) for z in vals]
sage: line(L, rgbcolor=(3/4,1/2,5/8))
```

Graphics object consisting of 1 graphics primitive

A red, blue, and green “cool cat”:

```
sage: G = plot(-cos(x), -2, 2, thickness=5, rgbcolor=(0.5,1,0.5))
sage: P = polygon([[1,2], [5,6], [5,0]], rgbcolor=(1,0,0))
sage: Q = polygon([-x,y] for x,y in P[0]], rgbcolor=(0,0,1))
```

(continues on next page)
4.8 Points

class sage.plot.point.Point(xdata, ydata, options)
Bases: sage.plot.primitive.GraphicPrimitive_xydata

Primitive class for the point graphics type. See point?, point2d? or point3d? for information about actually plotting points.

INPUT:
- xdata – list of x values for points in Point object
- ydata – list of y values for points in Point object
- options – dict of valid plot options to pass to constructor

EXAMPLES:
Note this should normally be used indirectly via point and friends:

```sage
def from sage.plot.point import Point
sage: P = Point([(1,2),[2,3],{'alpha':.5})
sage: P
Point set defined by 2 point(s)
sage: P.options()['alpha']
0.500000000000000
sage: P.xdata
[1, 2]
```

plot3d(z=0, **kwds)

Plots a two-dimensional point in 3-D, with default height zero.

INPUT:
- z - optional 3D height above \(xy\)-plane. May be a list if self is a list of points.

EXAMPLES:
One point:

```sage:A=point((1,1))
sage: a=A[0];a
Point set defined by 1 point(s)
sage: b=a.plot3d()
```

One point with a height:

```sage:A=point((1,1))
sage: a=A[0];a
Point set defined by 1 point(s)
sage: b=a.plot3d(z=3)
sage: b.loc[2]
3.0
```
Multiple points:

```
sage: P=point([(0,0), (1,1)])
sage: p=P[0]; p
Point set defined by 2 point(s)
sage: q=p.plot3d(size=22)
```

Multiple points with different heights:

```
sage: P=point([(0,0), (1,1)])
sage: p=P[0]
sage: q=p.plot3d(z=[2,3])
sage: q.all[0].loc[2]
2.0
sage: q.all[1].loc[2]
3.0
```

Note that keywords passed must be valid point3d options:

```
sage: A=point((1,1),size=22)
sage: a=A[0];a
Point set defined by 1 point(s)
sage: b=a.plot3d()
sage: b.size
22
sage: b=a.plot3d(pointsize=23) # only 2D valid option
sage: b.size
22
sage: b=a.plot3d(size=23) # correct keyword
sage: b.size
23
```

`sage.plot.point.point(points, **kwds)`
Return either a 2-dimensional or 3-dimensional point or sum of points.

**INPUT:**

- `points` - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either `point2d?` or `point3d?`.

**See also:**
`sage.plot.point.point2d(), sage.plot.plot3d.shapes2.point3d()`

**EXAMPLES:**

```
sage: point((1,2))
Graphics object consisting of 1 graphics primitive
```

```
sage: point((1,2,3))
Graphics3d Object
```

```
sage: point([(0,0), (1,1)])
Graphics object consisting of 1 graphics primitive
```
**sage**:
```
point([[0,0,1], (1,1,1)])
Graphics3d Object
```

Extra options will get passed on to `show()`, as long as they are valid:

```
sage: point([[cos(theta), sin(theta)] for theta in srange(0, 2*pi, pi/8)], frame=True)
Graphics object consisting of 1 graphics primitive
sage: point([[cos(theta), sin(theta)] for theta in srange(0, 2*pi, pi/8)]).show(frame=True)  # These are equivalent
```

```
sage.plot.point.point2d(points, alpha=1, aspect_ratio='automatic', faceted=False, legend_color=None, legend_label=None, marker='o', markeredgecolor=None, rgbcolor=(0, 0, 1), size=10, **options)
```

A point of size `size` defined by `point = (x, y)`.

**INPUT:**
- `points` – either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers
- `alpha` – how transparent the point is
- `faceted` – if `True`, color the edge of the point (only for 2D plots)
- `hue` – the color given as a hue
- `legend_color` – the color of the legend text
- `legend_label` – the label for this item in the legend
- `marker` – the marker symbol for 2D plots only (see documentation of `plot()` for details)
- `markeredgecolor` – the color of the marker edge (only for 2D plots)
- `rgbcolor` – the color as an RGB tuple
- `size` – how big the point is (i.e., area in points\(^2\)=(1/72 \text{ inch})^2\)
- `zorder` – the layer level in which to draw

**EXAMPLES:**

A purple point from a single tuple of coordinates:

```
sage: point((0.5, 0.5), rgbcolor=hue(0.75))
Graphics object consisting of 1 graphics primitive
```

Points with customized markers and edge colors:

```
sage: r = [(random(), random()) for _ in range(10)]
sage: point(r, marker='d', markeredgecolor='red', size=20)
Graphics object consisting of 1 graphics primitive
```

Passing an empty list returns an empty plot:

```
sage: point([])
Graphics object consisting of 0 graphics primitives
```

```
sage: import numpy; point(numpy.array([]))
Graphics object consisting of 0 graphics primitives
```
If you need a 2D point to live in 3-space later, this is possible:

```
sage: A = point((1, 1))
sage: a = A[0]; a
Point set defined by 1 point(s)
sage: b = a.plot3d(z=3)
```

This is also true with multiple points:

```
sage: P = point([(0, 0), (1, 1)])
sage: p = P[0]
sage: q = p.plot3d(z=[2,3])
```

Here are some random larger red points, given as a list of tuples:

```
sage: point(((0.5, 0.5), (1, 2), (0.5, 0.9), (-1, -1)), rgbcolor=hue(1), size=30)
Graphics object consisting of 1 graphics primitive
```

And an example with a legend:

```
sage: point((0, 0), rgbcolor='black', pointsize=40, legend_label='origin')
Graphics object consisting of 1 graphics primitive
```

The legend can be colored:

```
sage: P = points([(0, 0), (1, 0)], pointsize=40, legend_label='origin', legend_color='red')
sage: P + plot(x^2, (x, 0, 1), legend_label='plot', legend_color='green')
```

Extra options will get passed on to show(), as long as they are valid:

```
sage: point([(cos(theta), sin(theta)) for theta in srange(0, 2*pi, pi/8)], frame=True)
Graphics object consisting of 1 graphics primitive
```

For plotting data, we can use a logarithmic scale, as long as we are sure not to include any nonpositive points in the logarithmic direction:

```
sage: point([(1, 2),(2, 4),(3, 4),(4, 8),(4.5, 32)], scale='semilogy', base=2)
```

Since Sage Version 4.4 (trac ticket #8599), the size of a 2d point can be given by the argument size instead of pointsize. The argument pointsize is still supported:

```
sage: point((3, 4), size=100)
Graphics object consisting of 1 graphics primitive
```

We can plot a single complex number:
We can also plot a list of complex numbers:

```
sage: point([1 + I, 1 + I, 2 + 2*I], pointsize=100)
Graphics object consisting of 1 graphics primitive
```

`sage.plot.point.points(points, **kwds)`

Return either a 2-dimensional or 3-dimensional point or sum of points.

**INPUT:**

- `points` - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either `point2d?` or `point3d?`.

**See also:**

`sage.plot.point.point2d()`, `sage.plot.plot3d.shapes2.point3d()`

**EXAMPLES:**

```
sage: point((1,2))
Graphics object consisting of 1 graphics primitive
```

```
sage: point((1,2,3))
Graphics3d Object
```

```
sage: point([(0,0), (1,1)])
Graphics object consisting of 1 graphics primitive
```

```
sage: point([(0,0,1), (1,1,1)])
Graphics3d Object
```

Extra options will get passed on to `show()`, as long as they are valid:

```
sage: point([(cos(theta), sin(theta)) for theta in srange(0, 2*pi, pi/8)], frame=True)
Graphics object consisting of 1 graphics primitive
```

```
sage: point([(cos(theta), sin(theta)) for theta in srange(0, 2*pi, pi/8)]).show(frame=True) # These are equivalent
```
4.9 Polygons

```python
class sage.plot.polygon.Polygon(xdata, ydata, options)
  Bases: sage.plot.primitive.GraphicPrimitive_xydata

  Primitive class for the Polygon graphics type. For information on actual plotting, please see polygon(), polygon2d(), or polygon3d().

  INPUT:
  • xdata – list of x-coordinates of points defining Polygon
  • ydata – list of y-coordinates of points defining Polygon
  • options – dict of valid plot options to pass to constructor

  EXAMPLES:
  Note this should normally be used indirectly via polygon():

  sage: from sage.plot.polygon import Polygon
  sage: P = Polygon([1,2,3],[2,3,2],{'alpha':.5})
  sage: P
  Polygon defined by 3 points
  sage: P.options()['alpha']
  0.500000000000000
  sage: P.ydata
  [2, 3, 2]

  plot3d(z=0, **kwds)
  Plots a 2D polygon in 3D, with default height zero.

  INPUT:
  • z - optional 3D height above x-y-plane, or a list of heights corresponding to the list of 2D polygon points.

  EXAMPLES:
  A pentagon:

  sage: polygon(((cos(t), sin(t)) for t in srange(0, 2*pi, 2*pi/5))).plot3d()
  Graphics3d Object

  Showing behavior of the optional parameter z:

  sage: P = polygon(((0,0), (1,2), (0,1), (-1,2)))
  sage: p = P[0]; p
  Polygon defined by 4 points
  sage: q = p.plot3d()
  sage: q.obj_repr(q.testing_render_params())[2]
  ['v 0 0 0', 'v 1 2 0', 'v 0 1 0', 'v -1 2 0']
  sage: r = p.plot3d(z=3)
  sage: r.obj_repr(r.testing_render_params())[2]
  ['v 0 0 3', 'v 1 2 3', 'v 0 1 3', 'v -1 2 3']
  sage: s = p.plot3d(z=[0,1,2,3])
  sage: s.obj_repr(s.testing_render_params())[2]
  ['v 0 0 0', 'v 1 2 1', 'v 0 1 2', 'v -1 2 3']
```

4.9. Polygons
sage.plot.polygon.polygon(points, **options)
Return either a 2-dimensional or 3-dimensional polygon depending on value of points.

For information regarding additional arguments, see either polygon2d() or polygon3d(). Options may be
found and set using the dictionaries polygon2d.options and polygon3d.options.

EXAMPLES:

sage: polygon([(0,0), (1,1), (0,1)])
Graphics object consisting of 1 graphics primitive

sage: polygon([(0,0,1), (1,1,1), (2,0,1)])
Graphics3d Object

Extra options will get passed on to show(), as long as they are valid:

sage: polygon([(0,0), (1,1), (0,1)], axes=False)
Graphics object consisting of 1 graphics primitive
sage: polygon([(0,0), (1,1), (0,1)]).show(axes=False) # These are equivalent

sage.plot.polygon.polygon2d(points, alpha=1, rgbcolor=(0, 0, 1), edgecolor=None, thickness=None, legend_label=None, legend_color=None, aspect_ratio=1.0, fill=True, **options)
Return a 2-dimensional polygon defined by points.
Type `polygon2d.options` for a dictionary of the default options for polygons. You can change this to change the defaults for all future polygons. Use `polygon2d.reset()` to reset to the default options.

**EXAMPLES:**

We create a purple-ish polygon:

```
sage: polygon2d([[1,2], [5,6], [5,0]], rgbcolor=(1,0,1))
```

By default, polygons are filled in, but we can make them without a fill as well:

```
sage: polygon2d([[1,2], [5,6], [5,0]], fill=False)
```

In either case, the thickness of the border can be controlled:

```
sage: polygon2d([[1,2], [5,6], [5,0]], fill=False, thickness=4, color='orange')
```

For filled polygons, one can use different colors for the border and the interior as follows:

```
sage: L = [[0,0]]+[[i/100, 1.1+cos(i/20)] for i in range(100)]+[[1,0]]
sage: polygon2d(L, color="limegreen", edgecolor="black", axes=False)
```

By default, polygons are filled in, but we can make them without a fill as well:

```
sage: polygon2d([[1,2], [5,6], [5,0]], fill=False)
```
4.9. Polygons
Some modern art – a random polygon, with legend:

```sage
v = [(randrange(-5,5), randrange(-5,5)) for _ in range(10)]
sage: polygon2d(v, legend_label='some form')
```

A purple hexagon:

```sage
L = [[cos(pi*i/3),sin(pi*i/3)] for i in range(6)]
sage: polygon2d(L, rgbcolor=(1,0,1))
```

A green deltoid:

```sage
L = [-1+cos(pi*i/100)*(1+cos(pi*i/100)),2*sin(pi*i/100)*(1-cos(pi*i/100))] for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,3/4,1/2))
```

A blue hypotrochoid:

```sage
L = [6*cos(pi*i/100)+5*cos((6/2)*pi*i/100),6*sin(pi*i/100)-5*sin((6/2)*pi*i/100)) for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,1/4,1/2))
```
4.9. Polygons

4.9.1. Polygons in 2D Graphics
Another one:

```sage	n = 4; h = 5; b = 2
sage: L = [(n*cos(pi*i/100)+h*cos((n/b)*pi*i/100),n*sin(pi*i/100)-h*sin((n/b)*pi*i/100)) for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,1/4,3/4))
```

A purple epicycloid:

```sage
m = 9; b = 1
sage: L = [(m*cos(pi*i/100)+b*cos((m/b)*pi*i/100),m*sin(pi*i/100)-b*sin((m/b)*pi*i/100)) for i in range(200)]
sage: polygon2d(L, rgbcolor=(7/8,1/4,3/4))
```

A brown astroid:

```sage:L = [(cos(pi*i/100)^3,sin(pi*i/100)^3) for i in range(200)]sage: polygon2d(L, rgbcolor=(3/4,1/4,1/4))```

4.9. Polygons
4.9. Polygons
And, my favorite, a greenish blob:

```
sage: L = [[cos(pi*i/100)*(1+cos(pi*i/50)), sin(pi*i/100)*(1+sin(pi*i/50))] for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,3/4,1/2))
```

This one is for my wife:

```
sage: L = [[sin(pi*i/100)+sin(pi*i/50),-(1+cos(pi*i/100)+cos(pi*i/50))] for i in range(-100,100)]
sage: polygon2d(L, rgbcolor=(1,1/4,1/2))
```

One can do the same one with a colored legend label:

```
sage: polygon2d(L, color='red', legend_label='For you!', legend_color='red')
```

Polygons have a default aspect ratio of 1.0:

```
sage: polygon2d([[1,2], [5,6], [5,0]]).aspect_ratio()
```

AUTHORS:

4.9. Polygons
• David Joyner (2006-04-14): the long list of examples above.

4.10 Arcs in hyperbolic geometry

AUTHORS:
• Hartmut Monien (2011 - 08)

class sage.plot.hyperbolic_arc.HyperbolicArc(A, B, options)
Bases: sage.plot.bezier_path.BezierPath

Primitive class for hyperbolic arc type. See hyperbolic_arc? for information about plotting a hyperbolic arc in the complex plane.

INPUT:
• a, b - coordinates of the hyperbolic arc in the complex plane
• options - dict of valid plot options to pass to constructor

EXAMPLES:
Note that constructions should use hyperbolic_arc:

```
sage: from sage.plot.hyperbolic_arc import HyperbolicArc
sage: print(HyperbolicArc(0, 1/2+I*sqrt(3)/2, {}))
Hyperbolic arc (0.000000000000000, 0.500000000000000 + 0.866025403784439*I)
```

sage.plot.hyperbolic_arc.hyperbolic_arc(a, b, alpha=1, fill=False, thickness=1, rgbcolor='blue', zorder=2, linestyle='solid', **options)

Plot an arc from a to b in hyperbolic geometry in the complex upper half plane.

INPUT:
• a, b - complex numbers in the upper half complex plane connected by the arc

OPTIONS:
• alpha - default: 1
• thickness - default: 1
• rgbcolor - default: ‘blue’
• linestyle - (default: 'solid') The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or ':', '-', '.', respectively.

Examples:
Show a hyperbolic arc from 0 to 1:

```
sage: hyperbolic_arc(0, 1)
Graphics object consisting of 1 graphics primitive
```

Show a hyperbolic arc from 1/2 to i with a red thick line:

```
sage: hyperbolic_arc(1/2, I, color='red', thickness=2)
Graphics object consisting of 1 graphics primitive
```

Show a hyperbolic arc from i to 2i with dashed line:
4.11 Polygons and triangles in hyperbolic geometry

AUTHORS:

• Hartmut Monien (2011-08)
• Vincent Delecroix (2014-11)

class sage.plot.hyperbolic_polygon.HyperbolicPolygon(pts, options)

Bases: sage.plot.bezier_path.BezierPath

Primitive class for hyperbolic polygon type.

See hyperbolic_polygon? for information about plotting a hyperbolic polygon in the complex plane.

INPUT:

• pts – coordinates of the polygon (as complex numbers)
• options – dict of valid plot options to pass to constructor

EXAMPLES:

Note that constructions should use hyperbolic_polygon() or hyperbolic_triangle():

```
sage: from sage.plot.hyperbolic_polygon import HyperbolicPolygon
g sage: print(HyperbolicPolygon([0, 1/2, I], {}}))
Hyperbolic polygon (0.000000000000000, 0.500000000000000, 1.000000000000000*I)
```

sage.plot.hyperbolic_polygon.hyperbolic_polygon(pts, alpha=1, fill=False, thickness=1, rgbcolor='blue', zorder=2, linestyle='solid', **options)

Return a hyperbolic polygon in the hyperbolic plane with vertices pts.

Type ?hyperbolic_polygon to see all options.

INPUT:

• pts – a list or tuple of complex numbers

OPTIONS:

• alpha – default: 1
• fill – default: False
• thickness – default: 1
• rgbcolor – default: 'blue'
• linestyle – (default: 'solid') The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or ' - ', ':', '-', '.', respectively.

EXAMPLES:

Show a hyperbolic polygon with coordinates $-1, 3i, 2 + 2i, 1 + i$: 
sage: hyperbolic_polygon([-1,3*I,2+2*I,1+I])
Graphics object consisting of 1 graphics primitive

With more options:

sage: hyperbolic_polygon([-1,3*I,2+2*I,1+I], fill=True, color='red')
Graphics object consisting of 1 graphics primitive

sage.plot.hyperbolic_polygon.hyperbolic_triangle(a, b, c, **options)
Return a hyperbolic triangle in the hyperbolic plane with vertices (a, b, c).
Type ?hyperbolic_polygon to see all options.

INPUT:
• a, b, c – complex numbers in the upper half complex plane

OPTIONS:
• alpha – default: 1
• fill – default: False
• thickness – default: 1
• rgbcolor – default: 'blue'
4.11. Polygons and triangles in hyperbolic geometry

```python
# Code for plotting a hyperbolic polygon

# Plotting the hyperbolic polygon
```

403
• linestyle - (default: 'solid') The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '---', ':', '-', '--', respectively.

EXAMPLES:

Show a hyperbolic triangle with coordinates $0, 1/2 + \frac{i}{2} + \sqrt{3}/2$ and $-1/2 + \frac{i}{2} + \sqrt{3}/2$:

```sage
sage: hyperbolic_triangle(0, -1/2+I*sqrt(3)/2, 1/2+I*sqrt(3)/2)
Graphics object consisting of 1 graphics primitive
```

A hyperbolic triangle with coordinates $0, 1, 2 + i$ and a dashed line:

```sage
sage: hyperbolic_triangle(0, 1, 2+i, fill=true, rgbcolor='red', linestyle='--')
Graphics object consisting of 1 graphics primitive
```
4.11. Polygons and triangles in hyperbolic geometry
4.12 Regular polygons in the upper half model for hyperbolic plane

AUTHORS:
- Javier Honrubia (2016-01)

class sage.plot.hyperbolic_regular_polygon.HyperbolicRegularPolygon(sides, i_angle, center, options)

Bases: sage.plot.hyperbolic_polygon.HyperbolicPolygon

Primitive class for regular hyperbolic polygon type.

See hyperbolic_regular_polygon? for information about plotting a hyperbolic regular polygon in the upper complex halfplane.

INPUT:
- sides – number of sides of the polygon
- i_angle – interior angle of the polygon
- center – center point as a complex number of the polygon

EXAMPLES:

Note that constructions should use hyperbolic_regular_polygon():

```
sage: from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
sage: print(HyperbolicRegularPolygon(5, pi/2, I, {}))
Hyperbolic regular polygon (sides=5, i_angle=1/2*pi, center=1.00000000000000*I)
```

The code verifies there exists a compact hyperbolic regular polygon with the given data, checking

\[ A(P) = \pi(s - 2) - s \cdot \alpha > 0, \]

where \( s \) is sides and \( \alpha \) is i_angle’. This raises an error if the ‘i_angle is less than the minimum to generate a compact polygon:

```
sage: from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
sage: P = HyperbolicRegularPolygon(4, pi/2, I, {})  
Traceback (most recent call last):
  ... 
ValueError: there exists no hyperbolic regular compact polygon, 
for sides=4 the interior angle must be less than 1/2*pi
```

It is an error to give a center outside the upper half plane in this model

```
sage: from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
sage: P = HyperbolicRegularPolygon(4, pi/4, 1-I, {})  
Traceback (most recent call last):
  ... 
ValueError: center: 1.00000000000000 - 1.00000000000000*I is not 
a valid point in the upper half plane model of the hyperbolic plane
```

sage.plot.hyperbolic_regular_polygon.HyperbolicRegularPolygon(sides, i_angle, 
center=1.00000000000000 * I, 
alpha=1, fill=False, thickness=1, 
rgbcolor='blue', zorder=2, 
linestyle='solid', **options)

Return a hyperbolic regular polygon in the upper half model of Hyperbolic plane given the number of sides,
interior angle and possibly a center.

Type \texttt{?hyperbolic\_regular\_polygon} to see all options.

**INPUT:**
- \texttt{sides} – number of sides of the polygon
- \texttt{i\_angle} – interior angle of the polygon
- \texttt{center} – (default: \texttt{i}) hyperbolic center point (complex number) of the polygon

**OPTIONS:**
- \texttt{alpha} – default: 1
- \texttt{fill} – default: \texttt{False}
- \texttt{thickness} – default: 1
- \texttt{rgbcolor} – default: \texttt{'blue'}
- \texttt{linestyle} – (default: \texttt{'solid'}) the style of the line, which can be one of the following:
  - \texttt{'dashed'} or \texttt{'--'}
  - \texttt{'dotted'} or \texttt{:}
  - \texttt{'solid'} or \texttt{-}
  - \texttt{'dashdot'} or \texttt{-.}'

**EXAMPLES:**

Show a hyperbolic regular polygon with 6 sides and square angles:

```python
sage: g = hyperbolic_regular_polygon(6, pi/2)
sage: g.plot()
Graphics object consisting of 1 graphics primitive
```

With more options:

```python
sage: g = hyperbolic_regular_polygon(6, pi/2, center=3+2*I, fill=True, color='red')
sage: g.plot()
Graphics object consisting of 1 graphics primitive
```

The code verifies if there exists a hyperbolic regular polygon with the given data, checking

\[ A(P) = \pi(s - 2) - s \cdot \alpha > 0, \]

where \( s \) is \texttt{sides} and \( \alpha \) is \texttt{i\_angle}'. This raises an error if the \texttt{``i\_angle'} is less than the minimum to generate a compact polygon:

```python
sage: hyperbolic_regular_polygon(4, pi/2)
Traceback (most recent call last):
...
ValueError: there exists no hyperbolic regular compact polygon, for sides=4 the interior angle must be less than 1/2*pi
```

It is an error to give a center outside the upper half plane in this model:
4.12. Regular polygons in the upper half model for hyperbolic plane
| sage: from sage.plot.hyperbolic_regular_polygon import hyperbolic_regular_polygon |
| sage: hyperbolic_regular_polygon(4, pi/4, 1-I) |
| Traceback (most recent call last): |
| ... |
| ValueError: center: 1.00000000000000 - 1.00000000000000*I is not |
| a valid point in the upper half plane model of the hyperbolic plane |
5.1 Graphics objects

This file contains the definition of the class Graphics. Usually, you don’t call the constructor of this class directly (although you can do it), you would use plot() instead.

AUTHORS:

- Jeroen Demeyer (2012-04-19): split off this file from plot.py (trac ticket #12857)
- Punarbasu Purkayastha (2012-05-20): Add logarithmic scale (trac ticket #4529)
- Emily Chen (2013-01-05): Add documentation for show() figsize parameter (trac ticket #5956)
- Eric Gourgoulhon (2015-03-19): Add parameter axes_labels_size (trac ticket #18004)

class sage.plot.graphics.Graphics

    Bases: sage.misc.fast_methods.WithEqualityById, sage.structure.sage_object.SageObject

    The Graphics object is an empty list of graphics objects. It is useful to use this object when initializing a for loop where different graphics object will be added to the empty object.

EXAMPLES:

```
sage: G = Graphics(); print(G)
Graphics object consisting of 0 graphics primitives
sage: c = circle((1,1), 1)
sage: G+=c; print(G)
Graphics object consisting of 1 graphics primitive
```

Here we make a graphic of embedded isosceles triangles, coloring each one with a different color as we go:

```
sage: h=10; c=0.4; p=0.5
sage: G = Graphics()
sage: for x in srange(1,h+1):
    ....:     l = [[0,x*sqrt(3)],[x/2,-x*sqrt(3)/2],[x/2,-x*sqrt(3)/2],[0,x*sqrt(3)]]
    ....:     G+=line(l,color=hue(c + p*(x/h)))
sage: G.show(figsize=[5,5])
```

We can change the scale of the axes in the graphics before displaying:

```
sage: G = plot(exp, 1, 10) # long time
sage: G.show(scale='semilogy') # long time
```
_rich_repr_(display_manager, **kwds)
Rich Output Magic Method

See sage.repl.rich_output for details.

EXAMПLES:

```
sage: from sage.repl.rich_output import get_display_manager
sage: dm = get_display_manager()
```
```
sage: g = Graphics()
sage: g._rich_repr_(dm)
```
```
OutputImagePng container
```

add_primitive(primitive)
Adds a primitive to this graphics object.

EXAMПLES:

We give a very explicit example:

```
sage: G = Graphics()
sage: from sage.plot.line import Line
sage: from sage.plot.arrow import Arrow
sage: L = Line([3,4,2,7,-2],[1,2,e,4,5.],{'alpha':1,'thickness':2,'rgbcolor':(0, ←1,1),'legend_label':''})
sage: A = Arrow(2,-5,.1,.2,{′width':3,'head':0,'rgbcolor':(1,0,0),′linestyle':←′dashed′,'zorder':8,'legend_label':''})
sage: G.add Primitive(L)
sage: G.add Primitive(A)
sage: G
```
```
Graphics object consisting of 2 graphics primitives
```

aspect_ratio()
Get the current aspect ratio, which is the ratio of height to width of a unit square, or ‘automatic’.

OUTPUT: a positive float (height/width of a unit square), or ‘automatic’ (expand to fill the figure).

EXAMPLES:

The default aspect ratio for a new blank Graphics object is ‘automatic’:

```
sage: P = Graphics()
sage: P.aspect_ratio()
'automatic'
```

The aspect ratio can be explicitly set different than the object’s default:

```
sage: P = circle((1,1), 1)
sage: P.aspect_ratio()
1.0
```
```
sage: P.set_aspect_ratio(2)
sage: P.aspect_ratio()
2.0
```
```
sage: P.set_aspect_ratio('automatic')
sage: P.aspect_ratio()
'automatic'
```
axes\( (\text{show}=\text{None}) \)
Set whether or not the \( x \) and \( y \) axes are shown by default.

INPUT:

- \text{show} - bool

If called with no input, return the current axes setting.

EXAMPLES:

```
sage: L = line([[1,2], [3,-4], [2, 5], [1,2]])
```

By default the axes are displayed.

```
sage: L.axes()
True
```

But we turn them off, and verify that they are off.

```
sage: L.axes(False)
sage: L.axes()
False
```

Displaying \( L \) now shows a triangle but no axes.

```
sage: L
Graphics object consisting of 1 graphics primitive
```

**axes_color\( (c=\text{None}) \)**
Set the axes color.

If called with no input, return the current axes_color setting.

INPUT:

- \text{c} - an RGB color 3-tuple, where each tuple entry is a float between 0 and 1

EXAMPLES: We create a line, which has like everything a default axes color of black.

```
sage: L = line([[1,2], [3,-4], [2, 5], [1,2]])
sage: L.axes_color()
(0, 0, 0)
```

We change the axes color to red and verify the change.

```
sage: L.axes_color((1,0,0))
sage: L.axes_color()
(1.0, 0.0, 0.0)
```

When we display the plot, we’ll see a blue triangle and bright red axes.

```
sage: L
Graphics object consisting of 1 graphics primitive
```

**axes_label_color\( (c=\text{None}) \)**
Set the color of the axes labels.

The axes labels are placed at the edge of the \( x \) and \( y \) axes, and are not on by default (use the axes_labels command to set them; see the example below). This function just changes their color.
INPUT:
- $c$ - an RGB 3-tuple of numbers between 0 and 1

If called with no input, return the current axes_label_color setting.

EXAMPLES: We create a plot, which by default has axes label color black.

```python
sage: p = plot(sin, (-1,1))
sage: p.axes_label_color()
(0, 0, 0)
```

We change the labels to be red, and confirm this:

```python
sage: p.axes_label_color((1,0,0))
sage: p.axes_label_color()
(1.0, 0.0, 0.0)
```

We set labels, since otherwise we won’t see anything.

```python
sage: p.axes_labels(['$x$ axis', '$y$ axis'])
```

In the plot below, notice that the labels are red:

```python
sage: p
Graphics object consisting of 1 graphics primitive
```

`axes_labels(l=None)`

Set the axes labels.

INPUT:
- $l$ - (default: None) a list of two strings or None

OUTPUT: a 2-tuple of strings

If $l$ is None, returns the current axes_labels, which is itself by default None. The default labels are both empty.

EXAMPLES: We create a plot and put x and y axes labels on it.

```python
sage: p = plot(sin(x), (x, 0, 10))
sage: p.axes_labels(['$x$', '$y$'])
sage: p.axes_labels()
('x', 'y')
```

Now when you plot $p$, you see $x$ and $y$ axes labels:

```python
sage: p
Graphics object consisting of 1 graphics primitive
```

Notice that some may prefer axes labels which are not typeset:

```python
sage: plot(sin(x), (x, 0, 10), axes_labels=['x','y'])
```

`axes_labels_size(s=None)`

Set the relative size of axes labels w.r.t. the axes tick marks.

INPUT:
• $s$ - float, relative size of axes labels w.r.t. to the tick marks, the size of the tick marks being set by `fontsize()`.

If called with no input, return the current relative size.

EXAMPLES:

```
sage: p = plot(sin(x^2), (x, -3, 3), axes_labels=['$x$', '$y$'])
sage: p.axes_labels_size() # default value
1.6
sage: p.axes_labels_size(2.5)
sage: p.axes_labels_size()
2.5
```

Now the axes labels are large w.r.t. the tick marks:

```
sage: p
Graphics object consisting of 1 graphics primitive
```

### axes_range

Set the ranges of the $x$ and $y$ axes.

**INPUT:**

- `xmin`, `xmax`, `ymin`, `ymax` - floats

**EXAMPLES:**

```
sage: L = line([[1,2], [3,-4], [2, 5], [1,2]])
sage: L.set_axes_range(-1, 20, 0, 2)
sage: d = L.get_axes_range()
sage: d['xmin'], d['xmax'], d['ymin'], d['ymax']
(-1.0, 20.0, 0.0, 2.0)
```

### axes_width

Set the axes width. Use this to draw a plot with really fat or really thin axes.

**INPUT:**

- `w` - a float

If called with no input, return the current axes_width setting.

**EXAMPLES:** We create a plot, see the default axes width (with funny Python float rounding), then reset the width to 10 (very fat).

```
sage: p = plot(cos, (-3,3))
sage: p.axes_width()
0.8
sage: p.axes_width(10)
sage: p.axes_width()
10.0
```

Finally we plot the result, which is a graph with very fat axes.

```
sage: p
Graphics object consisting of 1 graphics primitive
```

### description

Print a textual description to stdout.
This method is mostly used for doctests.

EXAMPLES:

```
sage: print(polytopes.hypercube(2).plot().description())
Polygon defined by 4 points: [(-1.0, -1.0), (1.0, -1.0), (1.0, 1.0), (-1.0, 1.0)]
Line defined by 2 points: [(-1.0, 1.0), (-1.0, -1.0)]
Line defined by 2 points: [(1.0, -1.0), (-1.0, -1.0)]
Line defined by 2 points: [(1.0, -1.0), (1.0, 1.0)]
Line defined by 2 points: [(1.0, 1.0), (-1.0, 1.0)]
Point set defined by 4 point(s): [(1.0, -1.0), (1.0, 1.0), (-1.0, 1.0), (-1.0, -1.0)]
```

```
flip(flip_x=False, flip_y=False)
```

Get the flip options and optionally mirror this graphics object.

INPUT:

- `flip_x` – boolean (default: False); if True, replace the current `flip_x` option by its opposite
- `flip_y` – boolean (default: False); if True, replace the current `flip_y` option by its opposite

OUTPUT: a tuple containing the new flip options

EXAMPLES:

When called without arguments, this just returns the current flip options:

```
sage: L = line([(1, 0), (2, 3)])
sage: L.flip()
(False, False)
```

Otherwise, the specified options are changed and the new options are returned:

```
sage: L.flip(flip_y=True)
(False, True)
sage: L.flip(True, True)
(True, False)
```

```
fontsize(s=None)
```

Set the font size of axes labels and tick marks.

Note that the relative size of the axes labels font w.r.t. the tick marks font can be adjusted via `axes_labels_size()`.

INPUT:

- `s` - integer, a font size in points.

If called with no input, return the current fontsize.

EXAMPLES:

```
sage: L = line([(1, 2), (3, -4), (2, 5), (1, 2)])
sage: L.fontsize()
10
sage: L.fontsize(20)
sage: L.fontsize()
20
```
All the numbers on the axes will be very large in this plot:

```python
sage: L
Graphics object consisting of 1 graphics primitive
```

**get_axes_range()**

Returns a dictionary of the range of the axes for this graphics object. This is fall back to the ranges in `get_minmax_data()` for any value which the user has not explicitly set.

**Warning:** Changing the dictionary returned by this function does not change the axes range for this object. To do that, use the `set_axes_range()` method.

**EXAMPLES:**

```python
sage: L = line([(1,2), (3,-4), (2,5), (1,2)])

sage: list(sorted(L.get_axes_range().items()))
[('xmax', 3.0), ('xmin', 1.0), ('ymax', 5.0), ('ymin', -4.0)]

sage: L.set_axes_range(xmin=-1)

sage: list(sorted(L.get_axes_range().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 5.0), ('ymin', -4.0)]
```

**get_minmax_data()**

Return the x and y coordinate minimum and maximum.

**Warning:** The returned dictionary is mutable, but changing it does not change the xmin/xmax/ymin/ymax data. The minmax data is a function of the primitives which make up this Graphics object. To change the range of the axes, call methods `xmin()`, `xmax()`, `ymin()`, `ymax()`, or `set_axes_range()`.

**OUTPUT:**

A dictionary whose keys give the xmin, xmax, ymin, and ymax data for this graphic.

**EXAMPLES:**

```python
sage: g = line([(1,2), (3,2)])

sage: list(sorted(g.get_minmax_data().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 2.0), ('ymin', 1.0)]
```

Note that changing ymax doesn’t change the output of `get_minmax_data`:

```python
sage: g.ymax(10)

sage: list(sorted(g.get_minmax_data().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 2.0), ('ymin', 1.0)]
```

The width/height ratio (in output units, after factoring in the chosen aspect ratio) of the plot is limited to $10^{-15} \ldots 10^{15}$, otherwise floating point errors cause problems in matplotlib:

```python
sage: l = line([(1e-19,-1), (-1e-19,+1)], aspect_ratio=1.0)

sage: l.get_minmax_data()
{'xmax': 1.00000000000000e-15,
 'xmin': -9.99900000000000e-16,
 (continues on next page)
```
inset\(\text{(graphics, pos=\text{None}, fontsize=\text{None})}\)
Add a graphics object as an inset.

INPUT:
• graphics – the graphics object (instance of \textit{Graphics}) to be added as an inset to the current graphics
• pos – (default: \text{None}) \text{4-tuple} (left, bottom, width, height) specifying the location and size of the inset on the final figure, all quantities being in fractions of the figure width and height; if \text{None}, the value \((0.7, \ 0.7, \ 0.2, \ 0.2)\) is used
• fontsize – (default: \text{None}) integer, font size (in points) for the inset; if \text{None}, the value of 6 points is used, unless \text{fontsize} has been explicitly set in the construction of \text{graphics} (in this case, it is not overwritten here)

OUTPUT:
• instance of \textit{MultiGraphics}

EXAMPLES:
\begin{verbatim}
sage: f(x) = x^2*\sin(1/x)
sage: g1 = plot(f(x), (x, -2, 2), axes_labels=['$x$', '$y$'])
sage: g2 = plot(f(x), (x, -0.3, 0.3), axes_labels=['$x$', '$y$'],
           \text{: frame=True})
sage: g1.inset(g2)
Multigraphics with 2 elements
\end{verbatim}

Using non-default values for the position/size and the font size:
\begin{verbatim}
sage: g1.inset(g2, pos=\text{(0.15, 0.7, 0.25, 0.25)}, fontsize=8)
Multigraphics with 2 elements
\end{verbatim}

We can add another inset by invoking \texttt{inset} on the last output:
\begin{verbatim}
sage: g1g2 = _
sage: g3 = plot(f(x), (x, -0.05, 0.05), axes_labels=['$x$', '$y$'],
           \text{: frame=True})
sage: g1g2.inset(g3, pos=\text{(0.65, 0.12, 0.25, 0.25)})
Multigraphics with 3 elements
\end{verbatim}

legend\(\text{(show=\text{None})}\)
Set whether or not the legend is shown by default.

INPUT:
• show - (default: \text{None}) a boolean

If called with no input, return the current legend setting.

EXAMPLES:
By default no legend is displayed:
5.1. Graphics objects
5.1. Graphics objects
sage: P = plot(sin)
sage: P.legend()
False

But if we put a label then the legend is shown:

sage: P = plot(sin, legend_label='sin')
sage: P.legend()
True

We can turn it on or off:

sage: P.legend(False)
sage: P.legend()
False
sage: P.legend(True)
sage: P
# show with the legend
Graphics object consisting of 1 graphics primitive

Construct or modify a Matplotlib figure by drawing `self` on it.

**INPUT** (partial description, involving only Matplotlib objects; see `show()` for the other arguments):

- `figure` – (default: None) Matplotlib figure (class `matplotlib.figure.Figure`) on which `self` is to be displayed; if None, the figure will be created from the parameter `figsize`
- `figsize` – (default: None) width or [width, height] in inches of the Matplotlib figure in case `figure` is None; if `figsize` is None, Matplotlib’s default (6.4 x 4.8 inches) is used
- `sub` – (default: None) subpart of the figure, as an instance of Matplotlib “axes” (class `matplotlib.axes.Axes`) on which `self` is to be drawn; if None, the subpart will be created so as to cover the whole figure

**OUTPUT:**

- a `matplotlib.figure.Figure` object; if the argument `figure` is provided, this is the same object as `figure`.

**EXAMPLES:**

```
sage: c = circle((1,1),1)
sage: print(c.matplotlib())
Figure(640x480)
```

To obtain the first Matplotlib Axes object inside of the figure, you can do something like the following.

```sage
sage: p = plot(sin(x), (x, -2*pi, 2*pi))
sage: figure = p.matplotlib()
sage: axes = figure.axes[0]
```
plot()
Draw a 2D plot of this graphics object, which just returns this object since this is already a 2D graphics object.

EXAMPLES:

```
sage: S = circle((0,0), 2)
sage: S.plot() is S
True
```

It does not accept any argument (trac ticket #19539):

```
sage: S.plot(1)
Traceback (most recent call last):
  ... plot() takes 1 positional argument but 2 were given
sage: S.plot(hey="hou")
Traceback (most recent call last):
  ... plot() got an unexpected keyword argument 'hey'
```

plot3d(z=0, **kwds)
Returns an embedding of this 2D plot into the xy-plane of 3D space, as a 3D plot object. An optional parameter z can be given to specify the z-coordinate.

EXAMPLES:

```
sage: sum([plot(z*sin(x), 0, 10).plot3d(z) for z in range(6)]) # long time
Graphics3d Object
```

save(filename, legend_back_color='white', legend_borderpad=0.6, legend_borderaxespad=None,
legend_columnsspacing=None, legend_fancybox=False, legend_font_family='sans-serif',
legend_font_size='medium', legend_font_style='normal', legend_font_variant='normal',
legend_font_weight='medium', legend_handlelength=0.05, legend_handletextpad=0.5,
legend_labelsspacing=0.02, legend_loc='best', legend_markerscale=0.6, legend_ncol=1,
legend_numpoints=2, legend_shadow=True, legend_title=None, **kwds)
Save the graphics to an image file.

INPUT:

* filename – string. The filename and the image format given by the extension, which can be one of the following:
  - .eps,
  - .pdf,
  - .pgf,
  - .png,
  - .ps,
  - .sobj (for a Sage object you can load later),
  - .svg,
  - empty extension will be treated as .sobj.
All other keyword arguments will be passed to the plotter.

OUTPUT:

- none.

EXAMPLES:

```sage
c = circle((1,1), 1, color='red')
filename = os.path.join(SAGE_TMP, 'test.png')
c.save(filename, xmin=-1, xmax=3, ymin=-1, ymax=3)
```

To make a figure bigger or smaller, use figsize:

```sage
c.save(filename, figsize=5, xmin=-1, xmax=3, ymin=-1, ymax=3)
```

By default, the figure grows to include all of the graphics and text, so the final image may not be exactly the figure size you specified. If you want a figure to be exactly a certain size, specify the keyword fig_tight=False:

```sage
c.save(filename, figsize=[8,4], fig_tight=False, ....: xmin=-1, xmax=3, ymin=-1, ymax=3)
```

You can also pass extra options to the plot command instead of this method, e.g.

```sage
plot(x^2 - 5, (x, 0, 5), ymin=0).save(tmp_filename(ext='png'))
```

will save the same plot as the one shown by this command:

```sage
plot(x^2 - 5, (x, 0, 5), ymin=0)
```

Graphics object consisting of 1 graphics primitive

(This test verifies that trac ticket #8632 is fixed.)

**save_image**

```
filename=None, *args, **kwds)
```

Save an image representation of self.

The image type is determined by the extension of the filename. For example, this could be .png, .jpg, .gif, .pdf, .svg. Currently this is implemented by calling the `save()` method of self, passing along all arguments and keywords.

**Note:** Not all image types are necessarily implemented for all graphics types. See `save()` for more details.

EXAMPLES:

```sage
c = circle((1,1), 1, color='red')
filename = os.path.join(SAGE_TMP, 'test.png')
c.save_image(filename, xmin=-1, xmax=3, ymin=-1, ymax=3)
```

**set_aspect_ratio**

```
ratio)
```

Set the aspect ratio, which is the ratio of height and width of a unit square (i.e., height/width of a unit square), or ‘automatic’ (expand to fill the figure).

**INPUT:**

- ratio - a positive real number or ‘automatic’
EXAMPLES: We create a plot of the upper half of a circle, but it doesn’t look round because the aspect ratio is off:

\[
\text{sage: } P = \text{plot}(\sqrt{1-x^2}, (x, -1, 1)); P
\]

Graphics object consisting of 1 graphics primitive

So we set the aspect ratio and now it is round:

\[
\begin{align*}
\text{sage: } & P.\text{set_aspect_ratio}(1) \\
\text{sage: } & P.\text{aspect_ratio}() \\
& 1.0 \\
\text{sage: } & P
\end{align*}
\]

Graphics object consisting of 1 graphics primitive

Note that the aspect ratio is inherited upon addition (which takes the max of aspect ratios of objects whose aspect ratio has been set):

\[
\begin{align*}
\text{sage: } & P + \text{plot}(\sqrt{4-x^2}, (x, -2, 2)) \\
\text{sage: } & (P + Q).\text{aspect_ratio}(); P+Q \\
& 2.0 \\
\text{sage: } & (Q + P).\text{aspect_ratio}(); Q+P \\
& 2.0
\end{align*}
\]

\begin{verbatim}
set_axes_range(xmin=None, xmax=None, ymin=None, ymax=None)
\end{verbatim}

Set the ranges of the \(x\) and \(y\) axes.

INPUT:

\[
\begin{align*}
\text{• } & \text{xmin, xmax, ymin, ymax - floats}
\end{align*}
\]

EXAMPLES:

\[
\begin{align*}
\text{sage: } & L = \text{line}(((1,2), (3,-4), (2,5), (1,2))) \\
\text{sage: } & L.\text{set_axes_range}(-1, 20, 0, 2) \\
\text{sage: } & d = L.\text{get_axes_range}() \\
\text{sage: } & d['\text{xmin}'], d['\text{xmax}'], d['\text{ymin}'], d['\text{ymax}'] \\
& (-1.0, 20.0, 0.0, 2.0)
\end{align*}
\]

\[
\begin{verbatim}
set_flip(flip_x=None, flip_y=None)
\end{verbatim}

Set the flip options for this graphics object.

INPUT:

\[
\begin{align*}
\text{• } & \text{flip_x - boolean (default: None); if not None, set the flip_x option to this value} \\
\text{• } & \text{flip_y - boolean (default: None); if not None, set the flip_y option to this value}
\end{align*}
\]

EXAMPLES:

\[
\begin{align*}
\text{sage: } & L = \text{line}(((1,0), (2,3))) \\
\text{sage: } & L.\text{set_flip}(\text{flip_y=True})
\end{align*}
\]
sage: L.flip()
(False, True)
sage: L.set_flip(True, False)
sage: L.flip()
(True, False)

\texttt{set\_legend\_options(**kwds)}

Set various legend options.

**INPUT:**

- **title** - (default: None) string, the legend title
- **ncol** - (default: 1) positive integer, the number of columns
- **columnspacing** - (default: None) the spacing between columns
- **borderaxespad** - (default: None) float, length between the axes and the legend
- **back_color** - (default: ‘white’) This parameter can be a string denoting a color or an RGB tuple. The string can be a color name as in (‘red’, ‘green’, ‘yellow’, …) or a floating point number like ‘0.8’ which gets expanded to (0.8, 0.8, 0.8). The tuple form is just a floating point RGB tuple with all values ranging from 0 to 1.
- **handlelength** - (default: 0.05) float, the length of the legend handles
- **handletextpad** - (default: 0.5) float, the pad between the legend handle and text
- **labelspacing** - (default: 0.02) float, vertical space between legend entries
- **loc** - (default: ‘best’) \textbf{May be a string, an integer or a tuple. String or integer inputs must be one of the following:}
  - 0, ‘best’
  - 1, ‘upper right’
  - 2, ‘upper left’
  - 3, ‘lower left’
  - 4, ‘lower right’
  - 5, ‘right’
  - 6, ‘center left’
  - 7, ‘center right’
  - 8, ‘lower center’
  - 9, ‘upper center’
  - 10, ‘center’
  - Tuple arguments represent an absolute (x, y) position on the plot in axes coordinates (meaning from 0 to 1 in each direction).
- **markerscale** - (default: 0.6) float, how much to scale the markers in the legend.
- **numpoints** - (default: 2) integer, the number of points in the legend for line
- **borderpad** - (default: 0.6) float, the fractional whitespace inside the legend border (between 0 and 1)

• **font_style** - (default: ‘normal’) string, one of ‘normal’, ‘italic’, ‘oblique’

• **font_variant** - (default: ‘normal’) string, one of ‘normal’, ‘small-caps’


• **font_size** - (default: ‘medium’) string, one of ‘xx-small’, ‘x-small’, ‘small’, ‘medium’, ‘large’, ‘x-large’, ‘xx-large’ or an absolute font size (e.g. 12)

• **shadow** - (default: True) boolean - draw a shadow behind the legend

• **fancybox** - (default: False) a boolean. If True, draws a frame with a round fancybox.

These are all keyword arguments.

OUTPUT: a dictionary of all current legend options

EXAMPLES:

By default, no options are set:

```python
sage: p = plot(tan, legend_label='tan')
sage: p.set_legend_options()
{}
```

We build a legend without a shadow:

```python
sage: p.set_legend_options(shadow=False)
sage: p.set_legend_options()['shadow']
False
```

To set the legend position to the center of the plot, all these methods are roughly equivalent:

```python
sage: p.set_legend_options(loc='center'); p
Graphics object consisting of 1 graphics primitive

sage: p.set_legend_options(loc=10); p
Graphics object consisting of 1 graphics primitive

sage: p.set_legend_options(loc=(0.5,0.5)); p  # aligns the bottom of the box to the center
Graphics object consisting of 1 graphics primitive
```

```python
show(legend_back_color='white', legend_borderpad=0.6, legend_borderticks='never', legend_columnspacing=0.1, legend_font_family='sans-serif', legend_font_size='medium', legend_font_style='normal', legend_font_variant='normal', legend_font_weight='medium', legend_handlelength=0.05, legend_handletextpad=0.5, legend_labelspacing=0.2, legend_loc='best', legend_markerscale=0.6, legend_ncol=1, legend_numpoints=2, legend_shadow=True, legend_title=None, **kwds)
```

Show this graphics image immediately.

This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

OPTIONAL INPUT:
• **dpi** - (default: 100) dots per inch

• **figsize** - (default: [6.4, 4.8]) [width, height] inches. The maximum value of each of the width and the height can be 327 inches, at the default dpi of 100 dpi, which is just shy of the maximum allowed value of 32768 dots (pixels).

• **fig_tight** - (default: True) whether to clip the drawing tightly around drawn objects. If True, then the resulting image will usually not have dimensions corresponding to figsize. If False, the resulting image will have dimensions corresponding to figsize.

• **aspect_ratio** - the perceived height divided by the perceived width. For example, if the aspect ratio is set to 1, circles will look round and a unit square will appear to have sides of equal length, and if the aspect ratio is set 2, vertical units will be twice as long as horizontal units, so a unit square will be twice as high as it is wide. If set to 'automatic', the aspect ratio is determined by figsize and the picture fills the figure.

• **axes** - (default: True)

• **axes_labels** - (default: None) list (or tuple) of two strings; the first is used as the label for the horizontal axis, and the second for the vertical axis.

• **axes_labels_size** - (default: current setting – 1.6) scale factor relating the size of the axes labels with respect to the size of the tick marks.

• **fontsize** - (default: current setting – 10) positive integer; used for axes labels; if you make this very large, you may have to increase figsize to see all labels.

• **frame** - (default: False) draw a frame around the image

• **gridlines** - (default: None) can be any of the following:
  - None, False: do not add grid lines.
  - True, “automatic”, “major”: add grid lines at major ticks of the axes.
  - “minor”: add grid at major and minor ticks.
  - [xlist,ylist]: a tuple or list containing two elements, where xlist (or ylist) can be any of the following.
    - None, False: don’t add horizontal (or vertical) lines.
    - True, “automatic”, “major”: add horizontal (or vertical) grid lines at the major ticks of the axes.
    - “minor”: add horizontal (or vertical) grid lines at major and minor ticks of axes.
    - an iterable yielding numbers n or pairs (n,opts), where n is the coordinate of the line and opt is a dictionary of MATPLOTLIB options for rendering the line.

• **gridlinesstyle, hgridlinesstyle, vgridlinesstyle** - (default: None) a dictionary of MATPLOTLIB options for the rendering of the grid lines, the horizontal grid lines or the vertical grid lines, respectively.

• **transparent** - (default: False) If True, make the background transparent.

• **axes_pad** - (default: 0.02 on "linear" scale, 1 on "log" scale).
  - In the "linear" scale, it determines the percentage of the axis range that is added to each end of each axis. This helps avoid problems like clipping lines because of line-width, etc. To get axes that are exactly the specified limits, set axes_pad to zero.
  - On the "log" scale, it determines the exponent of the fraction of the minimum (resp. maximum) that is subtracted from the minimum (resp. added to the maximum) value of the axis. For instance if the minimum is \( m \) and the base of the axis is \( b \) then the new minimum after padding the axis will be \( m - \frac{m}{b^{\text{axes_pad}}} \).
• **ticks_integer** - (default: False) guarantee that the ticks are integers (the **ticks** option, if specified, will override this)

• **ticks** - A matplotlib locator for the major ticks, or a number. There are several options. For more information about locators, type `from matplotlib import ticker` and then `ticker?`.

  – If this is a locator object, then it is the locator for the horizontal axis. A value of None means use the default locator.

  – If it is a list of two locators, then the first is for the horizontal axis and one for the vertical axis. A value of None means use the default locator (so a value of [None, my locator] uses my_locator for the vertical axis and the default for the horizontal axis).

  – If in either case above one of the entries is a number \( m \) (something which can be coerced to a float), it will be replaced by a MultipleLocator which places major ticks at integer multiples of \( m \). See examples.

  – If in either case above one of the entries is a list of numbers, it will be replaced by a FixedLocator which places ticks at the locations specified. This includes the case of of the empty list, which will give no ticks. See examples.

• **tick_formatter** - A matplotlib formatter for the major ticks. There are several options. For more information about formatters, type `from matplotlib import ticker` and then `ticker?`.

  If the value of this keyword is a single item, then this will give the formatting for the horizontal axis only (except for the "latex" option). If it is a list or tuple, the first is for the horizontal axis, the second for the vertical axis. The options are below:

  – If one of the entries is a formatter object, then it used. A value of None means to use the default locator (so using `tick_formatter=[None, my_formatter]` uses my_formatter for the vertical axis and the default for the horizontal axis).

  – If one of the entries is a symbolic constant such as \( \pi \), \( e \), or \( sqrt(2) \), ticks will be formatted nicely at rational multiples of this constant.

  **Warning:** This should only be used with the **ticks** option using nice rational multiples of that constant!

  – If one of the entries is the string "latex", then the formatting will be nice typesetting of the ticks. This is intended to be used when the tick locator for at least one of the axes is a list including some symbolic elements. This uses matplotlib's internal LaTeX rendering engine. If you want to use an external LaTeX compiler, then set the keyword option `typeset`. See examples.

• **title** - (default: None) The title for the plot

• **title_pos** - (default: None) The position of the title for the plot. It must be a tuple or a list of two real numbers \((x_pos, y_pos)\) which indicate the relative position of the title within the plot. The plot itself can be considered to occupy, in relative terms, the region within a unit square \([0, 1] \times [0, 1]\). The title text is centered around the horizontal factor \(x_pos\) of the plot. The baseline of the title text is present at the vertical factor \(y_pos\) of the plot. Hence, `title_pos=(0.5, 0.5)` will center the title in the plot, whereas `title_pos=(0.5, 1.1)` will center the title along the horizontal direction, but will place the title a fraction 0.1 times above the plot.

  – If the first entry is a list of strings (or numbers), then the formatting for the horizontal axis will be typeset with the strings present in the list. Each entry of the list of strings must be provided with a corresponding number in the first entry of **ticks** to indicate its position on the axis. To typeset the strings with "latex" enclose them within "$" symbols. To have similar custom formatting of the labels along the vertical axis, the second entry must be a list of strings and the second entry
of ticks must also be a list of numbers which give the positions of the labels. See the examples below.

- **show_legend** - (default: None) If True, show the legend
- **legend_*** - all the options valid for `set_legend_options()` prefixed with `legend_`
- **base** - (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple `(basex, basey)`. `basex` sets the base of the logarithm along the horizontal axis and `basey` sets the base along the vertical axis.
- **scale** – (default: "linear") string. The scale of the axes. Possible values are
  - "linear" – linear scaling of both the axes
  - "loglog" – sets both the horizontal and vertical axes to logarithmic scale
  - "semilogx" – sets only the horizontal axis to logarithmic scale.
  - "semilogy" – sets only the vertical axis to logarithmic scale.

The scale can be also be given as single argument that is a list or tuple `(scale, base)` or `(scale, basex, basey)`.

**Note:**
- If the `scale` is "linear", then irrespective of what `base` is set to, it will default to 10 and will remain unused.

- **xmin** – starting x value in the rendered figure.
- **xmax** – ending x value in the rendered figure.
- **ymin** – starting y value in the rendered figure.
- **ymax** – ending y value in the rendered figure.
- **flip_x** – (default: False) boolean. If True, flip the horizontal axis.
- **flip_y** – (default: False) boolean. If True, flip the vertical axis.
- **typeset** – (default: "default") string. The type of font rendering that should be used for the text. The possible values are
  - "default" – Uses matplotlib’s internal text rendering engine called Mathtext (see [https://matplotlib.org/users/mathtext.html](https://matplotlib.org/users/mathtext.html)). If you have modified the default matplotlib settings, for instance via a matplotlibrc file, then this option will not change any of those settings.
  - "latex" – LaTeX is used for rendering the fonts. This requires LaTeX, dvipng and Ghostscript to be installed.
  - "type1" – Type 1 fonts are used by matplotlib in the text in the figure. This requires LaTeX, dvipng and Ghostscript to be installed.

**OUTPUT:**
This method does not return anything. Use `save()` if you want to save the figure as an image.

**EXAMPLES:**
```python
sage: c = circle((1,1), 1, color='red')
sage: c.show(xmin=-1, xmax=3, ymin=-1, ymax=3)
```
You can make the picture larger by changing figsize with width, height each having a maximum value of 327 inches at default dpi:

```sage
sage: p = ellipse((0,0),4,1)
sage: p.show(figsize=[327,10],dpi=100)
sage: p.show(figsize=[328,10],dpi=80)
```

You can turn off the drawing of the axes:

```sage
sage: show(plot(sin,-4,4), axes=False)
```

You can also label the axes. Putting something in dollar signs formats it as a mathematical expression:

```sage
sage: show(plot(sin,-4,4), axes_labels=('x','y'))
```

You can add a title to a plot:

```sage
sage: show(plot(sin,-4,4), title=r'A plot of $\sin(x)$')
```

You can also provide the position for the title to the plot. In the plot below the title is placed on the bottom left of the figure:

```sage
sage: plot(sin, -4, 4, title='Plot sin(x)', title_pos=(0.05,-0.05))
```

If you want all the text to be rendered by using an external LaTeX installation then set the typeset to "latex". This requires that LaTeX, dvipng and Ghostscript be installed:

```sage
sage: plot(x, typeset='latex') # optional - latex
```

If you want all the text in your plot to use Type 1 fonts, then set the typeset option to "type1". This requires that LaTeX, dvipng and Ghostscript be installed:

```sage
sage: plot(x, typeset='type1') # optional - latex
```

You can turn on the drawing of a frame around the plots:

```sage
sage: show(plot(sin,-4,4), frame=True)
```

You can make the background transparent:

```sage
sage: plot(sin(x), (x, -4, 4), transparent=True)
```

Prior to trac ticket #19485, legends by default had a shadowless gray background. This behavior can be recovered by passing in certain legend_options:

```sage
sage: p = plot(sin(x), legend_label=r'$\sin(x)$')
sage: p.show(legend_options={
'back_color': (0.9,0.9,0.9),
'shadow': False})
```

We can change the scale of the axes in the graphics before displaying:
We can change the base of the logarithm too. The following changes the vertical axis to be on log scale, and with base 2. Note that the base argument will ignore any changes to the axis which is in linear scale:

```
sage: G.show(scale='semilogy', base=2) # long time # y axis as powers of 2
```

```
sage: G.show(scale='semilogy', base=(3,2)) # base ignored for x-axis
```

The scale can be also given as a 2-tuple or a 3-tuple:

```
sage: G.show(scale=('loglog', 2.1)) # long time # both x and y axes in base 2.1
```

```
sage: G.show(scale=('loglog', 2, 3)) # long time # x in base 2, y in base 3
```

The base need not be an integer, though it does have to be made a float:

```
sage: G.show(scale='semilogx', base=float(e)) # base is e
```

Logarithmic scale can be used for various kinds of plots. Here are some examples:

```
sage: G = list_plot([10**i for i in range(10)])
# long time
sage: G.show(scale='semilogy')
```

```
sage: G = parametric_plot((x, x**2), (x, 1, 10))
sage: G.show(scale='loglog')
```

```
sage: disk((5,5), 4, (0, 3*pi/2)).show(scale='loglog',base=2)
```

```
sage: u,v = var('u, v')
sage: f = exp(-(u**2+v**2))
sage: p = plot_vector_field(f.gradient(), (u,-2,2), (v,-2,2))
sage: p.show(gridlines='minor')
```

Flip the horizontal or vertical axis:

```
sage: G = plot(x^3, -2, 3)
sage: G.show(flip_x=True)
sage: G.show(flip_y=True)
```

Add grid lines at the major ticks of the axes:

```
sage: c = circle((0,0), 1)
sage: c.show(gridlines=True)
sage: c.show(gridlines="automatic")
sage: c.show(gridlines="major")
```

Add grid lines at the major and minor ticks of the axes:

```
sage: u,v = var('u, v')
sage: f = exp(-(-u^2+v^2))
sage: p = plot_vector_field(f.gradient(), (u,-2,2), (v,-2,2))
sage: p.show(gridlines="minor")
```
Add only horizontal or vertical grid lines.

```python
sage: p = plot(sin,-10,20)
sage: p.show(gridlines=[None, "automatic"])
sage: p.show(gridlines=["minor", False])
```

Add grid lines at specific positions (using lists/tuples).

```python
sage: x, y = var('x, y')
sage: p = implicit_plot((y^2-x^2)*(x-1)*(2*x-3)-4*(x^2+y^2-2*x)^2, 
.....: (x,-2,2), (y,-2,2), plot_points=1000)
sage: p.show(gridlines=[[1,0],[-1,0,1]])
```

Add grid lines at specific positions (using iterators).

```python
sage: def maple_leaf(t):
.....:     return 100/(100+(t-pi/2)^8))*(2-sin(7*t)-cos(30*t)/2)
sage: p = polar_plot(maple_leaf, -pi/4, 3*pi/2, color="red",plot_points=1000) # long time
sage: p.show(gridlines=([-3,-2.75,...,3], range(-1,5,2))) # long time
```

Add grid lines at specific positions (using functions).

```python
sage: y = x^5 + 4*x^4 - 10*x^3 - 40*x^2 + 9*x + 36
sage: p = plot(y, -4.1, 1.1)
sage: xlines = lambda a,b: [z for z,m in y.roots()]
sage: p.show(gridlines=[xlines, [0]], frame=True, axes=False)
```

Change the style of all the grid lines.

```python
sage: b = bar_chart([-3,5,-6,11], color='red')
sage: b.show(gridlines=([-1,-0.5,...,4],True),
.....: gridlinesstyle=dict(color="blue", linestyle=":"))
```

Change the style of the horizontal or vertical grid lines separately.

```python
sage: p = polar_plot(2 + 2*cos(x), 0, 2*pi, color=hue(0.3))
sage: p.show(gridlines= True,
.....: hgridlinesstyle=dict(color="orange", linewidth=1.0),
.....: vgridlinesstyle=dict(color="blue", linestyle=":"))
```

Change the style of each grid line individually.

```python
sage: x, y = var('x, y')
sage: p = implicit_plot((y^2-x^2)*(x-1)*(2*x-3)-4*(x^2+y^2-2*x)^2, 
.....: (x,-2,2), (y,-2,2), plot_points=1000)
sage: p.show(gridlines=(
.....: [1,"color":"red","linestyle":"-"],
.....: [0,"color":"blue","linestyle":"--"],
.....: [-1,"color":"red","linestyle":"-"],
.....: [0,"color":"blue","linestyle":"--"],
.....: [1,"color":"red","linestyle":"-"],
```

(continues on next page)
Grid lines can be added to contour plots.

```
sage: f = sin(x^2 + y^2)*cos(x)*sin(y)
sage: c = contour_plot(f, (x, -4, 4), (y, -4, 4), plot_points=100)
sage: c.show(gridlines=True, gridlinesstyle={'linestyle':':', 'linewidth':1, 'color':'red'})
```

Grid lines can be added to matrix plots.

```
sage: M = MatrixSpace(QQ,10).random_element()
sage: matrix_plot(M).show(gridlines=True)
```

By default, Sage increases the horizontal and vertical axes limits by a certain percentage in all directions. This is controlled by the `axes_pad` parameter. Increasing the range of the axes helps avoid problems with lines and dots being clipped because the linewidth extends beyond the axes. To get axes limits that are exactly what is specified, set `axes_pad` to zero. Compare the following two examples.

```
sage: plot(sin(x), (x, -pi, pi), thickness=2)+point((pi, -1), pointsize=15)
Graphics object consisting of 2 graphics primitives
sage: plot(sin(x), (x, -pi, pi), thickness=2, axes_pad=0)+point((pi, -1), pointsize=15)
Graphics object consisting of 2 graphics primitives
```

The behavior of the `axes_pad` parameter is different if the axis is in the "log" scale. If $b$ is the base of the axis, the minimum value of the axis, is decreased by the factor $1/b$ of the minimum and the maximum value of the axis is increased by the same factor of the maximum value. Compare the axes in the following two plots to see the difference.

```
sage: plot_loglog(x, (1.1*10**-2, 9990))
Graphics object consisting of 1 graphics primitive
sage: plot_loglog(x, (1.1*10**-2, 9990), axes_pad=0)
Graphics object consisting of 1 graphics primitive
```

Via matplotlib, Sage allows setting of custom ticks. See above for more details.

Here the labels are not so useful:

```
sage: plot(sin(pi*x), (x, -8, 8))
Graphics object consisting of 1 graphics primitive
```

Now put ticks at multiples of 2:

```
sage: plot(sin(pi*x), (x, -8, 8), ticks=2)
Graphics object consisting of 1 graphics primitive
```

Or just choose where you want the ticks:

```
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[[-7,-3,0,3,7],[-1/2,0,1/2]])
Graphics object consisting of 1 graphics primitive
```
Or no ticks at all:

```python
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[[],[]])
Graphics object consisting of 1 graphics primitive
```

This can be very helpful in showing certain features of plots.

```python
sage: plot(1.5/(1+e^(-x)), (x, -10, 10)) # doesn't quite show value of
˓→inflexion point
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(1.5/(1+e^(-x)), (x, -10, 10), ticks=[None, 1.5/4]) # It's right at
˓→f(x)=0.75!
Graphics object consisting of 1 graphics primitive
```

But be careful to leave enough room for at least two major ticks, so that the user can tell what the scale is:

```python
sage: plot(x^2,(x,1,8),ticks=6).show()
Traceback (most recent call last):
...
ValueError: Expand the range of the independent variable to
allow two multiples of your tick locator (option 'ticks').
```

We can also do custom formatting if you need it. See above for full details:

```python
sage: plot(2*x+1,(x,0,5),ticks=[[0,1,e,pi,sqrt(20)],2],tick_formatter="latex")
Graphics object consisting of 1 graphics primitive
```

This is particularly useful when setting custom ticks in multiples of \( \pi \).

```python
sage: plot(sin(x),(x,0,2*pi),ticks=pi/3,tick_formatter=pi)
Graphics object consisting of 1 graphics primitive
```

But keep in mind that you will get exactly the formatting you asked for if you specify both formatters. The first syntax is recommended for best style in that case.

```python
sage: plot(arcsin(x),(x,-1,1),ticks=[None,pi/6],tick_formatter=["latex",pi]) # Nice-looking!
˓→Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(arcsin(x),(x,-1,1),ticks=[None,pi/6],tick_formatter=[None,pi]) # Not so nice-looking
˓→Graphics object consisting of 1 graphics primitive
```

Custom tick labels can be provided by providing the keyword `tick_formatter` with the list of labels, and simultaneously providing the keyword `ticks` with the positions of the labels.

```python
sage: plot(x, (x,0,3), ticks=[[1,2.5],[0.5,1,2]], tick_formatter=["$x_1$","$x_2$
˓→$\rightarrow$"],["$y_1$","$y_2$","$y_3$"])
Graphics object consisting of 1 graphics primitive
```

The following sets the custom tick labels only along the horizontal axis.

5.1. Graphics objects

435

```python
sage: plot(x**2, (x,0,2), ticks=[[1,2], None], tick_formatter=[["x_1","x_2"], None])
Graphics object consisting of 1 graphics primitive
```

If the number of tick labels do not match the number of positions of tick labels, then it results in an error:

```python
sage: plot(x**2, (x,0,2), ticks=[[2], None], tick_formatter=[["x_1","x_2"], None]).show()
Traceback (most recent call last):
...
ValueError: If the first component of the list `tick_formatter` is a list then the first component of `ticks` must also be a list of equal length.
```

When using logarithmic scale along the axis, make sure to have enough room for two ticks so that the user can tell what the scale is. This can be effected by increasing the range of the independent variable, or by changing the base, or by providing enough tick locations by using the `ticks` parameter.

By default, Sage will expand the variable range so that at least two ticks are included along the logarithmic axis. However, if you specify ticks manually, this safety measure can be defeated:

```python
sage: list_plot_loglog([(1,2),(2,3)], plotjoined=True, ticks=[[1],[1]])
doctest:...: UserWarning: The x-axis contains fewer than 2 ticks; the logarithmic scale of the plot may not be apparent to the reader.
doctest:...: UserWarning: The y-axis contains fewer than 2 ticks; the logarithmic scale of the plot may not be apparent to the reader.
Graphics object consisting of 1 graphics primitive
```

This one works, since the horizontal axis is automatically expanded to contain two ticks and the vertical axis is provided with two ticks:

```python
sage: list_plot_loglog([(1,2),(2,3)], plotjoined=True, ticks=[None,[1,10]])
Graphics object consisting of 1 graphics primitive
```

Another example in the log scale where both the axes are automatically expanded to show two major ticks:

```python
sage: list_plot_loglog([(2,0.5), (3, 4)], plotjoined=True)              
Graphics object consisting of 1 graphics primitive
```

When using `title_pos`, it must be ensured that a list or a tuple of length two is used. Otherwise, a warning is raised:

```python
sage: plot(x, -4, 4, title='Plot x', title_pos=0.05)
doctest:...: ...RichReprWarning: Exception in _rich_repr_ while displaying...
object: 'title_pos' must be a list or tuple of two real numbers.
Graphics object consisting of 1 graphics primitive
```

**tick_label_color**

Set the color of the axes tick labels.

**INPUT:**

- c - an RGB 3-tuple of numbers between 0 and 1

If called with no input, return the current tick_label_color setting.

**EXAMPLES:**
```python
sage: p = plot(cos, (-3,3))
sage: p.tick_label_color()
(0, 0, 0)
sage: p.tick_label_color((1,0,0))
sage: p.tick_label_color()
(1.0, 0.0, 0.0)
sage: p
Graphics object consisting of 1 graphics primitive
```

### `xmax(xmax=None)`

**EXAMPLES:**

```python
sage: g = line([(-1,1), (3,2)])
sage: g.xmax()
3.0
sage: g.xmax(10)
sage: g.xmax()
10.0
```

### `xmin(xmin=None)`

**EXAMPLES:**

```python
sage: g = line([(-1,1), (3,2)])
sage: g.xmin()
-1.0
sage: g.xmin(-3)
sage: g.xmin()
-3.0
```

### `ymax(ymax=None)`

**EXAMPLES:**

```python
sage: g = line([(-1,1), (3,2)])
sage: g.ymax()
2.0
sage: g.ymax(10)
sage: g.ymax()
10.0
```

### `ymin(ymin=None)`

**EXAMPLES:**

```python
sage: g = line([(-1,1), (3,2)])
sage: g.ymin()
1.0
sage: g.ymin(-3)
sage: g.ymin()
-3.0
```

`sage.plot.graphics.GraphicsArray(*args, **kwargs)`

This is deprecated (see trac ticket #28675). Use `sage.plot.multigraphics.GraphicsArray` instead.

`sage.plot.graphics.is_Graphics(x)`

Return True if `x` is a Graphics object.

### 5.1. Graphics objects 437
EXAMPLES:

```
sage: from sage.plot.graphics import is_Graphics
sage: is_Graphics(1)
False
sage: is_Graphics(disk((0.0, 0.0), 1, (0, pi/2)))
True
```

### 5.2 Graphics arrays and insets

This module defines the classes `MultiGraphics` and `GraphicsArray`. The class `MultiGraphics` is the base class for 2-dimensional graphical objects that are composed of various `Graphics` objects, arranged in a given canvas. The subclass `GraphicsArray` is for `Graphics` objects arranged in a regular array.

AUTHORS:

- Eric Gourgoulhon (2019-05-24): initial version, refactoring the class `GraphicsArray` that was defined in the module `graphics`.

```
class sage.plot.multigraphics.GraphicsArray(array)
    Bases: sage.plot.multigraphics.MultiGraphics

    This class implements 2-dimensional graphical objects that constitute an array of `Graphics` drawn on a single canvas.

    The user interface is through the function `graphics_array()`.

    INPUT:

    - `array` – either a list of lists of `Graphics` elements (generic case) or a single list of `Graphics` elements (case of a single-row array)

    EXAMPLES:

    An array made of four graphics objects:

    ```
sage: g1 = plot(sin(x^2), (x, 0, 6), axes_labels=['$x$', '$y$'],
        ....:     axes=False, frame=True, gridlines='minor')
sage: y = var('y')
sage: g2 = streamline_plot((sin(x), cos(y)), (x,-3,3), (y,-3,3),
        ....:     aspect_ratio=1)
sage: g3 = graphs.DodecahedralGraph().plot()
sage: g4 = polar_plot(sin(5*x)^2, (x, 0, 2*pi), color='green',
        ....:     fontsize=8) \
        ....:     + circle((0,0), 0.5, rgbcolor='red', fill=True, alpha=0.1,
        ....:     legend_label='pink')
sage: g4.set_legend_options(loc='upper right')
sage: G = graphics_array([[g1, g2], [g3, g4]])
sage: G
    Graphics Array of size 2 x 2
```

If one constructs the graphics array from a single list of graphics objects, one obtains a single-row array:

```
sage: G = graphics_array([g1, g2, g3, g4])
sage: G
    Graphics Array of size 1 x 4
```
We note that the overall aspect ratio of the figure is 4/3 (the default), which makes $g_1$ elongated, while the aspect ratio of $g_2$, which has been specified with the parameter `aspect_ratio=1` is preserved. To get a better aspect ratio for the whole figure, one can use the option `figsize` in the method `show()`:

```
sage: G.show(figsize=[8, 3])
```

We can access individual elements of the graphics array with the square-bracket operator:

```
sage: G = graphics_array([[$g_1$, $g_2$], [$g_3$, $g_4$]])  # back to the 2x2 array
sage: print(G)
Graphics Array of size 2 x 2
sage: G[0] is $g_1$
True
sage: G[1] is $g_2$
True
sage: G[2] is $g_3$
True
sage: G[3] is $g_4$
True
```

Note that with respect to the square-bracket operator, $G$ is considered as a flattened list of graphics objects, not as an array. For instance, $G[0, 1]$ throws an error:

```
sage: G[0, 1]
Traceback (most recent call last):
...
TypeError: list indices must be integers or slices, not tuple
```

$G[:]$ returns the full (flattened) list of graphics objects composing $G$:

```
sage: G[:]
[Graphics object consisting of 1 graphics primitive,
Graphics object consisting of 1 graphics primitive,
Graphics object consisting of 51 graphics primitives,
Graphics object consisting of 2 graphics primitives]
```

The total number of Graphics objects composing the array is returned by the function `len`:
sage: len(G)
4

The square-bracket operator can be used to replace elements in the array:

sage: G[0] = g4
sage: G
Graphics Array of size 2 x 2

 append(g)
Append a graphics to the array.
Currently not implemented.

ncols()
Number of columns of the graphics array.

EXAMPLES:

sage: R = rainbow(6)
sage: L = [plot(x^n, (x,0,1), color=R[n]) for n in range(6)]
sage: G = graphics_array(L, 2, 3)
sage: G.ncols()
3

(continues on next page)

(continued from previous page)

```python
sage: graphics_array(L).ncols()
6
```

**nrows()**

Number of rows of the graphics array.

**EXAMPLES:**

```python
sage: R = rainbow(6)
sage: L = [plot(x^n, (x,0,1), color=R[n]) for n in range(6)]
sage: G = graphics_array(L, 2, 3)
sage: G.nrows()
2
sage: graphics_array(L).nrows()
1
```

**position(index)**

Return the position and relative size of an element of *self* on the canvas.

**INPUT:**

- **index** – integer specifying which element of *self*

**OUTPUT:**

- a 4-tuple (*left*, *bottom*, *width*, *height*) giving the location and relative size of the element on the canvas, all quantities being expressed in fractions of the canvas width and height

**EXAMPLES:**

```python
sage: g1 = plot(sin(x), (x, -pi, pi))
sage: g2 = circle((0,1), 1.)
sage: G = graphics_array([g1, g2])
sage: G.position(0)  # tol 5.0e-3
(0.025045451349937315, 0.03415488992713045, 0.4489880779745068, 0.9345951100728696)
sage: G.position(1)  # tol 5.0e-3
(0.5170637412999687, 0.20212705964722733, 0.4489880779745068, 0.5986507706326758)
```

**class** `sage.plot.multigraphics.MultiGraphics(graphics_list)`

**Bases:** `sage.misc.fast_methods.WithEqualityById`, `sage.structure.sage_object.SageObject`

Base class for objects composed of *Graphics* objects.

Both the display and the output to a file of *MultiGraphics* objects are governed by the method `save()`, which is called by the rich output display manager, via `graphics_from_save()`.

The user interface is through the functions `multi_graphics()` (generic multi-graphics) and `graphics_array()` (subclass *GraphicsArray*).

**INPUT:**

- **graphics_list** – a list of graphics along with their positions on the common canvas; each element of `graphics_list` is either

5.2. Graphics arrays and insets 443
- a pair \((\text{graphics, position})\), where \text{graphics} is a \texttt{Graphics} object and \text{position} is the 4-tuple \((\text{left, bottom, width, height})\) specifying the location and size of the graphics on the canvas, all quantities being in fractions of the canvas width and height

- or a single \texttt{Graphics} object; its position is then assumed to occupy the whole canvas, except for some padding; this corresponds to the default position \((\text{left, bottom, width, height}) = (0.125, 0.11, 0.775, 0.77)\)

**EXAMPLES:**

A multi-graphics made from two graphics objects:

```
sage: g1 = plot(sin(x^3), (x, -pi, pi))
sage: g2 = circle((0,0), 1, color='red')
sage: G = multi_graphics([g1, (g2, (0.2, 0.55, 0.3, 0.3))])
sage: G
Multigraphics with 2 elements
```

Since no position was given for \(g1\), it occupies the whole canvas. Moreover, we note that \(g2\) has been drawn over \(g1\) with a white background. To have a transparent background instead, one has to construct \(g2\) with the keyword \texttt{transparent} set to \texttt{True}:

```
sage: g2 = circle((0,0), 1, color='red', transparent=True)
sage: G = multi_graphics([g1, (g2, (0.2, 0.55, 0.3, 0.3))])
```

(continues on next page)
We can add a new graphics object to G via the method `append()`:  

```
sage: g3 = complex_plot(zeta, (-20, 10), (-20, 20),
    ....:     axes_labels=['$x$', '$y$'], frame=True)
sage: G.append(g3, pos=(0.63, 0.12, 0.3, 0.3))
sage: G
```

Multigradics with 3 elements

We can access the individual elements composing G with the square-bracket operator:

```
sage: print(G[0])
Graphics object consisting of 1 graphics primitive
sage: G[0] is g1
True
sage: G[1] is g2
True
sage: G[2] is g3
True
```

G[:] returns the full list of graphics objects composing G:
append\((graphics, pos=None)\)

Append a graphics object to self.

INPUT:

- \(\text{graphics}\) – the graphics object (instance of \texttt{Graphics}) to be added to self
- \(\text{pos} = (\text{default: None})\) 4-tuple \((\text{left}, \text{bottom}, \text{width}, \text{height})\) specifying the location and size of graphics on the canvas, all quantities being in fractions of the canvas width and height; if None, graphics is assumed to occupy the whole canvas, except for some padding; this corresponds to the default position \((\text{left}, \text{bottom}, \text{width}, \text{height}) = (0.125, 0.11, 0.775, 0.77)\)

EXAMPLES:

Let us consider a multigraphics with 2 elements:

\begin{verbatim}
sage: g1 = plot(chebyshev_T(4, x), (x, -1, 1), title='n=4')
sage: g2 = plot(chebyshev_T(8, x), (x, -1, 1), title='n=8',
........: color='red')
sage: G = multi_graphics([(g1, (0.125, 0.2, 0.4, 0.4)),
........: (g2, (0.55, 0.4, 0.4, 0.4))])
sage: G
Multigraphics with 2 elements
\end{verbatim}

We append a third plot to it:

\begin{verbatim}
sage: g3 = plot(chebyshev_T(16, x), (x, -1, 1), title='n=16',
........: color='brown')
sage: G.append(g3, pos=(0.55, 0.11, 0.4, 0.15))
sage: G
Multigraphics with 3 elements
\end{verbatim}

We may use append to add a title:

\begin{verbatim}
sage: title = text("Chebyshev polynomials", (0, 0), fontsize=16,
........: axes=False)
sage: G.append(title, pos=(0.18, 0.8, 0.7, 0.1))
sage: G
Multigraphics with 4 elements
\end{verbatim}

See also:

\texttt{inset()}

inset\((graphics, pos=None, fontsize=None)\)

Add a graphics object as an inset.

INPUT:

- \(\text{graphics}\) – the graphics object (instance of \texttt{Graphics}) to be added as an inset
5.2. Graphics arrays and insets
Chebyshev polynomials

\[ n=4 \]

\[ n=8 \]

\[ n=16 \]
• **pos** – (default: None) 4-tuple (left, bottom, width, height) specifying the location and relative size of the inset on the canvas, all quantities being expressed in fractions of the canvas width and height; if None, the value (0.7, 0.7, 0.2, 0.2) is used.

• **fontsize** – (default: None) integer, font size (in points) for the inset; if None, the value of 6 points is used, unless fontsize has been explicitly set in the construction of graphics (in this case, it is not overwritten here).

**OUTPUT:**

• instance of `MultiGraphics`

**EXAMPLES:**

Let us consider a graphics array of 2 elements:

```python
sage: G = graphics_array([plot(sin, (0, 2*pi)),
                          plot(cos, (0, 2*pi))])
sage: G
Graphics Array of size 1 x 2
```

and add some inset at the default position:

```python
sage: c = circle((0,0), 1, color='red', thickness=2, frame=True)
sage: G.inset(c)
```

Multigraphics with 3 elements
We may customize the position and font size of the inset:

```
sage: G.inset(c, pos=(0.3, 0.7, 0.2, 0.2), fontsize=8)
```

Multigraphics with 3 elements

```
matplotlib(figure=None, figsize=None, **kwds)
```

Construct or modify a Matplotlib figure by drawing `self` on it.

**INPUT:**

- `figure` – (default: `None`) Matplotlib figure (class `matplotlib.figure.Figure`) on which `self` is to be displayed; if `None`, the figure will be created from the parameter `figsize`
- `figsize` – (default: `None`) width or [width, height] in inches of the Matplotlib figure in case `figure` is `None`; if `figsize` is `None`, Matplotlib’s default (6.4 x 4.8 inches) is used
- `kwds` – options passed to the `matplotlib()` method of each graphics object constituting `self`

**OUTPUT:**

- a `matplotlib.figure.Figure` object; if the argument `figure` is provided, this is the same object as `figure`.

**EXAMPLES:**

Let us consider a `GraphicsArray` object with 3 elements:
G = graphics_array([plot(sin(x^k), (x, 0, 3)) for k in range(1, 4)]

If `matplotlib()` is invoked without any argument, a Matplotlib figure is created and contains the 3 graphics element of the array as 3 Matplotlib Axes:

```
sage: fig = G.matplotlib()
sage: fig
<Figure size 640x480 with 3 Axes>
sage: type(fig)
<class 'matplotlib.figure.Figure'>
```

Specifying the figure size (in inches):

```
sage: G.matplotlib(figsize=(8., 5.))
<Figure size 800x500 with 3 Axes>
```

If a single number is provided for `figsize`, it is considered to be the width; the height is then computed according to Matplotlib’s default aspect ratio (4/3):

```
sage: G.matplotlib(figsize=8.)
<Figure size 800x600 with 3 Axes>
```

An example of use with a preexisting created figure, created by `pyplot`:

```
sage: import matplotlib.pyplot as plt
sage: fig1 = plt.figure(1)
sage: fig1
<Figure size 640x480 with 0 Axes>
sage: fig_out = G.matplotlib(figure=fig1)
sage: fig_out
<Figure size 640x480 with 3 Axes>
```

Note that the output figure is the same object as the input one:

```
sage: fig_out is fig1
True
```

It has however been modified by `G.matplotlib(figure=fig1)`, which has added 3 new Axes to it.

Another example, with a figure created from scratch, via Matplotlib’s `Figure`:

```
sage: from matplotlib.figure import Figure
sage: fig2 = Figure()
sage: fig2
<Figure size 640x480 with 0 Axes>
sage: G.matplotlib(figure=fig2)
<Figure size 640x480 with 3 Axes>
sage: fig2
<Figure size 640x480 with 3 Axes>
```

Return `self` since `self` is already a graphics object.

EXAMPLES:
sage: g1 = plot(cos, 0, 1)
sage: g2 = circle((0,0), 1)

sage: G = multi_graphics([g1, g2])

sage: G.plot() is G
True

**position(index)**

Return the position and relative size of an element of `self` on the canvas.

**INPUT:**
- `index` – integer specifying which element of `self`

**OUTPUT:**
- a 4-tuple `(left, bottom, width, height)` giving the location and relative size of the element on the canvas, all quantities being expressed in fractions of the canvas width and height

**EXAMPLES:**

```
sage: g1 = plot(sin(x^2), (x, 0, 4))
sage: g2 = circle((0,0), 1, rgbcolor='red', fill=True, axes=False)

sage: G = multi_graphics([g1, (g2, (0.15, 0.2, 0.1, 0.15))])

sage: G.position(0) # tol 1.0e-13
(0.125, 0.11, 0.775, 0.77)

sage: G.position(1) # tol 1.0e-13
(0.15, 0.2, 0.1, 0.15)
```

**save(filename, figsize=None, **kwds)**

Save `self` to a file, in various formats.

**INPUT:**
- `filename` – (string) the file name; the image format is given by the extension, which can be one of the following:
  - .eps,
  - .pdf,
  - .png,
  - .ps,
  - .sobj (for a Sage object you can load later),
  - .svg,
  - empty extension will be treated as .sobj.
- `figsize` – (default: None) width or [width, height] in inches of the Matplotlib figure; if none is provided, Matplotlib's default (6.4 x 4.8 inches) is used
- `kwds` – keyword arguments, like dpi=..., passed to the plotter, see `show()`

**EXAMPLES:**

```
sage: F = tmp_filename(ext='.png')
sage: L = [plot(sin(k*x), (x,-pi,pi)) for k in [1..3]]
sage: G = graphics_array(L)

sage: G.save(F, dpi=500, axes=False)
```
save_image(filename=None, *args, **kwds)

Save an image representation of self. The image type is determined by the extension of the filename. For example, this could be .png, .jpg, .gif, .pdf, .svg. Currently this is implemented by calling the save() method of self, passing along all arguments and keywords.

Note: Not all image types are necessarily implemented for all graphics types. See save() for more details.

EXAMPLES:

```python
sage: plots = [[plot(m*cos(x + n*pi/4), (x, 0, 2*pi))
....: for n in range(3)] for m in range(1,3)]
sage: G = graphics_array(plots)
sage: G.save_image(tmp_filename(ext='.png'))
```

show(**kwds)

Show self immediately.

This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

OPTIONAL INPUT:

- `dpi` – dots per inch
- `figsize` – width or [width, height] of the figure, in inches; the default is 6.4 x 4.8 inches
- `axes` – boolean; if True, all individual graphics are endowed with axes; if False, all axes are removed (this overrides the axes option set in each graphics)
- `frame` – boolean; if True, all individual graphics are drawn with a frame around them; if False, all frames are removed (this overrides the frame option set in each graphics)
- `fontsize` – positive integer, the size of fonts for the axes labels (this overrides the fontsize option set in each graphics)

OUTPUT:

This method does not return anything. Use save() if you want to save the figure as an image.

EXAMPLES:

This draws a graphics array with four trig plots and no axes in any of the plots and a figure width of 4 inches:

```python
sage: G = graphics_array([[plot(sin), plot(cos)],
....: [plot(tan), plot(sec)]])
sage: G.show(axes=False, figsize=4)
```

Same thing with a frame around each individual graphics:

```python
sage: G.show(axes=False, frame=True, figsize=4)
```

Actually, many options are possible; for instance, we may set fontsize and gridlines:

```python
sage: G.show(axes=False, frame=True, figsize=4, fontsize=8,
....: gridlines='major')
```
5.2. Graphics arrays and insets
5.3 Plotting primitives

class sage.plot.primitive.GraphicPrimitive(options)

Bases: sage.misc.fast_methods.WithEqualityById, sage.structure.sage_object.SageObject

Base class for graphics primitives, e.g., things that knows how to draw themselves in 2D.

EXAMPLES:
We create an object that derives from GraphicPrimitive:

```python
sage: P = line([(-1,-2), (3,5)])
sage: P[0]
Line defined by 2 points
sage: type(P[0])
<class 'sage.plot.line.Line'>
```

options()

Return the dictionary of options for this graphics primitive.

By default this function verifies that the options are all valid; if any aren’t, then a verbose message is printed with level 0.

EXAMPLES:

```python
sage: from sage.plot.primitive import GraphicPrimitive
sage: GraphicPrimitive({}).options()
{}
```

plot3d(**kwds)

Plots 3D version of 2D graphics object. Not implemented for base class.

EXAMPLES:

```python
sage: from sage.plot.primitive import GraphicPrimitive
sage: G=GraphicPrimitive({})
```
set_options(new_options)

Change the options to new_options.

EXAMPLES:

```python
sage: from sage.plot.circle import Circle
sage: c = Circle(0,0,1,{})
sage: c.set_options({'thickness': 0.6})
sage: c.options()
{'thickness': 0.6...}
```

set_zorder(zorder)

Set the layer in which to draw the object.

EXAMPLES:

```python
sage: P = line([(-2,-3), (3,4)], thickness=4)
sage: p=P[0]
sage: p.set_zorder(2)
sage: p.options()['zorder']
2
sage: Q = line([(-2,-4), (3,5)], thickness=4,zorder=1,hue=.5)
sage: P+Q  # blue line on top
Graphics object consisting of 2 graphics primitives
sage: q=Q[0]
sage: q.set_zorder(3)
sage: P+Q  # teal line on top
Graphics object consisting of 2 graphics primitives
```

class sage.plot.primitive.GraphicPrimitive_xydata(options)

Bases: sage.plot.primitive.GraphicPrimitive

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

```python
sage: d = polygon([[1,2], [5,6], [5,0]], rgbcolor=(1,0,1))[0].get_minmax_data()
sage: d['ymin']
0.0
sage: d['xmin']
1.0

sage: d = point((3, 3), rgbcolor=hue(0.75))[0].get_minmax_data()
sage: d['xmin']
3.0
sage: d['ymin']
3.0
```
5.4 Plotting utilities

sage.plot.misc.get_matplotlib_linestyle(linestyle, return_type)
Function which translates between matplotlib linestyle in short notation (i.e. ‘-’, ‘–’, ‘:’, ‘.’) and long notation (i.e. ‘solid’, ‘dashed’, ‘dotted’, ‘dashdot’).

If linestyle is none of these allowed options, the function raises a ValueError.

INPUT:

• linestyle - The style of the line, which is one of
  – "-" or "solid"
  – "--" or "dashed"
  – "-." or "dash dot"
  – ":" or "dotted"
  – "None" or " " or "" (nothing)
The linestyle can also be prefixed with a drawing style (e.g., "steps--")
  – "default" (connect the points with straight lines)
  – "steps" or "steps-pre" (step function; horizontal line is to the left of point)
  – "steps-mid" (step function; points are in the middle of horizontal lines)
  – "steps-post" (step function; horizontal line is to the right of point)

If linestyle is None (of type NoneType), then we return it back unmodified.

• return_type - The type of linestyle that should be output. This argument takes only two values - "long" or "short".

EXAMPLES:

Here is an example how to call this function:

sage: from sage.plot.misc import get_matplotlib_linestyle
sage: get_matplotlib_linestyle(':', return_type='short')
':'

sage: get_matplotlib_linestyle(':', return_type='long')
'dotted'

sage.plot.misc.setup_for_eval_on_grid(funcs, ranges, plot_points=None, return_vars=False)
Calculate the necessary parameters to construct a list of points, and make the functions fast_callable.

INPUT:

• funcs – a function, or a list, tuple, or vector of functions
• **ranges** – a list of ranges. A range can be a 2-tuple of numbers specifying the minimum and maximum, or a 3-tuple giving the variable explicitly.

• **plot_points** – a tuple of integers specifying the number of plot points for each range. If a single number is specified, it will be the value for all ranges. This defaults to 2.

• **return_vars** – (default False) If True, return the variables, in order.

**OUTPUT:**

• **fast_funcs** - if only one function passed, then a fast callable function. If funcs is a list or tuple, then a tuple of fast callable functions is returned.

• **range_specs** - a list of range_specs: for each range, a tuple is returned of the form (range_min, range_max, range_step) such that `srange(range_min, range_max, range_step, include_endpoint=True)` gives the correct points for evaluation.

**EXAMPLES:**

```sage
sage: x,y,z=var('x,y,z')
sage: f(x,y)=x+y-z
g(x,y)=x+y
h(y)=-y
sage: sage.plot.misc.setup_for_eval_on_grid(f, [(0, 2),(1,3),(-4,1)], plot_points=5)
((<sage.ext...>, [(0.0, 2.0, 0.5), (1.0, 3.0, 0.5), (-4.0, 1.0, 1.25)])
sage: sage.plot.misc.setup_for_eval_on_grid([g,h], [(0, 2),(-1,1)], plot_points=5)
((<sage.ext...>, <sage.ext...>), [(-1.0, 1.0, 0.5)])
sage: sage.plot.misc.setup_for_eval_on_grid([sin,cos], [(-1,1)], plot_points=9)
((<sage.ext...>, <sage.ext...>), [(-1.0, 1.0, 0.25)])
sage: sage.plot.misc.setup_for_eval_on_grid([lambda x: x^2,cos], [(-1,1)], plot_points=9)
((<function <lambda> ...>, <sage.ext...>), [(-1.0, 1.0, 0.25)])
sage: sage.plot.misc.setup_for_eval_on_grid([x+y], [(x,-1,1),(y,-1,1)], plot_points=[4,9])
((<sage.ext...>, [(x, -1.0, 1.0), (y, -1.0, 1.0)]), [(-2.0, 2.0, 4.0)])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,-1,1),(y,-1,1)], plot_points=[4,9,10])
Traceback (most recent call last):
  ... ValueError: plot_points must be either an integer or a list of integers, one for each range
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(1,-1),(y,-1,1)], plot_points=[4,9,10])
Traceback (most recent call last):
  ... ValueError: Some variable ranges specify variables while others do not
```

Beware typos: a comma which should be a period, for instance:

```sage
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x, 1, 2), (y, 0,1, 0.2)], plot_points=[4,9,10])
Traceback (most recent call last):
  ... ValueError: At least one variable range has more than 3 entries: each should either have 2 or 3 entries, with one of the forms (xmin, xmax) or (x, xmin, xmax)
```

(continues on next page)
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(y,1,-1),(x,-1,1)], plot_points=5)
(<sage.ext...>, [(1.0, -1.0, 0.5), (-1.0, 1.0, 0.5)])

sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,-1),(x,-1,1)], plot_points=5)
Traceback (most recent call last):
... ValueError: range variables should be distinct, but there are duplicates

sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,1),(y,-1,1)])
Traceback (most recent call last):
... ValueError: plot start point and end point must be different

sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,-1),(y,-1,1)], return_vars=True)
(<sage.ext...>, [(1.0, -1.0, 2.0), (-1.0, 1.0, 2.0)], [x, y])

sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(y,1,-1),(x,-1,1)], return_vars=True)
(<sage.ext...>, [(1.0, -1.0, 2.0), (-1.0, 1.0, 2.0)], [y, x])

sage.plot.misc.unify_arguments(funcs)
Return a tuple of variables of the functions, as well as the number of “free” variables (i.e., variables that defined in a callable function).

INPUT:
• funcs – a list of functions; these can be symbolic expressions, polynomials, etc

OUTPUT: functions, expected arguments
• A tuple of variables in the functions
• A tuple of variables that were “free” in the functions

EXAMPLES:

sage: x,y,z=var('x,y,z')
sage: f(x,y)=x+y-z
g(x,y)=x+y
h(y)=-y
sage: sage.plot.misc.unify_arguments((f,g,h))
((x, y, z), (z,))
sage: sage.plot.misc.unify_arguments((f,h))
((x, y), ())
sage: sage.plot.misc.unify_arguments((f,z))
((x, y, z), (z,))
sage: sage.plot.misc.unify_arguments((h,z))
((y, z), (z,))
sage: sage.plot.misc.unify_arguments((x+y,x-y))
((x, y), (x, y))
CHAPTER SIX

INDICES AND TABLES

• Index
• Module Index
• Search Page
g
sage.graphs.graph_plot, 287

p
sage.plot.animate, 165
sage.plot.arc, 329
sage.plot.arrow, 331
sage.plot.bar_chart, 285
sage.plot.bezier_path, 343
sage.plot.circle, 352
sage.plot.colors, 154
sage.plot.complex_plot, 177
sage.plot.contour_plot, 185
sage.plot.density_plot, 254
sage.plot.disk, 364
sage.plot.ellipse, 366
sage.plot.graphics, 411
sage.plot.histogram, 282
sage.plot.hyperbolic_arc, 400
sage.plot.hyperbolic_polygon, 401
sage.plot.hyperbolic_regular_polygon, 406
sage.plot.line, 373
sage.plot.matrix_plot, 321
sage.plot.misc, 460
sage.plot.multigraphics, 438
sage.plot.plot, 1
sage.plot.plot_field, 261
sage.plot.point, 378
sage.plot.polygon, 383
sage.plot.primitive, 458
sage.plot.scatter_plot, 281
sage.plot.step, 282
sage.plot.streamline_plot, 271
sage.plot.text, 141
INDEX

Symbols

_rich_repr_() (sage.plot.graphics.Graphics method), 411

A
adaptive_refinement() (in module sage.plot.plot), 27
add_frame() (sage.plot.animate.APngAssembler method), 167
add_primitive() (sage.plot.graphics.Graphics method), 412
animate() (in module sage.plot.animate), 175
Animation (class in sage.plot.animate), 167
APngAssembler (class in sage.plot.animate), 166
append() (sage.plot.multigraphics.GraphicsArray method), 442
append() (sage.plot.multigraphics.MultiGraphics method), 447
Arc (class in sage.plot.arc), 329
arc() (in module sage.plot.arc), 330
Arrow (class in sage.plot.arrow), 331
arrow() (in module sage.plot.arrow), 333
arrow2d() (in module sage.plot.arrow), 333
aspect_ratio() (sage.plot.graphics.Graphics method), 412
axes() (sage.plot.graphics.Graphics method), 412
axes_color() (sage.plot.graphics.Graphics method), 413
axes_label_color() (sage.plot.graphics.Graphics method), 413
axes_labels() (sage.plot.graphics.Graphics method), 414
axes_labels_size() (sage.plot.graphics.Graphics method), 414
axes_range() (sage.plot.graphics.Graphics method), 415
axes_width() (sage.plot.graphics.Graphics method), 415

B
bar_chart() (in module sage.plot.bar_chart), 285
BarChart (class in sage.plot.bar_chart), 285
bez_path() (in module sage.plot.bezier_path), 346
bez_path() (sage.plot.arc.Arc method), 329
BezierPath (class in sage.plot.bezier_path), 343
blend() (sage.plot.colors.Color method), 155

C
check_color_data() (in module sage.plot.colors), 158
Circle (class in sage.plot.circle), 352
circle() (in module sage.plot.circle), 354
Color (class in sage.plot.colors), 154
Colormaps (class in sage.plot.colors), 158
ColorsDict (class in sage.plot.colors), 158
density_plot() (in module sage.plot.density_plot), 186
description() (sage.plot.graphics.Graphics method), 415
disk() (class in sage.plot.disk), 364
disk() (in module sage.plot.disk), 365

D
darker() (sage.plot.colors.Color method), 155
density_plot() (in module sage.plot.density_plot), 255
DensityPlot (class in sage.plot.density_plot), 254
description() (sage.plot.graphics.Graphics method), 415
Disk (class in sage.plot.disk), 364

E
Ellipse (class in sage.plot.ellipse), 366
ellipse() (in module sage.plot.ellipse), 367
equify() (in module sage.plot.contour_plot), 221

F
ffmpeg() (sage.plot.animate.Animation method), 169
flip() (sage.plot.graphics.Graphics method), 416
float_to_html() (in module sage.plot.colors), 158
float_to_integer() (in module sage.plot.colors), 159
fontsize() (sage.plot.graphics.Graphics method), 416
G

generate_plot_points() (in module sage.plot.plot), 27
get_axes_range() (sage.plot.graphics.Graphics method), 417
get_cmap() (in module sage.plot.colors), 159
get_matplotlib_linestyle() (in module sage.plot.misc), 460
get_minmax_data() (sage.plot.arc.Arc method), 330
graphs() (in module sage.plot.graphics.Graphics), 411

H

Histogram (class in sage.plot.histogram), 282
histogram() (in module sage.plot.histogram), 283
hls() (sage.plot.colors.Color method), 156
hsl() (sage.plot.colors.Color method), 156
hsv() (sage.plot.colors.Color method), 156
html_color() (sage.plot.colors.Color method), 157
html_to_float() (in module sage.plot.colors), 160
hue() (in module sage.plot.colors), 160
hyperbolic_arc() (in module sage.plot.hyperbolic_arc), 400
hyperbolic_polygon() (in module sage.plot.hyperbolic_polygon), 401
hyperbolic_regular_polygon() (in module sage.plot.hyperbolic_regular_polygon), 406
hyperbolic_triangle() (in module sage.plot.hyperbolic_polygon), 402
HyperbolicArc (class in sage.plot.hyperbolic_arc), 400
HyperbolicPolygon (class in sage.plot.hyperbolic_polygon), 401
HyperbolicRegularPolygon (class in sage.plot.hyperbolic_regular_polygon), 406

I

implicit_plot() (in module sage.plot.contour_plot), 224
inset() (sage.plot.graphics.Graphics method), 418
inset() (sage.plot.multigraphics.MultiGraphics method), 447
interactive() (sage.plot.animate.Animation method), 171
is_Graphics() (in module sage.plot.graphics), 437

L

layout_tree() (sage.graphs.graph_plot.GraphPlot method), 292
legend() (sage.plot.graphics.Graphics method), 418
lighter() (sage.plot.colors.Color method), 418
Line (class in sage.plot.line), 373
line() (in module sage.plot.line), 374
line2d() (in module sage.plot.line), 374
list_plot() (in module sage.plot.plot), 31
list_plot_loglog() (in module sage.plot.plot), 45
list_plot_semilogx() (in module sage.plot.plot), 47
list_plot_semilogy() (in module sage.plot.plot), 49
load_maps() (sage.plot.colors.Colormaps method), 158

M

make_image() (sage.plot.animate.Animation method), 172
matplotlib() (sage.plot.graphics.Graphics method), 422

468 Index
matplotlib() (sage.plot.multigraphics.MultiGraphics method), 453
matrix_plot() (in module sage.plot.matrix_plot), 322
MatrixPlot (class in sage.plot.matrix_plot), 321
minmax_data() (in module sage.plot.plot), 51
mod_one() (in module sage.plot.colors), 161
module
  sage.graphs.graph_plot, 287
  sage.plot.animate, 165
  sage.plot.arc, 329
  sage.plot.arrow, 331
  sage.plot.bar_chart, 285
  sage.plot.bezier_path, 352
  sage.plot.colors, 154
  sage.plot.complex_plot, 177
  sage.plot.contour_plot, 185
  sage.plot.density_plot, 254
  sage.plot.disk, 364
  sage.plot.ellipse, 366
  sage.plot.graphics, 411
  sage.plot.histogram, 282
  sage.plot.hyperbolic_arc, 400
  sage.plot.hyperbolic_polygon, 401
  sage.plot.hyperbolic_regular_polygon, 406
  sage.plot.line, 373
  sage.plot.matrix_plot, 321
  sage.plot.misc, 460
  sage.plot.multigraphics, 438
  sage.plot.plot, 1
  sage.plot.plot_field, 261
  sage.plot.point, 378
  sage.plot.polygon, 383
  sage.plot.primitive, 458
  sage.plot.scatter_plot, 281
  sage.plot.step, 282
  sage.plot.streamline_plot, 271
  sage.plot.text, 141
multi_graphics() (in module sage.plot.plot), 52
MultiGraphics (class in sage.plot.multigraphics), 443
plot() (sage.graphs.graph_plot.GraphPlot method), 292
plot() (sage.plot.graphics.Graphics method), 422
plot() (sage.plot.multigraphics.MultiGraphics method), 454
plot3d() (sage.plot.arrow.Arrow method), 332
plot3d() (sage.plot.bezier_path.BezierPath method), 345
plot3d() (sage.plot.circle.Circle method), 352
plot3d() (sage.plot.disk.Disk method), 364
plot3d() (sage.plot.ellipse.Ellipse method), 367
plot3d() (sage.plot.graphics.Graphics method), 423
plot3d() (sage.plot.line.Line method), 374
plot3d() (sage.plot.point.Point method), 378
plot3d() (sage.plot.polygon.Polygon method), 383
plot3d() (sage.plot.primitive.GraphicPrimitive method), 458
plot3d() (sage.plot.text.Text method), 141
plot_loglog() (in module sage.plot.plot), 123
plot_semilogx() (in module sage.plot.plot), 126
plot_semilogy() (in module sage.plot.plot), 130
plot_slope_field() (in module sage.plot.plot_field), 263
plot_step_function() (in module sage.plot.plot), 282
plot_vector_field() (in module sage.plot.plot_field), 264
PlotField (class in sage.plot.plot_field), 261
png() (sage.plot.animate.Animation method), 172
Point (class in sage.plot.point), 378
point() (in module sage.plot.point), 379
point2d() (in module sage.plot.point), 380
points() (in module sage.plot.point), 382
polar_plot() (in module sage.plot.plot), 130
Polygon (class in sage.plot.polygon), 383
polygon() (in module sage.plot.polygon), 383
polygon2d() (in module sage.plot.polygon), 385
position() (sage.plot.multigraphics.GraphicsArray method), 443
position() (sage.plot.multigraphics.MultiGraphics method), 455

N
ncols() (sage.plot.multigraphics.GraphicsArray method), 442
nrows() (sage.plot.multigraphics.GraphicsArray method), 443

O
options() (sage.plot.primitive.GraphicPrimitive method), 458

P
parametric_plot() (in module sage.plot.plot), 52
plot() (in module sage.plot.plot), 60

R
rainbow() (in module sage.plot.colors), 162
reshape() (in module sage.plot.plot), 140
rgb() (sage.plot.colors.Color method), 157
rgbcolor() (in module sage.plot.colors), 162

S
sage.graphs.graph_plot module, 287
sage.plot.animate module, 165
sage.plot.arc module, 329
sage.plot.arrow
module, 331
sage.plot.bar_chart
module, 285
sage.plot.bezier_path
module, 343
sage.plot.circle
module, 352
sage.plot.colors
module, 154
sage.plot.complex_plot
module, 177
sage.plot.contour_plot
module, 185
sage.plot.density_plot
module, 254
sage.plot.disk
module, 364
sage.plot.ellipse
module, 366
sage.plot.graphics
module, 411
sage.plot.histogram
module, 282
sage.plot.hyperbolic_arc
module, 400
sage.plot.hyperbolic_polygon
module, 401
sage.plot.hyperbolic_regular_polygon
module, 406
sage.plot.line
module, 373
sage.plot.matrix_plot
module, 321
sage.plot.misc
module, 460
sage.plot.multigraphics
module, 438
sage.plot.plot
module, 1
sage.plot.plot_field
module, 261
sage.plot.point
module, 378
sage.plot.polygon
module, 383
sage.plot.primitive
module, 458
sage.plot.scatter_plot
module, 281
sage.plot.step
module, 282
sage.plot.streamline_plot
module, 271
sage.plot.text
module, 141
save() (sage.plot.animate.Animation method), 173
save() (sage.plot.graphics.Graphics method), 423
save() (sage.plot.multigraphics.MultiGraphics method), 455
save_image() (sage.plot.graphics.Graphics method), 424
save_image() (sage.plot.multigraphics.MultiGraphics method), 455
scatter_plot() (in module sage.plot.scatter_plot), 281
ScatterPlot (class in sage.plot.scatter_plot), 281
SelectiveFormatter() (in module sage.plot.plot), 27
set_aspect_ratio() (sage.plot.graphics.Graphics method), 424
set_axes_range() (sage.plot.graphics.Graphics method), 425
set_default() (sage.plot.animate.APngAssembler method), 167
set_edges() (sage.graphs.graph_plot.GraphPlot method), 308
set_flip() (sage.plot.graphics.Graphics method), 425
set_legend_options() (sage.plot.graphics.Graphics method), 426
set_options() (sage.plot.primitive.GraphicPrimitive method), 459
set_pos() (sage.graphs.graph_plot.GraphPlot method), 312
set_vertices() (sage.graphs.graph_plot.GraphPlot method), 314
set_zorder() (sage.plot.primitive.GraphicPrimitive method), 459
setup_for_eval_on_grid() (in module sage.plot.misc), 460
show() (sage.graphs.graph_plot.GraphPlot method), 319
show() (sage.plot.animate.Animation method), 173
show() (sage.plot.graphics.Graphics method), 427
show() (sage.plot.multigraphics.MultiGraphics method), 456
streamline_plot() (in module sage.plot.streamline_plot), 273
StreamlinePlot (class in sage.plot.streamline_plot), 271

T
Text (class in sage.plot.text), 141
text() (in module sage.plot.text), 141
tick_label_color() (sage.plot.graphics.Graphics method), 436
to_float_list() (in module sage.plot.plot), 141
to_mpl_color() (in module sage.plot.colors), 163

U
unify_arguments() (in module sage.plot.misc), 462
X
xmax() (sage.plot.graphics.Graphics method), 437
xmin() (sage.plot.graphics.Graphics method), 437
xydata_from_point_list() (in module sage.plot.plot), 141

Y
ymax() (sage.plot.graphics.Graphics method), 437
ymin() (sage.plot.graphics.Graphics method), 437