2D Graphics

*Release 10.1*

The Sage Development Team

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1.1 2D plotting

Sage provides extensive 2D plotting functionality. The underlying rendering is done using the matplotlib Python library.

The following graphics primitives are supported:

- `arrow()` - an arrow from a min point to a max point.
- `circle()` - a circle with given radius
- `ellipse()` - an ellipse with given radii and angle
- `arc()` - an arc of a circle or an ellipse
- `disk()` - a filled disk (i.e. a sector or wedge of a circle)
- `line()` - a line determined by a sequence of points (this need not be straight!)
- `point()` - a point
- `text()` - some text
- `polygon()` - a filled polygon

The following plotting functions are supported:

- `plot()` - plot of a function or other Sage object (e.g., elliptic curve).
- `parametric_plot()`
- `implicit_plot()`
- `polar_plot()`
- `region_plot()`
- `list_plot()`
- `scatter_plot()`
- `bar_chart()`
- `contour_plot()`
- `density_plot()`
- `plot_vector_field()`
- `plot_slope_field()`
- `matrix_plot()`
- `complex_plot()`
• `graphics_array()`
• `multi_graphics()`

• The following log plotting functions:
  – `plot_loglog()`
  – `plot_semilogx()` and `plot_semilogy()`
  – `list_plot_loglog()`
  – `list_plot_semilogx()` and `list_plot_semilogy()`

The following miscellaneous Graphics functions are included:

• `Graphics()`
• `is_Graphics()`
• `hue()`

Type `?` after each primitive in Sage for help and examples.

**EXAMPLES:**

We draw a curve:

```sage
sage: plot(x^2, (x,0,5))
Graphics object consisting of 1 graphics primitive
```

![Graph of a curve](image)
We draw a circle and a curve:

```
sage: circle((1,1), 1) + plot(x^2, (x,0,5))
Graphics object consisting of 2 graphics primitives
```

Notice that the aspect ratio of the above plot makes the plot very tall because the plot adopts the default aspect ratio of the circle (to make the circle appear like a circle). We can change the aspect ratio to be what we normally expect for a plot by explicitly asking for an ‘automatic’ aspect ratio:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), aspect_ratio='automatic')
```

The aspect ratio describes the apparently height/width ratio of a unit square. If you want the vertical units to be twice as big as the horizontal units, specify an aspect ratio of 2:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), aspect_ratio=2)
```

The figsize option adjusts the figure size. The default figsize is 4. To make a figure that is roughly twice as big, use figsize=8:

```
sage: show(circle((1,1), 1) + plot(x^2, (x,0,5)), figsize=8)
```

You can also give separate horizontal and vertical dimensions. Both will be measured in inches:
However, do not make the figsize too big (e.g. one dimension greater than 327 or both in the mid-200s) as this will lead to errors or crashes. See `show()` for full details.

Note that the axes will not cross if the data is not on both sides of both axes, even if it is quite close:

```python
sage: plot(x^3, (x,1,10))
```

When the labels have quite different orders of magnitude or are very large, scientific notation (the $e$ notation for powers of ten) is used:

```python
sage: plot(x^2, (x,480,500))  # no scientific notation
```

```python
sage: plot(x^2, (x,300,500))  # scientific notation on y-axis
```

But you can fix your own tick labels, if you know what to expect and have a preference:

```python
sage: plot(x^2, (x,300,500), ticks=[100,50000])
```
1.1. 2D plotting
To change the ticks on one axis only, use the following notation:

```
sage: plot(x^2, (x,300,500), ticks=[None,50000])
Graphics object consisting of 1 graphics primitive
```

You can even have custom tick labels along with custom positioning.

```
sage: plot(x^2, (x,0,3), ticks=[[1,2.5],pi/2], tick_formatter=[["x_1","x_2"],pi])  # long time
Graphics object consisting of 1 graphics primitive
```

We construct a plot involving several graphics objects:

```
sage: G = plot(cos(x), (x, -5, 5), thickness=5, color='green', title='A plot')
sage: P = polygon([[1,2], [5,6], [5,0]], color='red')
sage: G + P
Graphics object consisting of 2 graphics primitives
```

Next we construct the reflection of the above polygon about the \( y \)-axis by iterating over the list of first-coordinates of the first graphic element of \( P \) (which is the actual Polygon; note that \( P \) is a Graphics object, which consists of a single polygon):

```
sage: Q = polygon([(-x,y) for x,y in P[0]], color='blue')
sage: Q  # show it
```

(continues on next page)
1.1. 2D plotting
We combine together different graphics objects using “+”:

```
sage: H = G + P + Q
sage: print(H)
Graphics object consisting of 3 graphics primitives
sage: type(H)
<class 'sage.plot.graphics.Graphics'>
sage: H[1]
 Polygon defined by 3 points
sage: list(H[1])
[(1.0, 2.0), (5.0, 6.0), (5.0, 0.0)]
sage: H  # show it
Graphics object consisting of 3 graphics primitives
```

We can put text in a graph:

```
sage: L = [(cos(pi*i/100)^3, sin(pi*i/100)) for i in range(200)]
sage: p = line(L, rgbcolor=(1/4,1/8,3/4))
sage: tt = text('A Bulb', (1.5, 0.25))
sage: tx = text('x axis', (1.5,-0.2))
```

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```
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Graphics object consisting of 3 graphics primitives
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 Polygon defined by 3 points
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[(1.0, 2.0), (5.0, 6.0), (5.0, 0.0)]
sage: H  # show it
Graphics object consisting of 3 graphics primitives
```

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sage: tt = text('A Bulb', (1.5, 0.25))
sage: tx = text('x axis', (1.5,-0.2))
```
We can add a graphics object to another one as an inset:

```python
sage: g1 = plot(x^2*sin(1/x), (x, -2, 2), axes_labels=['$x$', '$y$'])
sage: g2 = plot(x^2*sin(1/x), (x, -0.3, 0.3), axes_labels=['$x$', '$y$'],
....: frame=True)
sage: g1.inset(g2, pos=(0.15, 0.7, 0.25, 0.25))
```

We can add a title to a graph:

```python
sage: plot(x^2, (x, -2, 2), title='A plot of $x^2$')
```

We can set the position of the title:

```python
sage: plot(x^2, (-2, 2), title='Plot of $x^2$', title_pos=(0.5, -0.05))
```

We plot the Riemann zeta function along the critical line and see the first few zeros:
A plot of $x^2$
Plot of $x^2$
```

```sage: i = CDF.0  # define i this way for maximum speed.
sage: p1 = plot(lambda t: arg(zeta(0.5+t*i)), 1, 27, rgbcolor=(0.8,0,0))
sage: p2 = plot(lambda t: abs(zeta(0.5+t*i)), 1, 27, color=hue(0.7))
sage: print(p1 + p2)
Graphics object consisting of 2 graphics primitives
sage: p1 + p2  # display it
Graphics object consisting of 2 graphics primitives

```

**Note:** Not all functions in Sage are symbolic. When plotting non-symbolic functions they should be wrapped in `lambda`:

```
```sage: plot(lambda x:fibonacci(round(x)), (x,1,10))
Graphics object consisting of 1 graphics primitive

```

Many concentric circles shrinking toward the origin:

```
```sage: show(sum(circle((i,0), i, hue=sin(i/10)) for i in [10,9,9,...,0]))  # long time

```

Here is a pretty graph:
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sage: g = Graphics()
sage: for i in range(60):
    ...:     p = polygon([(i*cos(i),i*sin(i)), (0,i), (i,0)],
    ...:                 color=hue(i/40+0.4), alpha=0.2)
    ...:     g = g + p
sage: g.show(dpi=200, axes=False)

Another graph:

sage: x = var('x')
sage: P = plot(sin(x)/x, -4, 4, color='blue') + 
    ...:     plot(x*cos(x), -4, 4, color='red') + 
    ...:     plot(tan(x), -4, 4, color='green')
sage: P.show(ymin=-pi, ymax=pi)

PYX EXAMPLES: These are some examples of plots similar to some of the plots in the PyX (http://pyx.sourceforge.net) documentation:

Symbloline:

sage: y(x) = x*sin(x^2)
sage: v = [(x, y(x)) for x in [-3,-2.95,..,3]]
sage: show(points(v, rgbcolor=(0.2,0.6, 0.1), pointsize=30) + plot(spline(v), -3.1, 3))
1.1. 2D plotting
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Cycliclink:
sage: g1 = plot(cos(20*x)*exp(-2*x), 0, 1)
sage: g2 = plot(2*exp(-30*x) - exp(-3*x), 0, 1)
sage: show(graphics_array([g1, g2], 2, 1))

1.0
0.5
0.2

0.4

0.6

0.8

1.0

0.2

0.4

0.6

0.8

1.0

0.5
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
Pi Axis:
sage: g1 = plot(sin(x), 0, 2*pi)
sage: g2 = plot(cos(x), 0, 2*pi, linestyle="--")
sage: (g1 + g2).show(ticks=pi/6,
# show their sum, nicely formatted
....:
tick_formatter=pi)

# long time

An illustration of integration:
sage: f(x) = (x-3)*(x-5)*(x-7)+40
sage: P = line([(2,0),(2,f(2))], color='black')
sage: P += line([(8,0),(8,f(8))], color='black')
sage: P += polygon([(2,0),(2,f(2))] + [(x, f(x)) for x in [2,2.1,..,8]] + [(8,0),(2,0)],
....:
rgbcolor=(0.8,0.8,0.8), aspect_ratio='automatic')
sage: P += text("$\\int_{a}^b f(x) dx$", (5, 20), fontsize=16, color='black')
sage: P += plot(f, (1, 8.5), thickness=3)
sage: P
# show the result
Graphics object consisting of 5 graphics primitives

1.1. 2D plotting

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\[ \int_{a}^{b} f(x) \, dx \]
NUMERICAL PLOTTING:

Sage includes Matplotlib, which provides 2D plotting with an interface that is a likely very familiar to people doing numerical computation. You can use `plt.clf()` to clear the current image frame and `plt.close()` to close it. For example,

```python
sage: import pylab as plt
sage: t = plt.arange(0.0, 2.0, 0.01)
sage: s = sin(2*pi*t)
sage: P = plt.plot(t, s, linewidth=1.0)
sage: xl = plt.xlabel('time (s)')
sage: yl = plt.ylabel('voltage (mV)')
sage: t = plt.title('About as simple as it gets, folks')
sage: plt.grid(True)
```

We test that `imshow` works as well, verifying that github issue #2900 is fixed (in Matplotlib).

```python
sage: plt.imshow([[1,2],[0,1]])
<matplotlib.image.AxesImage object at ...>
sage: plt.imshow([[0.0,0.0,0.0]])
<matplotlib.image.AxesImage object at ...>
```

Since the above overwrites many Sage plotting functions, we reset the state of Sage, so that the examples below work!

```python
sage: reset()
```

See [http://matplotlib.sourceforge.net](http://matplotlib.sourceforge.net) for complete documentation about how to use Matplotlib.

AUTHORS:

- Alex Clemesha and William Stein (2006-04-10): initial version
- David Joyner: examples
- Alex Clemesha (2006-05-04) major update
- William Stein (2006-05-29): fine tuning, bug fixes, better server integration
- Alex Clemesha (2006-09-29): added contour_plot, frame axes, misc polishing
- Bobby Moretti and William Stein (2008-01): Change plot to specify ranges using the (varname, min, max) notation.
• William Stein (2008-01-19): raised the documentation coverage from a miserable 12 percent to a ‘wopping’ 35 percent, and fixed and clarified numerous small issues.

• Jason Grout (2009-09-05): shifted axes and grid functionality over to matplotlib; fixed a number of smaller issues.

• Jason Grout (2010-10): rewrote aspect ratio portions of the code

• Jeroen Demeyer (2012-04-19): move parts of this file to graphics.py (github issue #12857)

• Aaron Lauve (2016-07-13): reworked handling of ‘color’ when passed a list of functions; now more in-line with other CAS’s. Added list functionality to linestyle and legend_label options as well. (github issue #12962)

• Eric Gourgoulhon (2019-04-24): add multi_graphics() and insets

sage.plot.plot.SelectiveFormatter(formatter, skip_values)

This matplotlib formatter selectively omits some tick values and passes the rest on to a specified formatter.

EXAMPLES:

This example is almost straight from a matplotlib example.

```
sage: # needs numpy
sage: from sage.plot.plot import SelectiveFormatter
sage: import matplotlib.pyplot as plt
sage: import numpy
sage: fig = plt.figure()
sage: ax = fig.add_subplot(111)
sage: t = numpy.arange(0.0, 2.0, 0.01)
sage: s = numpy.sin(2*numpy.pi*t)
sage: p = ax.plot(t, s)
sage: formatter = SelectiveFormatter(ax.xaxis.get_major_formatter(),
... skip_values=[0,1])
sage: ax.xaxis.set_major_formatter(formatter)
```

sage.plot.plot.adaptive_refinement(f, p1, p2, adaptive_tolerance, adaptive_recursion=0.01, level=5, excluded=0)

The adaptive refinement algorithm for plotting a function $f$. See the docstring for plot for a description of the algorithm.

INPUT:

• $f$ – a function of one variable
• $p1$, $p2$ – two points to refine between
• adaptive_recursion – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.
• adaptive_tolerance – (default: 0.01); how large a relative difference should be before the adaptive refinement code considers it significant; see documentation for generate_plot_points for more information. See the documentation for plot() for more information on how the adaptive refinement algorithm works.
• excluded – (default: False); also return locations where it has been discovered that the function is not defined (y-value will be 'NaN' in this case)

OUTPUT:
A list of points to insert between $p_1$ and $p_2$ to get a better linear approximation between them. If excluded, also x-values for which the calculation failed are given with 'NaN' as y-value.

`sage.plot.plot.generate_plot_points(f, xrange, plot_points, adaptive_tolerance, adaptive_recursion=5, randomize=0.01, initial_points=5, excluded=True, imaginary_tolerance=None)`

Calculate plot points for a function $f$ in the interval $xrange$. The adaptive refinement algorithm is also automatically invoked with a relative adaptive tolerance of adaptive_tolerance; see below.

**INPUT:**

- $f$ – a function of one variable
- $p_1$, $p_2$ – two points to refine between
- `plot_points` – (default: 5); the minimal number of plot points. (Note however that in any actual plot a number is passed to this, with default value 200.)
- `adaptive_recursion` – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.
- `adaptive_tolerance` – (default: 0.01); how large the relative difference should be before the adaptive refinement code considers it significant. If the actual difference is greater than adaptive_tolerance*delta, where delta is the initial subinterval size for the given xrange and plot_points, then the algorithm will consider it significant.
- `initial_points` – (default: None); a list of x-values that should be evaluated.
- `excluded` – (default: False); add a list of discovered x-values, for which $f$ is not defined
- `imaginary_tolerance` – (default: 1e-8); if an imaginary number arises (due, for example, to numerical issues), this tolerance specifies how large it has to be in magnitude before we raise an error. In other words, imaginary parts smaller than this are ignored in your plot points.

**OUTPUT:**

- a list of points (x, f(x)) in the interval $xrange$, which approximate the function $f$.
- if `excluded` a tuple consisting of the above and a list of x-values at which $f$ is not defined

`sage.plot.plot.graphics_array(array, nrows=None, ncols=None)`

Plot a list of lists (or tuples) of graphics objects on one canvas, arranged as an array.

**INPUT:**

- `array` – either a list of lists of Graphics elements or a single list of Graphics elements
- `nrows`, `ncols` – (optional) integers. If both are given then the input array is flattened and turned into an `nrows x ncols` array, with blank graphics objects padded at the end, if necessary. If only one is specified, the other is chosen automatically.

**OUTPUT:** an instance of GraphicsArray

**EXAMPLES:**

Make some plots of sin functions:

```
sage: # long time
sage: f(x) = sin(x)
sage: g(x) = sin(2*x)
sage: h(x) = sin(4*x)
sage: p1 = plot(f, (-2*pi,2*pi), color=hue(0.5))
sage: p2 = plot(g, (-2*pi,2*pi), color=hue(0.9))
```

(continues on next page)
Now make a graphics array out of the plots:

```sage
sage: graphics_array(((p1,p2), (p3,p4)))
# long time
Graphics Array of size 2 x 2
```

One can also name the array, and then use `show()` or `save()`:

```sage
sage: ga = graphics_array(((p1,p2), (p3,p4)))  # long time
sage: ga.show()  # long time; same output as above
```

Here we give only one row:

```sage
sage: p1 = plot(sin, (-4,4))
sage: p2 = plot(cos, (-4,4))
sage: ga = graphics_array([p1, p2]); ga
Graphics Array of size 1 x 2
sage: ga.show()
```

It is possible to use `figsize` to change the size of the plot as a whole:
```python
sage: L = [plot(sin(k*x), (x,-pi,pi)) for k in [1..3]]
sage: ga = graphics_array(L)
sage: ga.show(figsize=[5,3])  # smallish and compact
```

```
1.0
1.0
1.0

-1.0
-0.5
0.5
1.0

sage: ga.show(figsize=[5,7])  # tall and thin; long time
```

```
sage: ga.show(figsize=4)  # width=4 inches, height fixed from default aspect ratio
```

Specifying only the number of rows or the number of columns computes the other dimension automatically:

```python
sage: ga = graphics_array([plot(sin)] * 10, nrows=3)
sage: ga.nrows(), ga.ncols()
(3, 4)
sage: ga = graphics_array([plot(sin)] * 10, ncols=3)
sage: ga.nrows(), ga.ncols()
(4, 3)
sage: ga = graphics_array([plot(sin)] * 4, nrows=2)
sage: ga.nrows(), ga.ncols()
(2, 2)
sage: ga = graphics_array([plot(sin)] * 6, ncols=2)
sage: ga.nrows(), ga.ncols()
(3, 2)
```

The options like fontsize, scale or frame passed to individual plots are preserved:

```python
sage: p1 = plot(sin(x^2), (x, 0, 6),  
.....:    axes_labels=[r'$\theta$', r'$\sin(\theta^2)$'], fontsize=16)
sage: p2 = plot(x^3, (x, 1, 100), axes_labels=[r'$x$', r'$y$'],  
.....:    scale='semilogy', frame=True, gridlines='minor')
sage: ga = graphics_array([p1, p2])
sage: ga.show()
```

See also:

`GraphicsArray` for more examples
\[ \sin(\theta^2) \]
sage.plot.plot.list_plot(data, plotjoined=False, aspect_ratio='automatic', **kwargs)

list_plot takes either a list of numbers, a list of tuples, a numpy array, or a dictionary and plots the corresponding points.

If given a list of numbers (that is, not a list of tuples or lists), list_plot forms a list of tuples \((i, x_i)\) where \(i\) goes from 0 to \(\text{len}(data)-1\) and \(x_i\) is the \(i\)-th data value, and puts points at those tuple values.

list_plot will plot a list of complex numbers in the obvious way; any numbers for which \(\text{CC}()\) makes sense will work.

list_plot also takes a list of tuples \((x_i, y_i)\) where \(x_i\) and \(y_i\) are the \(i\)-th values representing the \(x\)- and \(y\)-values, respectively.

If given a dictionary, list_plot interprets the keys as \(x\)-values and the values as \(y\)-values.

The plotjoined=True option tells list_plot to plot a line joining all the data.

For other keyword options that the list_plot function can take, refer to plot().

It is possible to pass empty dictionaries, lists, or tuples to list_plot. Doing so will plot nothing (returning an empty plot).

EXAMPLES:

```
sage: list_plot([i^2 for i in range(5)])  # long time
Graphics object consisting of 1 graphics primitive
```

Here are a bunch of random red points:
sage: r = [(random(), random()) for _ in range(20)]
sage: list_plot(r, color='red')
Graphics object consisting of 1 graphics primitive

This gives all the random points joined in a purple line:

sage: list_plot(r, plotjoined=True, color='purple')
Graphics object consisting of 1 graphics primitive

You can provide a numpy array:

sage: import numpy

# needs numpy
sage: list_plot(numpy.arange(10))
# needs numpy
Graphics object consisting of 1 graphics primitive

sage: list_plot(numpy.array([[1, 2], [2, 3], [3, 4]]))
# needs numpy
Graphics object consisting of 1 graphics primitive

Plot a list of complex numbers:
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sage: list_plot([1, I, pi + I/2, CC(.25, .25)])
Graphics object consisting of 1 graphics primitive

Note that if your list of complex numbers are all actually real, they get plotted as real values, so this

sage: list_plot([CDF(1), CDF(1/2), CDF(1/3)])
Graphics object consisting of 1 graphics primitive

is the same as list_plot([1, 1/2, 1/3]) – it produces a plot of the points (0, 1), (1, 1/2), and (2, 1/3).

If you have separate lists of \(x\) values and \(y\) values which you want to plot against each other, use the \texttt{zip} command to make a single list whose entries are pairs of \((x, y)\) values, and feed the result into \texttt{list_plot}:

sage: x_coords = [cos(t)^3 for t in srange(0, 2*pi, 0.02)]
sage: y_coords = [sin(t)^3 for t in srange(0, 2*pi, 0.02)]
sage: list_plot(list(zip(x_coords, y_coords)))
Graphics object consisting of 1 graphics primitive

If instead you try to pass the two lists as separate arguments, you will get an error message:
1.1. 2D plotting
Dictionaries with numeric keys and values can be plotted:

```sage
```

Graphics object consisting of 1 graphics primitive

Plotting in logarithmic scale is possible for 2D list plots. There are two different syntaxes available:

```sage
# long time # log axis on vertical
list_plot_semilogy(yl)  # same
```

Graphics object consisting of 1 graphics primitive
**Warning:** If `plotjoined` is `False` then the axis that is in log scale must have all points strictly positive. For instance, the following plot will show no points in the figure since the points in the horizontal axis starts from \((0, 1)\). Further, matplotlib will display a user warning.

```python
sage: list_plot(yl, scale='loglog') # both axes are log
doctest:warning
...
Graphics object consisting of 1 graphics primitive
```

Instead this will work. We drop the point \((0, 1)\):

```python
sage: list_plot(list(zip(range(1, len(yl)), yl[1:])), scale='loglog') # long time
Graphics object consisting of 1 graphics primitive
```

We use `list_plot_loglog()` and plot in a different base:

```python
sage: list_plot_loglog(list(zip(range(1, len(yl)), yl[1:])), base=2) # long time
Graphics object consisting of 1 graphics primitive
```

We can also change the scale of the axes in the graphics just before displaying:

```python
sage: G = list_plot(yl) # long time
sage: G.show(scale=('semilogy', 2)) # long time
```
sage.plot.plot.list_plot_loglog(data, plotjoined=False, base=10, **kwds)

Plot the data in ‘loglog’ scale, that is, both the horizontal and the vertical axes will be in logarithmic scale.

**INPUT:**

- `base` – (default: 10); the base of the logarithm. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.

For all other inputs, look at the documentation of `list_plot()`.

**EXAMPLES:**

```python
sage: yl = [5**k for k in range(10)]; xl = [2**k for k in range(10)]
sage: list_plot_loglog(list(zip(xl, yl))) # use loglog scale with base 10
```

Graphics object consisting of 1 graphics primitive

```python
sage: list_plot_loglog(list(zip(xl, yl)), base=2.1)  # with base 2.1 on both axes
```

Graphics object consisting of 1 graphics primitive
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```python
sage: list_plot_loglog(list(zip(xl, yl)), base=(2,5))
˓→long time
Graphics object consisting of 1 graphics primitive
```

**Warning:** If `plotjoined` is `False` then the axis that is in log scale must have all points strictly positive. For instance, the following plot will show no points in the figure since the points in the horizontal axis starts from `(0,1)`.

```python
sage: yl = [2**k for k in range(20)]
sage: list_plot_loglog(yl)
Graphics object consisting of 1 graphics primitive
```

Instead this will work. We drop the point `(0,1):`

```python
sage: list_plot_loglog(list(zip(range(1,len(yl)), yl[1:])))
Graphics object consisting of 1 graphics primitive
```

```python
sage.plot.plot.list_plot_loglog(data, plotjoined=False, base=10, **kwds)
```

Plot data in ‘loglog’ scale, that is, both axes will be in logarithmic scale.

**INPUT:**

- `base` – (default: 10); the base of the logarithm. This must be greater than 1.

For all other inputs, look at the documentation of `list_plot()`.

**EXAMPLES:**

```python
sage: yl = [2**k for k in range(12)]
sage: list_plot_loglog(list(zip(yl,yl)))
Graphics object consisting of 1 graphics primitive
```

```python
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogx(yl)  # plot empty due to (0,1)
Graphics object consisting of 1 graphics primitive
```

We remove `(0,1)` to fix this:

```python
sage: list_plot_semilogx(list(zip(range(1, len(yl)), yl[1:])))
Graphics object consisting of 1 graphics primitive
```

```python
sage: list_plot_semilogx([(1,2),(3,4),(3,-1),(25,3)], base=2)  # with base 2
Graphics object consisting of 1 graphics primitive
```

```python
sage.plot.plot.list_plot_semilogx(data, plotjoined=False, base=10, **kwds)
```

Plot data in ‘semilogx’ scale, that is, the horizontal axis will be in logarithmic scale.

**INPUT:**

- `base` – (default: 10); the base of the logarithm. This must be greater than 1.

For all other inputs, look at the documentation of `list_plot()`.

**EXAMPLES:**

```python
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogx(list(zip(yl,yl)))
Graphics object consisting of 1 graphics primitive
```

```python
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogx(yl)  # plot empty due to (0,1)
Graphics object consisting of 1 graphics primitive
```

We remove `(0,1)` to fix this:

```python
sage: list_plot_semilogx(list(zip(range(1, len(yl)), yl[1:])))
Graphics object consisting of 1 graphics primitive
```

```python
sage: list_plot_semilogx([(1,2),(3,4),(3,-1),(25,3)], base=2)  # with base 2
Graphics object consisting of 1 graphics primitive
```

```python
sage.plot.plot.list_plot_semilogy(data, plotjoined=False, base=10, **kwds)
```

Plot data in ‘semilogy’ scale, that is, the vertical axis will be in logarithmic scale.

**INPUT:**

- `base` – (default: 10); the base of the logarithm. This must be greater than 1.
1.1. 2D plotting
For all other inputs, look at the documentation of `list_plot()`.

EXAMPLES:

```sage
sage: yl = [2**k for k in range(12)]
sage: list_plot_semilogy(yl)  # plot in semilogy scale, base 10
Graphics object consisting of 1 graphics primitive
```

![Graph showing points on a semilogarithmic scale.]

**Warning:** If `plotjoined` is False then the vertical axis must have all points strictly positive. Otherwise the plot will come up empty. For instance the following plot contains a point at (1,0). Further, matplotlib will display a user warning.

```sage
sage: xl = [2**k for k in range(12)]; yl = range(len(xl))
sage: list_plot_semilogy(list(zip(xl, yl)))  # plot empty due to (1,0)
doctest:warning
...
Graphics object consisting of 1 graphics primitive
```

We remove (1,0) to fix this:

```sage
sage: list_plot_semilogy(list(zip(xl[1:], yl[1:])))
Graphics object consisting of 1 graphics primitive
```
sage: list_plot_semilogy([2, 4, 6, 8, 16, 31], base=2)  # with base 2
Graphics object consisting of 1 graphics primitive

sage.plot.plot.minmax_data(xdata, ydata, dict=False)
Return the minimums and maximums of xdata and ydata.
If dict is False, then minmax_data returns the tuple (xmin, xmax, ymin, ymax); otherwise, it returns a dictionary whose keys are ‘xmin’, ‘xmax’, ‘ymin’, and ‘ymax’ and whose values are the corresponding values.

EXAMPLES:

```python
sage: from sage.plot.plot import minmax_data
sage: minmax_data([], [])
(-1, 1, -1, 1)
sage: minmax_data([-1, 2], [4, -3])
(-1, 2, -3, 4)
sage: minmax_data([1, 2], [4, -3])
(1, 2, -3, 4)
sage: d = minmax_data([-1, 2], [4, -3], dict=True)
sage: list(sorted(d.items()))
[('xmax', 2), ('xmin', -1), ('ymax', 4), ('ymin', -3)]
sage: d = minmax_data([1, 2], [3, 4], dict=True)
sage: list(sorted(d.items()))
[('xmax', 2), ('xmin', 1), ('ymax', 4), ('ymin', 3)]
```
sage.plot.plot.multi_graphics(graphics_list)

Plot a list of graphics at specified positions on a single canvas.

If the graphics positions define a regular array, use graphics_array() instead.

INPUT:

• graphics_list — a list of graphics along with their positions on the canvas; each element of
  graphics_list is either
  – a pair (graphics, position), where graphics is a Graphics object and position is the 4-tuple
    (left, bottom, width, height) specifying the location and size of the graphics on the canvas,
    all quantities being in fractions of the canvas width and height
  – or a single Graphics object; its position is then assumed to occupy the whole canvas, except for some
    padding; this corresponds to the default position (left, bottom, width, height) = (0.125, 0.11, 0.775, 0.77)

OUTPUT: an instance of MultiGraphics

EXAMPLES:

multi_graphics is to be used for plot arrangements that cannot be achieved with graphics_array(), for instance:

```sage
sage: g1 = plot(sin(x), (x, -10, 10), frame=True)
sage: g2 = EllipticCurve([0,0,1,-1,0]).plot(color='red', thickness=2,
  ....:     axes_labels=['$x$', '$y$']) \
  ....:     + text(r"$y^2 + y = x^3 - x$", (1.2, 2), color='red')
sage: g3 = matrix_plot(matrix([[1,3,5,1], [2,4,5,6], [1,3,5,7]]))
sage: G = multi_graphics([(g1, (0.125, 0.65, 0.775, 0.3)),
  ....:          (g2, (0.125, 0.11, 0.4, 0.4)),
  ....:          (g3, (0.55, 0.18, 0.4, 0.3))])
sage: G
Multigraphics with 3 elements
```

An example with a list containing a graphics object without any specified position (the graphics, here g3, occupies then the whole canvas):

```sage
sage: G = multi_graphics([g3, (g1, (0.4, 0.4, 0.2, 0.2))])
sage: G
Multigraphics with 2 elements
```

See also:

MultiGraphics for more examples

sage.plot.plot.parametric_plot(funcs, aspect_ratio=1.0, *args, **kwargs)

Plot a parametric curve or surface in 2d or 3d.

parametric_plot() takes two or three functions as a list or a tuple and makes a plot with the first function giving the $x$ coordinates, the second function giving the $y$ coordinates, and the third function (if present) giving the $z$ coordinates.

In the 2d case, parametric_plot() is equivalent to the plot() command with the option parametric=True. In the 3d case, parametric_plot() is equivalent to parametric_plot3d(). See each of these functions for more help and examples.

INPUT:

• funcs – 2 or 3-tuple of functions, or a vector of dimension 2 or 3.
\[ y^2 + y = x^3 - x \]
1.1. 2D plotting
• other options – passed to plot() or parametric_plot3d()

EXAMPLES: We draw some 2d parametric plots. Note that the default aspect ratio is 1, so that circles look like circles.

```sage
sage: t = var('t')
sage: parametric_plot( (cos(t), sin(t)), (t, 0, 2*pi))
Graphics object consisting of 1 graphics primitive
```

```sage
sage: parametric_plot((sin(t), sin(2*t)), (t, 0, 2*pi), color=hue(0.6))
Graphics object consisting of 1 graphics primitive
```

```sage
sage: parametric_plot((1, t), (t, 0, 4))
Graphics object consisting of 1 graphics primitive
```

Note that in parametric_plot, there is only fill or no fill.

```sage
sage: parametric_plot((t, t^2), (t, -4, 4), fill=True)
Graphics object consisting of 2 graphics primitives
```

A filled Hypotrochoid:
1.1. 2D plotting
1.1. 2D plotting
```python
sage: parametric_plot(
    [cos(x) + 2 * cos(x/4), sin(x) - 2 * sin(x/4)],
    (x, 0, 8*pi),
    fill=True
)
Graphics object consisting of 2 graphics primitives
```

```python
sage: parametric_plot((5*cos(x), 5*sin(x), x), (x, -12, 12), plot_points=150, color='red')
# long time
Graphics3d Object
```

```python
sage: y = var('y')
sage: parametric_plot((5*cos(x), x*y, cos(x*y)), (x, -4, 4), (y, -4, 4))  # long time
Graphics3d Object
```

```python
sage: t = var('t')
sage: parametric_plot(vector((sin(t), sin(2*t))), (t, 0, 2*pi), color='green')  # long time
Graphics object consisting of 1 graphics primitive
```

```python
sage: t = var('t')
sage: parametric_plot(vector([t, t+1, t^2]), (t, 0, 1))  # long time
Graphics3d Object
```
1.1. 2D plotting
Plotting in logarithmic scale is possible with 2D plots. The keyword `aspect_ratio` will be ignored if the scale is not 'loglog' or 'linear':

```sage
sage: parametric_plot((x, x**2), (x, 1, 10), scale='loglog')
Graphics object consisting of 1 graphics primitive
```

We can also change the scale of the axes in the graphics just before displaying. In this case, the `aspect_ratio` must be specified as 'automatic' if the scale is set to 'semilogx' or 'semilogy'. For other values of the scale parameter, any `aspect_ratio` can be used, or the keyword need not be provided:

```sage
sage: p = parametric_plot((x, x**2), (x, 1, 10))
sage: p.show(scale='semilogy', aspect_ratio='automatic')
```

Use plot by writing

```
plot(X, ...)  
```

where `X` is a Sage object (or list of Sage objects) that either is callable and returns numbers that can be coerced to floats, or has a plot method that returns a `GraphicPrimitive` object.

There are many other specialized 2D plot commands available in Sage, such as `plot_slope_field`, as well as various graphics primitives like `Arrow`; type `sage.plot.plot?` for a current list.
Type `plot.options` for a dictionary of the default options for plots. You can change this to change the defaults for all future plots. Use `plot.reset()` to reset to the default options.

**PLOT OPTIONS:**

- `plot_points` – (default: 200); the minimal number of plot points.
- `adaptive_recursion` – (default: 5); how many levels of recursion to go before giving up when doing adaptive refinement. Setting this to 0 disables adaptive refinement.
- `adaptive_tolerance` – (default: 0.01); how large a difference should be before the adaptive refinement code considers it significant. See the documentation further below for more information, starting at “the algorithm used to insert”.
- `imaginary_tolerance` – (default: 1e-8); if an imaginary number arises (due, for example, to numerical issues), this tolerance specifies how large it has to be in magnitude before we raise an error. In other words, imaginary parts smaller than this are ignored in your plot points.
- `base` – (default: 10); the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple `(basex, basey)`. `basex` sets the base of the logarithm along the horizontal axis and `basey` sets the base along the vertical axis.
- `scale` – string (default: "linear"); scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy". The scale can be also be given as single argument that is a list or tuple `(scale, base)` or `(scale, basex, basey)`. The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when `Graphics` is initialized.
- `xmin` – starting x value in the rendered figure. This parameter is passed directly to the `show` procedure and it could be overwritten.
- `xmax` – ending x value in the rendered figure. This parameter is passed directly to the `show` procedure and it could be overwritten.
- `ymin` – starting y value in the rendered figure. This parameter is passed directly to the `show` procedure and it could be overwritten.
- `ymax` – ending y value in the rendered figure. This parameter is passed directly to the `show` procedure and it could be overwritten.
- `detect_poles` – (default: False) If set to True poles are detected. If set to “show” vertical asymptotes are drawn.
- `legend_label` – a (TeX) string serving as the label for X in the legend. If X is a list, then this option can be a single string, or a list or dictionary with strings as entries/values. If a dictionary, then keys are taken from `range(len(X))`.

**Note:**

- If the `scale` is "linear", then irrespective of what `base` is set to, it will default to 10 and will remain unused.
- If you want to limit the plot along the horizontal axis in the final rendered figure, then pass the `xmin` and `xmax` keywords to the `show()` method. To limit the plot along the vertical axis, `ymin` and `ymax` keywords can be provided to either this `plot` command or to the `show` command.
• This function does NOT simply sample equally spaced points between xmin and xmax. Instead it computes equally spaced points and adds small perturbations to them. This reduces the possibility of, e.g., sampling \( \sin \) only at multiples of \( 2\pi \), which would yield a very misleading graph.

• If there is a range of consecutive points where the function has no value, then those points will be excluded from the plot. See the example below on automatic exclusion of points.

• For the other keyword options that the \texttt{plot} function can take, refer to the method \texttt{show()} and the further options below.

COLOR OPTIONS:

• \texttt{color} - (Default: ‘blue’) One of:
  – an RGB tuple (r,g,b) with each of r,g,b between 0 and 1.
  – a color name as a string (e.g., ‘purple’).
  – an HTML color such as ‘#aaff0b’.
  – a list or dictionary of colors (valid only if \( X \) is a list): if a dictionary, keys are taken from \texttt{range(len(X))}; the entries/values of the list/dictionary may be any of the options above.
  – ‘automatic’ – maps to default (‘blue’) if \( X \) is a single Sage object; and maps to a fixed sequence of regularly spaced colors if \( X \) is a list.

• \texttt{legend_color} - the color of the text for \( X \) (or each item in \( X \)) in the legend.
  Default color is ‘black’. Options are as in \texttt{color} above, except that the choice ‘automatic’ maps to ‘black’ if \( X \) is a single Sage object.

• \texttt{fillcolor} - The color of the fill for the plot of \( X \) (or each item in \( X \)).
  Default color is ‘gray’ if \( X \) is a single Sage object or if \texttt{color} is a single color. Otherwise, options are as in \texttt{color} above.

APPEARANCE OPTIONS:

The following options affect the appearance of the line through the points on the graph of \( X \) (these are the same as for the line function):

INPUT:

• \texttt{alpha} – how transparent the line is
• \texttt{thickness} – how thick the line is
• \texttt{rgbcolor} – the color as an RGB tuple
• \texttt{hue} – the color given as a hue

LINE OPTIONS:

Any \texttt{MATPLOTLIB} line option may also be passed in. E.g.,

• \texttt{linestyle} - (default: “-“) The style of the line, which is one of
  – "-" or "solid"
  – "--" or "dashed"
  – "-." or "dash dot"
  – ":" or "dotted"
  – "None" or " " or "" (nothing)
  – a list or dictionary (see below)
The linestyle can also be prefixed with a drawing style (e.g., "steps--")
- "default" (connect the points with straight lines)
- "steps" or "steps-pre" (step function; horizontal line is to the left of point)
- "steps-mid" (step function; points are in the middle of horizontal lines)
- "steps-post" (step function; horizontal line is to the right of point)

If $X$ is a list, then linestyle may be a list (with entries taken from the strings above) or a dictionary (with keys in range(len(X)) and values taken from the strings above).

• **marker** - The style of the markers, which is one of
  - "None" or " " or "" (nothing) – default
  - "," (pixel), ",." (point)
  - ",-" (horizontal line), "|" (vertical line)
  - "o" (circle), "p" (pentagon), "s" (square), "x" (x), "+" (plus), "*" (star)
  - "D" (diamond), "d" (thin diamond)
  - "H" (hexagon), "h" (alternative hexagon)
  - "<" (triangle left), ">" (triangle right), "^" (triangle up), "v" (triangle down)
  - "1" (tri down), "2" (tri up), "3" (tri left), "4" (tri right)
  - 0 (tick left), 1 (tick right), 2 (tick up), 3 (tick down)
  - 4 (caret left), 5 (caret right), 6 (caret up), 7 (caret down), 8 (octagon)
  - "$...$" (math TeX string)
  - (numsides, style, angle) to create a custom, regular symbol
    * numsides – the number of sides
    * style – 0 (regular polygon), 1 (star shape), 2 (asterisk), 3 (circle)
    * angle – the angular rotation in degrees

• **markersize** - the size of the marker in points
• **markeredgecolor** – the color of the marker edge
• **markerfacecolor** – the color of the marker face
• **markeredgewidth** - the size of the marker edge in points

• **exclude** - (Default: None) values which are excluded from the plot range. Either a list of real numbers, or an equation in one variable.

**FILLING OPTIONS:**

• **fill** - (default: False) One of:
  - "axis" or True: Fill the area between the function and the x-axis.
  - "min": Fill the area between the function and its minimal value.
  - "max": Fill the area between the function and its maximal value.
  - a number c: Fill the area between the function and the horizontal line $y = c$.
  - a function g: Fill the area between the function that is plotted and g.
- a dictionary \( d \) (only if a list of functions are plotted): The keys of the dictionary should be integers. The value of \( d[i] \) specifies the fill options for the \( i \)-th function in the list. If \( d[i] == [j] \): Fill the area between the \( i \)-th and the \( j \)-th function in the list. (But if \( d[i] == j \): Fill the area between the \( i \)-th function in the list and the horizontal line \( y = j \).)

- \texttt{fillalpha} - (default: 0.5) How transparent the fill is. A number between 0 and 1.

**MATPLOTLIB STYLE SHEET OPTION:**

- \texttt{stylesheet} - (Default: classic) Support for loading a full matplotlib style sheet. Any style sheet listed in \texttt{matplotlib.pyplot.style.available} is acceptable. If a non-existing style is provided the default classic is applied.

**EXAMPLES:**

We plot the \( \sin \) function:

```
sage: P = plot(sin, (0,10)); print(P)
Graphics object consisting of 1 graphics primitive
sage: len(P)  # number of graphics primitives
1
sage: len(P[0])  # how many points were computed (random)
225
sage: P  # render
Graphics object consisting of 1 graphics primitive
```

![Graph of \( \sin(x) \) from 0 to 10]
We plot with `randomize=False`, which makes the initial sample points evenly spaced (hence always the same). Adaptive plotting might insert other points, however, unless `adaptive_recursion=0`.

```sage
sage: p = plot(1, (x,0,3), plot_points=4, randomize=False, adaptive_recursion=0)
sage: list(p[0])
[(0.0, 1.0), (1.0, 1.0), (2.0, 1.0), (3.0, 1.0)]
```

Some colored functions:

```sage
sage: plot(sin, 0, 10, color='purple')
Graphics object consisting of 1 graphics primitive
```

```sage
sage: plot(sin, 0, 10, color='#ff00ff')
Graphics object consisting of 1 graphics primitive
```

We plot several functions together by passing a list of functions as input:
By default, color will change from one primitive to the next. This may be controlled by modifying the \texttt{color} option:

\begin{Verbatim}
\texttt{sage: g1 = plot([x*exp(-n*x^2)/.4 for n in [1..3]], (0, 2), \ldots: \quad \text{color='blue', aspect_ratio=.8}); g1} \end{Verbatim}

Graphics object consisting of 3 graphics primitives

\begin{Verbatim}
\texttt{sage: g2 = plot([x\times exp(-n\times x^2)/.4 for n in [1..3]], (0, 2), \ldots: \quad \text{color=['red', 'red', 'green'], linestyle=['-', '--', '-.'], \ldots: \quad \text{aspect_ratio=.8}); g2} \end{Verbatim}

Graphics object consisting of 3 graphics primitives

While plotting real functions, imaginary numbers that are “almost real” will inevitably arise due to numerical issues. By tweaking the \texttt{imaginary_tolerance}, you can decide how large an imaginary part you’re willing to sweep under the rug in order to plot the corresponding point. If a particular value’s imaginary part has magnitude larger than \texttt{imaginary_tolerance}, that point will not be plotted. The default tolerance is $1 \times 10^{-8}$, so the imaginary part in the first example below is ignored, but the second example “fails,” emits a warning, and produces an empty graph:

\begin{Verbatim}
\texttt{sage: f = x + I*1e-12} \\
\texttt{sage: plot(f, x, -1, 1)} \end{Verbatim}

Graphics object consisting of 1 graphics primitive
We can also build a plot step by step from an empty plot:

```
sage: a = plot([]); a                        # passing an empty list returns an empty plot
    (Graphics() object)
sage: a += plot(x**2); a                   # append another plot
    Graphics object consisting of 1 graphics primitive
```

The function \(\sin(1/x)\) wiggles wildly near 0. Sage adapts to this and plots extra points near the origin.

```
sage: plot(sin(1/x), (x, -1, 1))
    Graphics object consisting of 1 graphics primitive
```

Via the matplotlib library, Sage makes it easy to tell whether a graph is on both sides of both axes, as the axes only cross if the origin is actually part of the viewing area:
Chapter 1. General
Another thing to be aware of with axis labeling is that when the labels have quite different orders of magnitude or are very large, scientific notation (the $e$ notation for powers of ten) is used:

```
sage: plot(x^2, (x,480,500))  # this one has no scientific notation
Graphics object consisting of 1 graphics primitive
```

```
sage: plot(x^2, (x,300,500))  # this one has scientific notation on y-axis
Graphics object consisting of 1 graphics primitive
```

You can put a legend with `legend_label` (the legend is only put once in the case of multiple functions):

```
sage: plot(exp(x), 0, 2, legend_label='$e^x$')
Graphics object consisting of 1 graphics primitive
```

Sage understands TeX, so these all are slightly different, and you can choose one based on your needs:
1.1. 2D plotting
1.1. 2D plotting
It is possible to use a different color for the text of each label:

```python
sage: p1 = plot(sin, legend_label='sin', legend_color='red')
sage: p2 = plot(cos, legend_label='cos', legend_color='green')
sage: p1 + p2
```

Graphics object consisting of 2 graphics primitives

Prior to github issue #19485, legends by default had a shadowless gray background. This behavior can be recovered by setting the legend options on your plot object:

```python
sage: p = plot(sin(x), legend_label='\sin(x)')
sage: p.set_legend_options(back_color=(0.9,0.9,0.9), shadow=False)
```

If $X$ is a list of Sage objects and `legend_label` is ‘automatic’, then Sage will create labels for each function...
1.1. 2D plotting
Chapter 1. General
according to their internal representation:

```
sage: plot([sin(x), tan(x), 1 - x^2], legend_label='automatic')
Graphics object consisting of 3 graphics primitives
```

If `legend_label` is any single string other than ‘automatic’, then it is repeated for all members of $X$:

```
sage: plot([sin(x), tan(x)], color='blue', legend_label='trig')
Graphics object consisting of 2 graphics primitives
```

Note that the independent variable may be omitted if there is no ambiguity:

```
sage: plot(sin(1.0/x), (-1, 1))
Graphics object consisting of 1 graphics primitive
```

Plotting in logarithmic scale is possible for 2D plots. There are two different syntaxes supported:

```
sage: plot(exp, (1, 10), scale='semilogy') # log axis on vertical
Graphics object consisting of 1 graphics primitive
```

```
sage: plot_semilogy(exp, (1, 10)) # same thing
Graphics object consisting of 1 graphics primitive
```
Chapter 1. General
1.1. 2D plotting
We can also change the scale of the axes in the graphics just before displaying:

```python
sage: G = plot(exp, 1, 10)  # long time
sage: G.show(scale=('semilogy', 2))  # long time
```

The algorithm used to insert extra points is actually pretty simple. On the picture drawn by the lines below:

```python
sage: p = plot(x^2, (-0.5, 1.4)) + line([(0,0), (1,1)], color='green')
sage: p += line([(0.5, 0.5), (0.5, 0.5^2)], color='purple')
sage: p += point(((0, 0), (0.5, 0.5), (0.5, 0.5^2), (1, 1)), color='red', pointsize=20)
sage: p += text('A', (-0.05, 0.1), color='red')
sage: p += text('B', (1.01, 1.1), color='red')
sage: p += text('C', (0.48, 0.57), color='red')
sage: p += text('D', (0.53, 0.18), color='red')
sage: p.show(axes=False, xmin=-0.5, xmax=1.4, ymin=0, ymax=2)
```
1.1. 2D plotting
You have the function (in blue) and its approximation (in green) passing through the points A and B. The algorithm finds the midpoint C of AB and computes the distance between C and D. If that distance exceeds the adaptive_tolerance threshold (relative to the size of the initial plot subintervals), the point D is added to the curve. If D is added to the curve, then the algorithm is applied recursively to the points A and D, and D and B. It is repeated adaptive_recursion times (5, by default).

The actual sample points are slightly randomized, so the above plots may look slightly different each time you draw them.

We draw the graph of an elliptic curve as the union of graphs of 2 functions.

```sage
def h1(x):
    return abs(sqrt(x^3 - 1))
def h2(x):
    return -abs(sqrt(x^3 - 1))
P = plot([h1, h2], 1,4)
P  # show the result
```

It is important to mention that when we draw several graphs at the same time, parameters xmin, xmax, ymin and ymax are just passed directly to the show procedure. In fact, these parameters would be overwritten:

```sage
p=plot(x^3, x, xmin=-1, xmax=1,ymin=-1, ymax=1)
q=plot(exp(x), x, xmin=-2, xmax=2, ymin=0, ymax=4)
(p+q).show()
```

As a workaround, we can perform the trick:
We can also directly plot the elliptic curve:

```python
sage: E = EllipticCurve([0,-1])
# needs sage.schemes
sage: plot(E, (1, 4), color=hue(0.6))  # needs sage.schemes
Graphics object consisting of 1 graphics primitive
```

We can change the line style as well:

```python
sage: plot(sin(x), (x, 0, 10), linestyle='-.')
Graphics object consisting of 1 graphics primitive
```

If we have an empty linestyle and specify a marker, we can see the points that are actually being plotted:

```python
sage: plot(sin(x), (x,0,10), plot_points=20, linestyle='', marker='.'
Graphics object consisting of 1 graphics primitive
```
The marker can be a TeX symbol as well:

```python
sage: plot(sin(x), (x, 0, 10), plot_points=20, linestyle='', marker=r'$\checkmark$')
```

Graphics object consisting of 1 graphics primitive

Sage currently ignores points that cannot be evaluated

```python
sage: from sage.misc.verbose import set_verbose
sage: set_verbose(-1)
sage: plot(-x*log(x), (x, 0, 1))  # this works fine since the failed endpoint is just skipped.
```

Graphics object consisting of 1 graphics primitive

```python
sage: set_verbose(0)
```

This prints out a warning and plots where it can (we turn off the warning by setting the verbose mode temporarily to -1.)

```python
sage: set_verbose(-1)
sage: plot(x^(1/3), (x, -1, 1))
```

Graphics object consisting of 1 graphics primitive

```python
sage: set_verbose(0)
```

Plotting the real cube root function for negative input requires avoiding the complex numbers one would usually get. The easiest way is to use `real_nth_root(x, n)`.
We can also get the same plot in the following way:

```plaintext
sage: plot(sign(x)*abs(x)^(1/3), (x, -1, 1))
Graphics object consisting of 1 graphics primitive
```

A way to plot other functions without symbolic variants is to use lambda functions:

```plaintext
sage: plot(lambda x : RR(x).nth_root(3), (x,-1, 1))
Graphics object consisting of 1 graphics primitive
```

We can detect the poles of a function:

```plaintext
sage: plot(gamma, (-3, 4), detect_poles=True).show(ymin=-5, ymax=5)
```

We draw the Gamma-Function with its poles highlighted:

```plaintext
sage: plot(gamma, (-3, 4), detect_poles='show').show(ymin=-5, ymax=5)
```

The basic options for filling a plot:
1.1. 2D plotting
1.1. 2D plotting
The basic options for filling a list of plots:

```python
sage: (f1, f2) = x*exp(-1*x^2)/.35, x*exp(-2*x^2)/.35
sage: p1 = plot([f1, f2], -pi, pi, fill={1: [0]},
              fillcolor='blue', fillalpha=.25, color='blue')
sage: p2 = plot([f1, f2], -pi, pi, fill={0: x/3, 1:[0]}, color='blue')
sage: p3 = plot([f1, f2], -pi, pi, fill=[0, [0]],
              fillcolor=['orange', 'red'], fillalpha=1, color=[1: 'blue'])
sage: p4 = plot([f1, f2], (x,-pi, pi), fill=[x/3, 0],
              fillcolor=['grey', color=['red', 'blue'])
sage: graphics_array([[p1, p2],
                   [p3, p4]]).show(frame=True, axes=False)
```

![Diagram showing different fill options for plots.]
1.1. 2D plotting
A example about the growth of prime numbers:

```
sage: plot(1.13*log(x), 1, 100,
       ....:       fill=lambda x: nth_prime(x)/floor(x), fillcolor='red')
```

Fill the area between a function and its asymptote:

```
sage: f = (2*x^3+2*x-1)/((x-2)*(x+1))
sage: plot([f, 2*x+2], -7, 7, fill={0: [1]}, fillcolor='#ccc').show(ymin=-20, ymax=20)
```

Fill the area between a list of functions and the x-axis:

```
sage: def b(n):
    return lambda x: bessel_J(n, x)
sage: plot([b(n) for n in [1..5]], 0, 20, fill='axis')
```

Note that to fill between the ith and jth functions, you must use the dictionary key-value syntax i:j; using key-value pairs like i:j will fill between the ith function and the line y=j:

```
sage: def b(n):
    return lambda x: bessel_J(n, x) + 0.5*(n-1)
sage: plot([b(c) for c in [1..5]], 0, 20, fill={i: [i-1] for i in [1..4]},
       ....:       color={i: 'blue' for i in [1..5], aspect_ratio=3, ymax=3})
```

(continues on next page)
1.1. 2D plotting
Graphics object consisting of 9 graphics primitives

```python
sage: plot([b(c) for c in [1..5]], 0, 20, fill={i: i-1 for i in [1..4]},
˓→long time
˓→....: color='blue', aspect_ratio=3)
Graphics object consisting of 9 graphics primitives
```

Extra options will get passed on to `show()`, as long as they are valid:

```python
sage: plot(sin(x^2), (x, -3, 3), # These labels will be nicely typeset
˓→....: title=r'Plot of $\sin(x^2)$', axes_labels=['$x$', '$y$'])
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(sin(x^2), (x, -3, 3), # These will not
˓→....: title='Plot of sin(x^2)', axes_labels=['x', 'y'])
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(sin(x^2), (x, -3, 3), # Large axes labels (w.r.t. the tick marks)
˓→....: axes_labels=['x', 'y'], axes_labels_size=2.5)
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(sin(x^2), (x, -3, 3), figsize=[8,2])
```

(continues on next page)
Plot of $\sin(x^2)$
Plot of \( \sin(x^2) \)
Graphics object consisting of 1 graphics primitive

```
sage: plot(sin(x^2), (x, -3, 3)).show(figsize=[8,2])  # These are equivalent
```

This includes options for custom ticks and formatting. See documentation for `show()` for more details.

```
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[0, -7, 0, 7], [-1/2, 0, 1/2]])  
```

```
sage: plot(2*x + 1, (x, 0, 5),  
...:       ticks=[[0, 1, e, pi, sqrt(20)],  
...:       [1, 3, 2*e + 1, 2*pi + 1, 2*sqrt(20) + 1]],
```

(continues on next page)
tick_formatter="latex")
Graphics object consisting of 1 graphics primitive

This is particularly useful when setting custom ticks in multiples of $\pi$.
sage: plot(sin(x), (x,0,2"pi), ticks=pi/3, tick_formatter=pi)
Graphics object consisting of 1 graphics primitive

You can even have custom tick labels along with custom positioning.
sage: plot(x**2, (x,0,3), ticks=[[1,2.5], [0.5,1,2]],
       tick_formatter=['$x_1$','$x_2$'],
       ['$y_1$','$y_2$','$y_3$'])
Graphics object consisting of 1 graphics primitive

You can force Type 1 fonts in your figures by providing the relevant option as shown below. This also requires that LaTeX, dvipng and Ghostscript be installed:
sage: plot(x, typeset='type1') # optional - latex
Graphics object consisting of 1 graphics primitive

A example with excluded values:
1.1. 2D plotting
We exclude all points where $\text{PrimePi}$ makes a jump:

```sage
sage: jumps = [n for n in [1..100] if prime_pi(n) != prime_pi(n-1)]
sage: plot(lambda x: prime_pi(x), (x, 1, 100), exclude=jumps)
```

Excluded points can also be given by an equation:

```sage
g(x) = x^2 - 2x - 2
sage: plot(1/g(x), (x, -3, 4), exclude=g(x)==0, ymin=-5, ymax=5)  # long time
```

`exclude` and `detect_poles` can be used together:

```sage
f(x) = (floor(x)+0.5) / (1-(x-0.5)^2)
sage: plot(f, (x, -3.5, 3.5), detect_poles='show', exclude=[-3..3],
.....: ymin=-5, ymax=5)
```

Regions in which the plot has no values are automatically excluded. The regions thus excluded are in addition to the exclusion points present in the `exclude` keyword argument.
1.1. 2D plotting
sage: set_verbose(-1)
sage: plot(arcsec, (x, -2, 2))  # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives

sage: plot(arcsec, (x, -2, 2), exclude=[1.5])  # x=1.5 is also excluded
Graphics object consisting of 3 graphics primitives

sage: plot(arcsec(x/2), (-2, 2))  # plot should be empty; no valid points
Graphics object consisting of 0 graphics primitives
sage: plot(sqrt(x^2-1), (-2, 2))  # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives

sage: plot(arccsc, (-2, 2))  # [-1, 1] is excluded automatically
Graphics object consisting of 2 graphics primitives
sage: set_verbose(0)

sage.plot.plot.plot_loglog(funcs, base=10, *args, **kwds)

Plot graphics in ‘loglog’ scale, that is, both the horizontal and the vertical axes will be in logarithmic scale.

INPUT:

- base – (default: 10); the base of the logarithm. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey
1.1. 2D plotting
sets the base along the vertical axis.

- **funcs** – any Sage object which is acceptable to the `plot()`.

For all other inputs, look at the documentation of `plot()`.

**EXAMPLES:**

```python
sage: plot_loglog(exp, (1,10))  # plot in loglog scale with base 10
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot_loglog(exp, (1,10), base=2.1)  # with base 2.1 on both axes  # long time
Graphics object consisting of 1 graphics primitive
```

```python
sage: plot_loglog(exp, (1,10), base=(2,3))
Graphics object consisting of 1 graphics primitive
```

`sage.plot.plot.plot_semilogx(funcs, base=10, *args, **kwds)`

Plot graphics in ‘semilogx’ scale, that is, the horizontal axis will be in logarithmic scale.

**INPUT:**

- **base** – (default: 10); the base of the logarithm. This must be greater than 1.
- **funcs** – any Sage object which is acceptable to the `plot()`.
1.1. 2D plotting
For all other inputs, look at the documentation of \texttt{plot()}. 

EXAMPLES:

\begin{Verbatim}
\texttt{sage: plot_semilogx(exp, (1,10))  \# plot in semilogx scale, base 10  \# long time}
Graphics object consisting of 1 graphics primitive
\end{Verbatim}

\begin{Verbatim}
\texttt{sage: plot_semilogx(exp, (1,10), base=2)}  \# with base 2
Graphics object consisting of 1 graphics primitive
\end{Verbatim}

\begin{Verbatim}
\texttt{sage: s = var('s')}  \# Samples points logarithmically so graph is smooth
\texttt{sage: f = 4000000/(4000000 + 4000*s*i - s*s)}
\texttt{sage: plot_semilogx(20*log(abs(f), 10), (s, 1, 1e6))}
Graphics object consisting of 1 graphics primitive
\end{Verbatim}

\begin{verbatim}
\texttt{sage.plot.plot.plot_semilogy(funcs, base=10, *args, **kwds)}
Plot graphics in 'semilogy' scale, that is, the vertical axis will be in logarithmic scale.

INPUT:

- \texttt{base} – (default: 10); the base of the logarithm. This must be greater than 1.
- \texttt{funcs} – any Sage object which is acceptable to the \texttt{plot()}.

For all other inputs, look at the documentation of \texttt{plot()}.
\end{verbatim}
EXAMPLES:

```
sage: plot_semilogy(exp, (1,10)) # long time # plot in semilogy scale, base 10
Graphics object consisting of 1 graphics primitive
```

```
sage: plot_semilogy(exp, (1,10), base=2) # long time # with base 2
Graphics object consisting of 1 graphics primitive
```

```
sage.plot.plot.polar_plot(funcs, aspect_ratio=1.0, *args, **kwds)
```
polar_plot takes a single function or a list or tuple of functions and plots them with polar coordinates in the given domain.

This function is equivalent to the `plot()` command with the options `polar=True` and `aspect_ratio=1`. For more help on options, see the documentation for `plot()`.

**INPUT:**

- `funcs` – a function
- other options are passed to `plot`

**EXAMPLES:**

Here is a blue 8-leaved petal:
A red figure-8:

```
sage: polar_plot(abs(sqrt(1 - sin(x)^2)), (x, 0, 2*pi), color='red')
Graphics object consisting of 1 graphics primitive
```

A green limacon of Pascal:

```
sage: polar_plot(2 + 2*cos(x), (x, 0, 2*pi), color=hue(0.3))
Graphics object consisting of 1 graphics primitive
```

Several plots:

```
sage: polar_plot([2*sin(x), 2*cos(x)], (x, 0, 2*pi))
Graphics object consisting of 2 graphics primitives
```

A filled spiral:

```
sage: polar_plot(sqrt, 0, 2*pi, fill=True)
Graphics object consisting of 2 graphics primitives
```

Fill the area between two functions:
1.1. 2D plotting
Fill the area between several spirals:

```
sage: polar_plot([(1.2+k*0.2)*log(x) for k in range(6)], 1, 3*pi, fill={0: [1], 2: [3], 4: [5]})
```

Graphics object consisting of 9 graphics primitives

Exclude points at discontinuities:

```
sage: polar_plot(log(floor(x)), (x, 1, 4*pi), exclude=[1..12])
```

Graphics object consisting of 12 graphics primitives

`sage.plot.plot.reshape(v, n, m)`

Helper function for creating graphics arrays.

The input array is flattened and turned into an `nimesm` array, with blank graphics object padded at the end, if necessary.

**INPUT:**

- `v` – a list of lists or tuples
1.1. 2D plotting
* n, m – integers

OUTPUT: a list of lists of graphics objects

EXAMPLES:

```python
sage: L = [plot(sin(k*pi), (x,-pi,pi)) for k in range(10)]
sage: graphics_array(L,3,4)  # long time (up to 4s on sage.math, 2012)
Graphics Array of size 3 x 4
```

```python
sage: M = [[plot(sin(k*pi), (x,-pi,pi)) for k in range(3)],
       [plot(cos(j*pi),(x,-pi,pi)) for j in [3..5]]]
sage: graphics_array(M,6,1)  # long time (up to 4s on sage.math, 2012)
Graphics Array of size 6 x 1
```

```python
sage.plot.plot.to_float_list(v)
```

Given a list or tuple or iterable v, coerce each element of v to a float and make a list out of the result.

EXAMPLES:

```python
sage: from sage.plot.plot import to_float_list
sage: to_float_list([1,1/2,3])
[1.0, 0.5, 3.0]
```

```python
sage.plot.plot.xydata_from_point_list(points)
```

Return two lists (xdata, ydata), each coerced to a list of floats, which correspond to the x-coordinates and the y-coordinates of the points.

The points parameter can be a list of 2-tuples or some object that yields a list of one or two numbers.

This function can potentially be very slow for large point sets.

### 1.2 Text in plots

```python
class sage.plot.text.Text(string, point, options)
```

Bases: `GraphicPrimitive`

Base class for Text graphics primitive.

```python
get_minmax_data()
```

Return a dictionary with the bounding box data. Notice that, for text, the box is just the location itself.

EXAMPLES:

```python
sage: T = text("Where am I?",(1,1))
sage: t=T[0]
sage: t.get_minmax_data()['ymin']
1.0
sage: t.get_minmax_data()['ymax']
1.0
```

```python
plot3d(**kwds)
```

Plot 2D text in 3D.

EXAMPLES:
I like Fibonacci
sage: T = text("ABC", (1, 1))
sage: t = T[0]
sage: s = t.plot3d()
sage: s.jmol_repr(s.testing_render_params())[0][2]
'label "ABC"

sage: s._trans(1.0, 1.0, 0)

sage.plot.text.text(string, xy, fontsize=10, rgbcolor=(0, 0, 1), horizontal_alignment='center',
vertical_alignment='center', axis_coords=False, clip=False, **options)

Return a 2D text graphics object at the point (x, y).

Type text.options for a dictionary of options for 2D text.

2D OPTIONS:

• fontsize - How big the text is. Either an integer that specifies the size in points or a string which specifies a size (one of ‘xx-small’, ‘x-small’, ‘small’, ‘medium’, ‘large’, ‘x-large’, ‘xx-large’)

• fontstyle - A string either ‘normal’, ‘italic’ or ‘oblique’


• rgbcolor - The color as an RGB tuple

• hue - The color given as a hue

• background_color - The background color

• rotation - How to rotate the text: angle in degrees, vertical, horizontal

• vertical_alignment - How to align vertically: top, center, bottom

• horizontal_alignment - How to align horizontally: left, center, right

• zorder - The layer level in which to draw

• clip - (default: False) Whether to clip or not

• axis_coords - (default: False) If True, use axis coordinates, so that (0,0) is the lower left and (1,1) upper right, regardless of the x and y range of plotted values.

• bounding_box - A dictionary specifying a bounding box. Currently the text location.

EXAMPLES:

sage: text("Sage graphics are really neat because they use matplotlib!", (2,12))
Graphics object consisting of 1 graphics primitive

Larger font, bold, colored red and transparent text:

sage: text("I had a dream!", (2,12), alpha=0.3,
.....:      fontsize='large', fontweight='bold', color='red')
Graphics object consisting of 1 graphics primitive

By setting horizontal_alignment to ‘left’ the text is guaranteed to be in the lower left no matter what:
Sage graphics are really neat because they use matplotlib!
I had a dream!
sage: text("I got a horse and he lives in a tree", (0,0),
....:     axis_coords=True, horizontal_alignment='left')
Graphics object consisting of 1 graphics primitive

Various rotations:

sage: text("noitator", (0,0), rotation=45.0,
....:     horizontal_alignment='left', vertical_alignment='bottom')
Graphics object consisting of 1 graphics primitive

sage: text("Sage is really neat!!",(0,0), rotation="vertical")
Graphics object consisting of 1 graphics primitive

You can also align text differently:

sage: t1 = text("Hello", (1,1), vertical_alignment="top")
sage: t2 = text("World", (1,0.5), horizontal_alignment="left")
sage: t1 + t2  # render the sum
Graphics object consisting of 2 graphics primitives

You can save text as part of PDF output:
1.2. Text in plots
Sage is really neat!!
Hello
World
```python
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix=".pdf") as f:
    ....:     text("sage", (0,0), rgbcolor=(0,0,0)).save(f.name)
```

Some examples of bounding box:

```python
sage: bbox = {'boxstyle': "rarrow,pad=0.3", 'fc': "cyan", 'ec': "b", 'lw': 2}
sage: text("I feel good", (1,2), bounding_box=bbox)
```


A text with a background color:

```python
sage: text("So good", (0,0), bounding_box={'boxstyle': 'round', 'fc': 'w'})
```

Use dollar signs for LaTeX and raw strings to avoid having to escape backslash characters:
1.2. Text in plots
So good
sage: A = arc((0, 0), 1, sector=(0.0, RDF.pi()))
sage: a = sqrt(1./2.)
sage: PQ = point2d([(-a, a), (a, a)])
sage: botleft = dict(horizontal_alignment='left', vertical_alignment='bottom')
sage: botright = dict(horizontal_alignment='right', vertical_alignment='bottom')
sage: tp = text(r'$z_P = e^{3i\pi/4}$', (-a, a), **botright)
sage: tq = text(r'$Q = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$', (a, a), **botleft)
sage: A + PQ + tp + tq
Graphics object consisting of 4 graphics primitives

Text coordinates must be 2D, an error is raised if 3D coordinates are passed:

sage: t = text("hi", (1, 2, 3))
Traceback (most recent call last):
...
ValueError: use text3d instead for text in 3d

Use the text3d function for 3D text:

sage: t = text3d("hi", (1, 2, 3))

Or produce 2D text with coordinates $(x, y)$ and plot it in 3D (at $z = 0$):
sage: t = text("hi", (1, 2))
sage: t.plot3d()  # text at position (1, 2, 0)
Graphics3d Object

Extra options will get passed on to show(), as long as they are valid. Hence this

sage: text("MATH IS AWESOME", (0, 0), fontsize=40, axes=False)
Graphics object consisting of 1 graphics primitive

is equivalent to

sage: text("MATH IS AWESOME", (0, 0), fontsize=40).show(axes=False)

## 1.3 Colors

This module defines a `Color` object and helper functions (see, e.g., `hue()`, `rainbow()`), as well as a set of colors and colormaps to use with `Graphics` objects in Sage.

For a list of pre-defined colors in Sage, evaluate:

sage: sorted(colors)

`['aliceblue', 'antiquewhite', 'aqua', 'aquamarine', 'automatic', ...]`

Apart from ‘automatic’ which just an alias for ‘lightblue’, this list comprises the “official” W3C CSS3 / SVG colors.

For a list of color maps in Sage, evaluate:

sage: sorted(colormaps)

`['Accent', ...]`

These are imported from matplotlib’s colormaps collection.

```python
class sage.plot.colors.Color(r=#0000ff, g=None, b=None, space='rgb')
    Bases: object

    An Red-Green-Blue (RGB) color model color object. For most consumer-grade devices (e.g., CRTs, LCDs, and printers), as well as internet applications, this is a point in the sRGB absolute color space. The Hue-Saturation-Lightness (HSL), Hue-Lightness-Saturation (HLS), and Hue-Saturation-Value (HSV) spaces are useful alternate representations, or coordinate transformations, of this space. Coordinates in all of these spaces are floating point values in the interval [0.0, 1.0].

Note: All instantiations of `Color` are converted to an internal RGB floating point 3-tuple. This is likely to degrade precision.
```

INPUT:

- r, g, b - either a triple of floats between 0 and 1, OR r - a color name string or HTML color hex string
- space - a string (default: ‘rgb’); the coordinate system (other choices are ‘hsl’, ‘hls’, and ‘hsv’) in which to interpret a triple of floats

EXAMPLES:
```
sage: Color('purple')
RGB color (0.5019607843137255, 0.0, 0.5019607843137255)
sage: Color('#8000ff')
RGB color (0.5019607843137255, 0.0, 1.0)
sage: Color(0.5,0,1)
RGB color (0.5, 0.0, 1.0)
sage: Color(0.5, 1.0, 1, space='hsv')
RGB color (0.0, 1.0, 1.0)
sage: Color(0.25, 0.5, 0.5, space='hls')
RGB color (0.5000000000000001, 0.75, 0.25)
sage: Color(1, 0, 1/3, space='hsl')
RGB color (0.3333333333333333, 0.3333333333333333, 0.3333333333333333)
sage: from sage.plot.colors import chocolate
sage: Color(chocolate)
RGB color (0.8235294117647058, 0.4117647058823529, 0.11764705882352941)
```

**blend**(color, fraction=0.5)

Return a color blended with the given color by a given fraction. The algorithm interpolates linearly between the colors' corresponding R, G, and B coordinates.

**INPUT:**

- color - a Color instance or float-convertible 3-tuple/list; the color with which to blend this color
- fraction - a float-convertible number; the fraction of color to blend with this color

**OUTPUT:**

- a new Color instance

**EXAMPLES:**

```
sage: from sage.plot.colors import red, blue, lime
sage: red.blend(blue)
RGB color (0.5, 0.0, 0.5)
sage: red.blend(blue, fraction=0.0)
RGB color (1.0, 0.0, 0.0)
sage: red.blend(blue, fraction=1.0)
RGB color (0.0, 0.0, 1.0)
sage: lime.blend((0.3, 0.5, 0.7))
RGB color (0.15, 0.75, 0.35)
sage: blue.blend(blue)
RGB color (0.0, 0.0, 1.0)
sage: red.blend(lime, fraction=0.3)
RGB color (0.7, 0.3, 0.0)
sage: blue.blend((0.0, 0.9, 0.2), fraction=0.2)
RGB color (0.0, 0.18000000000000002, 0.8400000000000001)
sage: red.blend(0.2)
Traceback (most recent call last):
...
TypeError: 0.200000000000000 must be a Color or float-convertible 3-tuple/list
```

**darker**(fraction=0.3333333333333333)

Return a darker “shade” of this RGB color by blend()-ing it with black. This is not an inverse of lighter().

**INPUT:**
• fraction - a float (default: 1/3); blending fraction to apply

OUTPUT:
• a new instance of Color

EXAMPLES:
```
sage: from sage.plot.colors import black
dsage: vector(black.darker().rgb()) == vector(black.rgb())
    True
sage: Color(0.4, 0.6, 0.8).darker(0.1)
    RGB color (0.36000000000000004, 0.54, 0.72000000000000001)
sage: Color(1,2,3,space='hsl').darker()
    RGB color (0.24000000000000002, 0.20800000000000002, 0.16)
```

hls()

Return the Hue-Lightness-Saturation (HLS) coordinates of this color.

OUTPUT:
• a 3-tuple of floats

EXAMPLES:
```
sage: Color(0.3, 0.5, 0.7, space='hls').hls()
    (0.30000000000000004, 0.5, 0.7)
sage: Color(0.3, 0.5, 0.7, space='hls').hls() # abs tol 1e-15
    (0.30000000000000004, 0.5, 0.50000000000000001)
sage: Color('#aabbcc').hls() # abs tol 1e-15
    (0.5833333333333334, 0.7333333333333334, 0.250000000000000017)
sage: from sage.plot.colors import orchid
sage: orchid.hls() # abs tol 1e-15
    (0.8396226415094339, 0.6470588235294117, 0.5888888888888889)
```

hsl()

Return the Hue-Saturation-Lightness (HSL) coordinates of this color.

OUTPUT:
• a 3-tuple of floats

EXAMPLES:
```
sage: Color(1,0,0).hsl()
    (0.0, 1.0, 0.5)
sage: from sage.plot.colors import orchid
sage: orchid.hsl() # abs tol 1e-15
    (0.8396226415094339, 0.5888888888888889, 0.6470588235294117)
sage: Color('#aabbcc').hsl() # abs tol 1e-15
    (0.5833333333333334, 0.250000000000000017, 0.7333333333333334)
```

hsv()

Return the Hue-Saturation-Value (HSV) coordinates of this color.

OUTPUT:
• a 3-tuple of floats

EXAMPLES:
```python
from sage.plot.colors import red
sage: red.hsv()
(0.0, 1.0, 1.0)
sage: Color(1,1,1).hsv()
(0.0, 0.0, 1.0)
sage: Color('gray').hsv()
(0.0, 0.0, 0.5019607843137255)
```

### html_color()
Return a HTML hex representation for this color.

**OUTPUT:**
- a string of length 7.

**EXAMPLES:**
```
sage: Color('yellow').html_color()
'#ffff00'
sage: Color('#fedcba').html_color()
'#fedcba'
sage: Color(0.0, 1.0, 0.0).html_color()
'#00ff00'
sage: from sage.plot.colors import honeydew
color:
```
```
sage: honeydew.html_color()
'#f0fff0'
```

### lighter(fraction=0.3333333333333333)
Return a lighter “shade” of this RGB color by `blend()`-ing it with white. This is **not** an inverse of `darker()`.

**INPUT:**
- fraction - a float (default: 1/3); blending fraction to apply

**OUTPUT:**
- a new instance of `Color`

**EXAMPLES:**
```
sage: from sage.plot.colors import khaki
sage: khaki.lighter()
RGB color (0.9607843137254903, 0.934640522875817, 0.6993464052287582)
sage: Color('white').lighter().darker()
RGB color (0.6666666666666666, 0.6666666666666666, 0.6666666666666666)
sage: Color('#abcdef').lighter(1/4)
RGB color (0.7529411764705882, 0.8529411764705883, 0.9529411764705882)
sage: Color(1, 0, 8/9, space='hsv').lighter()
RGB color (0.925925925925926, 0.925925925925926, 0.925925925925926)
```

### rgb()
Return the underlying Red-Green-Blue (RGB) coordinates of this color.

**OUTPUT:**
- a 3-tuple of floats

**EXAMPLES:**
```
```
sage: Color(0.3, 0.5, 0.7).rgb()
    (0.3, 0.5, 0.7)
sage: Color('#8000ff').rgb()
    (0.5019607843137255, 0.0, 1.0)
sage: from sage.plot.colors import orange
sage: orange.rgb()
    (1.0, 0.6470588235294118, 0.0)
sage: Color('magenta').rgb()
    (1.0, 0.0, 1.0)
sage: Color(1, 0.7, 0.9, space='hsv').rgb()
    (0.9, 0.2700000000000001, 0.2700000000000001)

class sage.plot.colors.Colormaps

Bases: MutableMapping

A dict-like collection of lazily-loaded matplotlib color maps. For a list of map names, evaluate:

sage: sorted(colormaps)
['Accent', ...]

load_maps()

If it’s necessary, loads matplotlib's color maps and adds them to the collection.

EXAMPLES:

sage: from sage.plot.colors import colormaps
sage: maps = Colormaps()
sage: len(maps.maps)
0
sage: maps.load_maps()

EXAMPLES:

sage: sorted(colors)
['aliceblue', 'antiquewhite', 'aqua', 'aquamarine', ...]

sage.plot.colors.check_color_data(cfcm)

Make sure that the arguments are in order (coloring function, colormap).

This will allow users to use both possible orders.

EXAMPLES:

sage: from sage.plot.colors import check_color_data
sage: cf = lambda x,y : (x+y) % 1
sage: cm = colormaps.autumn
sage: check_color_data((cf, cm)) == (cf, cm)
True

(continues on next page)
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(continued from previous page)

```python
sage: check_color_data((cm, cf)) == (cf, cm)
True
```

```python
sage.plot.colors.float_to_html(r, g, b)
Convert a Red-Green-Blue (RGB) color tuple to a HTML hex color.

Each input value should be in the interval [0.0, 1.0]; otherwise, the values are first reduced modulo one (see `mod_one()`).

**INPUT:**

- `r` – a real number; the RGB color’s “red” intensity
- `g` – a real number; the RGB color’s “green” intensity
- `b` – a real number; the RGB color’s “blue” intensity

**OUTPUT:**

- a string of length 7, starting with ‘#’

**EXAMPLES:**

```python
sage: from sage.plot.colors import float_to_html
sage: float_to_html(1.,1.,0.)
'#ffffff'

sage: float_to_html(.03,.06,.02)
'#070f05'

sage: float_to_html(*Color('brown').rgb())
'#a52a2a'
```

```python
sage.plot.colors.float_to_integer(r, g, b)
Convert a Red-Green-Blue (RGB) color tuple to an integer.

Each input value should be in the interval [0.0, 1.0]; otherwise, the values are first reduced modulo one (see `mod_one()`).

**INPUT:**

- `r` – a real number; the RGB color’s “red” intensity
- `g` – a real number; the RGB color’s “green” intensity
- `b` – a real number; the RGB color’s “blue” intensity

**OUTPUT:**

- the integer $256^2 r_i nt + 256 g_i nt + b_i nt$, where $r_i nt$, $g_i nt$, and $b_i nt$ are obtained from $r$, $g$, and $b$ by converting from the real interval [0.0, 1.0] to the integer range 0, 1, ..., 255.

**EXAMPLES:**

```python
sage: from sage.plot.colors import float_to_integer
sage: float_to_integer(1.,1.,0.)
16776960

sage: float_to_integer(.03,.06,.02)
462597

sage: float_to_integer(*Color('brown').rgb())
10824234
```

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sage.plot.colors.get_cmap(cmap)

Returns a color map (actually, a matplotlib Colormap object), given its name or a [mixed] list/tuple of RGB list/tuples and color names. For a list of map names, evaluate:

```
sage: sorted(colormaps)
['Accent', ...
```

See rgbcolor() for valid list/tuple element formats.

INPUT:
- `cmap` - a string, list, tuple, or matplotlib.colors.Colormap; a string must be a valid color map name

OUTPUT:
- a matplotlib.colors.Colormap instance

EXAMPLES:

```
sage: from sage.plot.colors import get_cmap
sage: get_cmap('jet')
<matplotlib.colors.LinearSegmentedColormap object at 0x...>
sage: get_cmap(((0,0,0), (0.5,0.5,0.5), (1,1,1)))
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap(['green', 'lightblue', 'blue'])
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap(((0.5, 0.3, 0.2), [1.0, 0.0, 0.5], 'purple', Color(0.5,0.5,1, space='hsv')))
<matplotlib.colors.ListedColormap object at 0x...>
sage: get_cmap('jolies')
Traceback (most recent call last):
  ...
RuntimeError: Color map jolies not known (type "import matplotlib; list(matplotlib.colormaps.keys())" for valid names)
sage: get_cmap('mpl')
Traceback (most recent call last):
  ...
RuntimeError: Color map mpl not known (type "import matplotlib; list(matplotlib.colormaps.keys())" for valid names)
```

sage.plot.colors.html_to_float(c)

Convert a HTML hex color to a Red-Green-Blue (RGB) tuple.

INPUT:
- `c` - a string; a valid HTML hex color

OUTPUT:
- a RGB 3-tuple of floats in the interval [0.0, 1.0]

EXAMPLES:

```
sage: from sage.plot.colors import html_to_float
sage: html_to_float('#fff')
(1.0, 1.0, 1.0)
sage: html_to_float('#abcdef')
(0.6705882352941176, 0.803921568627451, 0.9372549019607843)
```
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sage: html_to_float('#123xyz')
Traceback (most recent call last):
...
ValueError: invalid literal for int() with base 16: '3x'

sage.plot.colors.hue(h, s=1, v=1)

Convert a Hue-Saturation-Value (HSV) color tuple to a valid Red-Green-Blue (RGB) tuple. All three inputs should lie in the interval [0.0, 1.0]; otherwise, they are reduced modulo 1 (see mod_one()). In particular \( h=0 \) and \( h=1 \) yield red, with the intermediate hues orange, yellow, green, cyan, blue, and violet as \( h \) increases.

This function makes it easy to sample a broad range of colors for graphics:

sage: p = Graphics()
sage: for phi in xsrange(0, 2 * pi, 1 / pi):
    ....:     p += plot(sin(x + phi), (x, -7, 7), rgbcolor = hue(phi))
sage: p
Graphics object consisting of 20 graphics primitives

INPUT:
- \( h \) - a number; the color's hue
- \( s \) - a number (default: 1); the color's saturation
- \( v \) - a number (default: 1); the color's value

OUTPUT:
- a RGB 3-tuple of floats in the interval [0.0, 1.0]

EXAMPLES:

sage: hue(0.6)
(0.0, 0.4000000000000036, 1.0)
sage: from sage.plot.colors import royalblue
sage: royalblue
RGB color (0.2549019607843137, 0.4117647058823529, 0.8823529411764706)
sage: hue(royalblue.hsv())
(0.2549019607843137, 0.4117647058823529, 0.8823529411764706)
sage: hue(.5, .5, .5)
(0.25, 0.5, 0.5)

Note: The HSV to RGB coordinate transformation itself is given in the source code for the Python library's colorsys module:

sage: from colorsys import hsv_to_rgb # not tested
sage: hsv_to_rgb? # not tested

sage.plot.colors.mod_one(x)

Reduce a number modulo 1.

INPUT:
- \( x \) - an instance of Integer, int, RealNumber, etc.; the number to reduce

OUTPUT:

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- a float

EXAMPLES:

```python
sage: from sage.plot.colors import mod_one
sage: mod_one(1)
1.0
sage: mod_one(7.0)
0.0
sage: mod_one(-11/7)
0.4285714285714286
sage: mod_one(pi) + mod_one(-pi)
1.0
```

`sage.plot.colors.rainbow(n, format='hex')`

Returns a list of colors sampled at equal intervals over the spectrum, from Hue-Saturation-Value (HSV) coordinates (0, 1, 1) to (1, 1, 1). This range is red at the extremes, but it covers orange, yellow, green, cyan, blue, violet, and many other hues in between. This function is particularly useful for representing vertex partitions on graphs.

INPUT:
- `n` - a number; the length of the list
- `format` - a string (default: ‘hex’); the output format for each color in the list; the other choice is ‘rgbtuple’

OUTPUT:
- a list of strings or RGB 3-tuples of floats in the interval [0.0, 1.0]

EXAMPLES:

```python
sage: from sage.plot.colors import rainbow
sage: rainbow(7)
['#ff0000', '#ffda00', '#48ff00', '#00ff91', '#0091ff', '#4800ff', '#ff00da']
sage: rainbow(int(7))
['#ff0000', '#ffda00', '#48ff00', '#00ff91', '#0091ff', '#4800ff', '#ff00da']
sage: rainbow(7, 'rgbtuple')
[(1.0, 0.0, 0.0), (1.0, 0.8571428571428571, 0.0), (0.2857142857142857, 1.0, 0.0), (0.0, 1.0, 0.5714285714285714), (0.0, 0.5714285714285714, 1.0), (0.0, 0.0, 1.0), (1.0, 0.0, 0.8571428571428577)]
```

AUTHORS:
- Robert L. Miller
- Karl-Dieter Crisman (directly use `hsv_to_rgb()` for hues)

`sage.plot.colors.rgbcolor(c, space='rgb')`

Convert a color (string, tuple, list, or `Color`) to a mod-one reduced (see `mod_one()`) valid Red-Green-Blue (RGB) tuple. The returned tuple is also a valid matplotlib RGB color.

INPUT:
- `c` - a `Color` instance, string (name or HTML hex), 3-tuple, or 3-list; the color to convert
- `space` - a string (default: ‘rgb’); the color space coordinate system (other choices are ‘hsl’, ‘hls’, and ‘hsv’) in which to interpret a 3-tuple or 3-list

OUTPUT:
- a RGB 3-tuple of floats in the interval [0.0, 1.0]
EXAMPLES:

```python
sage: from sage.plot.colors import rgbcolor
sage: rgbcolor(Color(0.25, 0.4, 0.9))
(0.25, 0.4, 0.9)
sage: rgbcolor('purple')
(0.5019607843137255, 0.0, 0.5019607843137255)
sage: rgbcolor('#fa0')
(1.0, 0.6666666666666666, 0.0)
sage: rgbcolor('ffffff')
(1.0, 1.0, 1.0)
sage: rgbcolor((1,1/2,1/3))
(1.0, 0.5, 0.3333333333333333)
sage: rgbcolor([1,1/2,1/3])
(1.0, 0.5, 0.3333333333333333)
sage: rgbcolor((1,1,1), space='hsv')
(1.0, 0.0, 0.0)
sage: rgbcolor((0.5,0.75,1), space='hls')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor((0.5,1.0,0.75), space='hsl')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor([1,2,255])  # WARNING -- numbers are reduced mod 1!!
(1.0, 0.0, 0.0)
sage: rgbcolor('#abcd')
Traceback (most recent call last):
... ValueError: color hex string (= 'abcd') must have length 3 or 6
sage: rgbcolor('fff')
Traceback (most recent call last):
... ValueError: unknown color 'fff'
sage: rgbcolor(1)
Traceback (most recent call last):
... TypeError: '1' must be a Color, list, tuple, or string
sage: rgbcolor((0.2,0.8,1), space='grassmann')
Traceback (most recent call last):
... ValueError: space must be one of 'rgb', 'hsv', 'hsl', 'hls'
sage: rgbcolor([0.4, 0.1])
Traceback (most recent call last):
... ValueError: color list or tuple '[0.4000000000000000, 0.1000000000000000]' must have →3 entries, one for each RGB, HSV, HLS, or HSL channel
```

\[ \text{sage.plot.colors.to_mpl_color}(c, \text{space}='rgb') \]

Convert a color (string, tuple, list, or \texttt{Color}) to a mod-one reduced (see \texttt{mod_one()}) valid Red-Green-Blue (RGB) tuple. The returned tuple is also a valid matplotlib RGB color.

**INPUT:**

- \texttt{c} - a \texttt{Color} instance, string (name or HTML hex), 3-tuple, or 3-list; the color to convert
- \texttt{space} - a string (default: ‘rgb’); the color space coordinate system (other choices are ‘hsl’, ‘hls’, and ‘hsv’) in which to interpret a 3-tuple or 3-list
OUTPUT:

- a RGB 3-tuple of floats in the interval $[0.0, 1.0]$

EXAMPLES:

```python
sage: from sage.plot.colors import rgbcolor
sage: rgbcolor(Color(0.25, 0.4, 0.9))
(0.25, 0.4, 0.9)
sage: rgbcolor('purple')
(0.5019607843137255, 0.0, 0.5019607843137255)
sage: rgbcolor('#fa0')
(1.0, 0.6666666666666666, 0.0)
sage: rgbcolor('#ffffff')
(1.0, 1.0, 1.0)
sage: rgbcolor((1,1/2,1/3))
(1.0, 0.5, 0.3333333333333333)
sage: rgbcolor((1,1/2,1/3))
(1.0, 0.5, 0.3333333333333333)
sage: rgbcolor((1,1,1), space='hsv')
(1.0, 0.0, 0.0)
sage: rgbcolor((0.5,0.75,1), space='hls')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor((0.5,1.0,0.75), space='hsl')
(0.5, 0.9999999999999999, 1.0)
sage: rgbcolor((1,2,255))  # WARNING -- numbers are reduced mod 1!!
(1.0, 0.0, 0.0)
sage: rgbcolor('#abcd')
Traceback (most recent call last):
  ...  % Value Error: color hex string (= 'abcd') must have length 3 or 6
sage: rgbcolor('fff')
Traceback (most recent call last):
  ...  % Value Error: unknown color 'fff'
```

```python
sage: rgbcolor(1)
Traceback (most recent call last):
  ...  % TypeError: '1' must be a Color, list, tuple, or string
sage: rgbcolor((0.2,0.8,1), space='grassmann')
Traceback (most recent call last):
  ...  % ValueError: space must be one of 'rgb', 'hsv', 'hsl', 'hls'
```

```python
sage: rgbcolor([0.4, 0.1])
Traceback (most recent call last):
  ...  % ValueError: color list or tuple '[0.4000000000000000, 0.1000000000000000]' must have 3 entries, one for each RGB, HSV, HLS, or HSL channel
```
1.4 Animated plots

Animations are generated from a list (or other iterable) of graphics objects. Images are produced by calling the `save_image` method on each input object, creating a sequence of PNG files. These are then assembled to various target formats using different tools. In particular, the `convert` program from ImageMagick can be used to generate an animated GIF file. FFmpeg (with the command line program `ffmpeg`) provides support for various video formats, but also an alternative method of generating animated GIFs. For browsers which support it, APNG can be used as another alternative which works without any extra dependencies.

**Warning:** Note that ImageMagick and FFmpeg are not included with Sage, and must be installed by the user. On unix systems, type `which convert` at a command prompt to see if `convert` (part of the ImageMagick suite) is installed. If it is, you will be given its location. Similarly, you can check for `ffmpeg` with `which ffmpeg`. See the websites of ImageMagick or FFmpeg for installation instructions.

**EXAMPLES:**

The sine function:

```python
def animate(sines):
    a = animate(sines)
    print(a)
    return a
```

```python
x = SR.var("x")
sines = [plot(c*sin(x), (-2*pi,2*pi), color=Color(c,0,0), ymin=-1, ymax=1)
    for c in sxrange(0,1,.2)]
a = animate(sines)
```

Animate using FFmpeg instead of ImageMagick:

```python
a.show(use_ffmpeg=True) # long time # optional -- FFmpeg
```

Animate as an APNG:

```python
E.show(use_ffmpeg=True) # long time # optional -- FFmpeg
```

An animated `sage.plot.multigraphics.GraphicsArray` of rotating ellipses:

```python
E = animate((graphics_array([[ellipse((0,0), a, b, angle=t, xmin=-3, xmax=3)
    + circle((0,0), 3, color='blue')
    for a in range(1,3)]
    for b in range(2,4)])
    for t in sxrange(0, pi/4, .15)))
```

A simple animation of a circle shooting up to the right:

```python
c = animate([circle((i,i), 1 - 1/(i+1), hue=i/10)
    for i in sxrange(0, 2, .2)],
    xmin=0, ymin=0, xmax=2, ymax=2, figsize=[2,2])
c.show() # long time # optional -- ImageMagick
```

Animations of 3d objects:
```
sage: s, t = SR.var("s, t")
sage: def sphere_and_plane(x):
    ....:     return (sphere((0,0,0), 1, color='red', opacity=.5)
    ....:        + parametric_plot3d([t,x,s], (s,-1,1), (t,-1,1),
    ....:            color='green', opacity=.7))
sage: sp = animate([sphere_and_plane(x)
    ....:     for x in sxrange(-1, 1, .3)])
sage: sp[0] # first frame
Graphics3d Object
sage: sp[-1] # last frame
Graphics3d Object
sage: sp.show() # long time # optional -- ImageMagick
```

```
sage: (x,y,z) = SR.var("x,y,z")
sage: def frame(t):
    ....:     return implicit_plot3d((x^2 + y^2 + z^2),
    ....:        (x, -2, 2), (y, -2, 2), (z, -2, 2),
    ....:        plot_points=60, contour=[1,3,5],
    ....:        region=lambda x,y,z: x<=t or y>=t or z<=t)
sage: a = animate([frame(t) for t in srange(0.01, 1.5, .2)])
sage: a[0] # long time
Graphics3d Object
sage: a.show() # long time # optional -- ImageMagick
```

If the input objects do not have a `save_image` method, then the animation object attempts to make an image by calling its internal method `sage.plot.animate.Animation.make_image()`. This is illustrated by the following example:

```
sage: t = SR.var("t")
sage: a = animate((sin(c*pi*t) for c in sxrange(1, 2, .2)))
sage: a.show() # long time # optional -- ImageMagick
```

AUTHORS:

- William Stein
- John Palmieri
- Niles Johnson (2013-12): Expand to animate more graphics objects
- Martin von Gagern (2014-12): Added APNG support
- Joshua Campbell (2020): interactive animation via Three.js viewer

REFERENCES:

- ImageMagick
- FFmpeg
- APNG
- browsers which support it

```
class sage.plot.animate.APngAssembler(out, num_frames, num_plays=0, delay=200,
delay_denominator=100)
```

Bases: object

Builds an APNG (Animated PNG) from a sequence of PNG files. This is used by the `sage.plot.animate.Animation.apng()` method.
This code is quite simple; it does little more than copying chunks from input PNG files to the output file. There is no optimization involved. This does not depend on external programs or libraries.

INPUT:

- `out` – a file opened for binary writing to which the data will be written
- `num_frames` – the number of frames in the animation
- `num_plays` – how often to iterate, 0 means infinitely
- `delay` – numerator of the delay fraction in seconds
- `delay_denominator` – denominator of the delay in seconds

EXAMPLES:

```python
sage: from sage.plot.animate import APngAssembler
sage: x = SR.var("x")
sage: def assembleAPNG():
....:     a = animate([sin(x + float(k)) for k in srange(0, 2*pi, 0.7)],
....:                  xmin=0, xmax=2*pi, figsize=[2, 1])
....:     pngdir = a.png()
....:     outfile = sage.misc.temporary_file.tmp_filename(ext='.png')
....:     with open(outfile, "wb") as f:
....:         apng = APngAssembler(f, len(a))
....:         for i in range(len(a)):
....:             png = os.path.join(pngdir, "{:08d}.png".format(i))
....:             apng.add_frame(png, delay=10*i + 10)
....:     return outfile
sage: assembleAPNG() # long time
'...png'
```

`add_frame(pngfile, delay=None, delay_denominator=None)`

Adds a single frame to the APNG file.

INPUT:

- `pngfile` – file name of the PNG file with data for this frame
- `delay` – numerator of the delay fraction in seconds
- `delay_denominator` – denominator of the delay in seconds

If the delay is not specified, the default from the constructor applies.

`magic = b'\x89PNG\r\n\x1a\n'`

`mustmatch = frozenset({b'IHDR', b'PLTE', b'BKGD', b'cHRM', b'gAMA', b'PHYS',
        b'sBIT', b'tRNS'})`

`set_default(pngfile)`

Adds a default image for the APNG file.

This image is used as a fallback in case some application does not understand the APNG format. This method must be called prior to any calls to the `add_frame` method, if it is called at all. If it is not called, then the first frame of the animation will be the default.

INPUT:

- `pngfile` – file name of the PNG file with data for the default image
class sage.plot.animate.Animation(v=None, **kwds)

Bases: WithEqualityById, SageObject

Return an animation of a sequence of plots of objects.

INPUT:

• v – iterable of Sage objects. These should preferably be graphics objects, but if they aren’t, then
  make_image() is called on them.

• xmin, xmax, ymin, ymax – the ranges of the x and y axes.

• **kwds – all additional inputs are passed onto the rendering command. E.g., use figsize to adjust the
  resolution and aspect ratio.

EXAMPLES:

sage: x = SR.var("x")
sage: a = animate([sin(x + float(k)) for k in srange(0, 2*pi, 0.3)],
               xmin=0, xmax=2*pi, figsize=[2,1])
sage: print(a)
Animation with 21 frames
sage: print(a[:5])  # long time # optional -- ImageMagick
Animation with 5 frames
sage: a.show()  # long time # optional -- ImageMagick
sage: a[:5].show()  # long time # optional -- ImageMagick

The show() method takes arguments to specify the delay between frames (measured in hundredths of a second, default value 20) and the number of iterations (default value 0, which means to iterate forever). To iterate 4 times with half a second between each frame:

sage: a.show(delay=50, iterations=4)  # long time # optional -- ImageMagick

An animation of drawing a parabola:

sage: step = 0.1
sage: L = Graphics()
sage: v = []
sage: for i in srange(0, 1, step):
    L += line([(i,i^2),(i+step,(i+step)^2)], rgbcolor=(1,0,0), thickness=2)
    v.append(L)
sage: a = animate(v, xmin=0, ymin=0)
sage: a.show()  # long time # optional -- ImageMagick
sage: show(L)

apng(savefile=None, show_path=False, delay=20, iterations=0)

Creates an animated PNG composed from rendering the graphics objects in self. Return the absolute path
to that file.

Notice that not all web browsers are capable of displaying APNG files, though they should still present the
first frame of the animation as a fallback.

The generated file is not optimized, so it may be quite large.

Input:

• delay - (default: 20) delay in hundredths of a second between frames

• savefile - file that the animated gif gets saved to
• **iterations** - integer (default: 0); number of iterations of animation. If 0, loop forever.
• **show_path** - boolean (default: False); if True, print the path to the saved file

**EXAMPLES:**

```sage
sage: x = SR.var("x")
sage: a = animate([sin(x + float(k))
.....: for k in srange(0, 2*pi, 0.7)],
.....: xmin=0, xmax=2*pi, figsize=[2,1])
sage: dir = tmp_dir()
sage: a.apng(show_path=True) # long time
Animation saved to .../my_animation.png.
sage: a.apng(savefile=dir + 'my_animation.png', delay=35, iterations=3) # long time
Animation saved to .../my_animation.png.
sage: a.apng(savefile=dir + 'my_animation.png', show_path=True) # long time
```

If the individual frames have different sizes, an error will be raised:

```sage
sage: a = animate([plot(sin(x), (x, 0, k))
.....: for k in range(1,4)],
.....: ymin=-1, ymax=1, aspect_ratio=1, figsize=[2,1])
sage: a.apng() # long time
Traceback (most recent call last):
... ValueError: Chunk IHDR mismatch
```

**ffmpeg**

```sage
ffmpeg(savefile=None, show_path=False, output_format=None, ffmpeg_options='', delay=None, iterations=0, pix_fmt='rgb24')
```

Return a movie showing an animation composed from rendering the frames in `self`.

This method will only work if ffmpeg is installed. See [https://www.ffmpeg.org](https://www.ffmpeg.org) for information about ffmpeg.

**INPUT:**

• **savefile** – file that the mpeg gets saved to.

**Warning:** This will overwrite `savefile` if it already exists.

• **show_path** – boolean (default: False); if True, print the path to the saved file
• **output_format** - string (default: None); format and suffix to use for the video. This may be 'mpg', 'mpeg', 'avi', 'gif', or any other format that ffmpeg can handle. If this is None and the user specifies `savefile` with a suffix, say `savefile='animation.avi'`, try to determine the format ('avi' in this case) from that file name. If no file is specified or if the suffix cannot be determined, 'mpg' is used.
• **ffmpeg_options** - string (default: ''); this string is passed directly to ffmpeg.
• **delay** - integer (default: None); delay in hundredths of a second between frames. The framerate is 100/delay. This is not supported for mpeg files: for mpegs, the frame rate is always 25 fps.
• **iterations** - integer (default: 0); number of iterations of animation. If 0, loop forever. This is only supported for animated gif output and requires ffmpeg version 0.9 or later. For older versions, set iterations=None.
• pix_fmt - string (default: ‘rgb24’); used only for gif output. Different values such as ‘rgb8’ or ‘pal8’ may be necessary depending on how ffmpeg was installed. Set pix_fmt=None to disable this option.

If savefile is not specified: in notebook mode, display the animation; otherwise, save it to a default file name. Use sage.misc.verbose.setVerbose() with level=1 to see additional output.

EXAMPLES:

```python
sage: x = SR.var("x")
sage: a = animate([sin(x + float(k))
    ....:     for k in srange(0, 2*pi, 0.7)],
    ....:     xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2,1])
sage: td = tmp_dir()
sage: a.ffmpeg(savefile=td + 'new.mpg')  # long time  #optional -- FFmpeg
˓→optional -- FFmpeg
sage: a.ffmpeg(savefile=td + 'new.avi')  # long time  #optional -- FFmpeg
˓→optional -- FFmpeg
sage: a.ffmpeg(savefile=td + 'new.gif')  # long time  #optional -- FFmpeg
˓→optional -- FFmpeg
sage: a.ffmpeg(savefile=td + 'new.mpg', show_path=True)  # long time  #optional -- FFmpeg
Animation saved to .../new.mpg.
```

Note: If ffmpeg is not installed, you will get an error message like this:

```
FeatureNotPresentError: ffmpeg is not available.
Executable 'ffmpeg' not found on PATH.
Further installation instructions might be available at https://www.ffmpeg.org/.
```

`gif`(delay=20, savefile=None, iterations=0, show_path=False, use_ffmpeg=False)

Returns an animated gif composed from rendering the graphics objects in self.

This method will only work if either (a) the ImageMagick software suite is installed, i.e., you have the convert command or (b) ffmpeg is installed. See the web sites of ImageMagick and FFmpeg for more details. By default, this produces the gif using convert if it is present. If this can’t find convert or if use_ffmpeg is True, then it uses ffmpeg instead.

INPUT:

• delay - (default: 20) delay in hundredths of a second between frames
• savefile - file that the animated gif gets saved to
• iterations - integer (default: 0); number of iterations of animation. If 0, loop forever.
• show_path - boolean (default: False); if True, print the path to the saved file
• use_ffmpeg - boolean (default: False); if True, use ‘ffmpeg’ by default instead of ‘convert’.

If savefile is not specified: in notebook mode, display the animation; otherwise, save it to a default file name.

EXAMPLES:

```python
sage: x = SR.var("x")
sage: a = animate([sin(x + float(k))
```
....:        for k in srange(0, 2*pi, 0.7),
....:            xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2, 1])
sage: td = tmp_dir()  
# not tested
sage: a.gif()  
# long time 
← optional -- ImageMagick
sage: a.gif(savefile=td + 'my_animation.gif',  
# long time 
← optional -- ImageMagick
delay=35, iterations=3)
sage: with open(td + 'my_animation.gif', 'rb') as f:  
# long time 
← optional -- ImageMagick
    print(b'GIF8' in f.read())
True
sage: a.gif(savefile=td + 'my_animation.gif',  
# long time 
← optional -- ImageMagick
    show_path=True)
Animation saved to .../my_animation.gif.
sage: a.gif(savefile=td + 'my_animation_2.gif',  
# long time 
← optional -- FFmpeg
delay=10,  
← FFmpeg
show_path=True, use_ffmpeg=True)
Animation saved to .../my_animation_2.gif.

Note: If neither ffmpeg nor ImageMagick is installed, you will get an error message like this:

Error: Neither ImageMagick nor ffmpeg appears to be installed. Saving an animation to a GIF file or displaying an animation requires one of these packages, so please install one of them and try again.


graphics_array(ncols=3)

Return a sage.plot.multigraphics.GraphicsArray with plots of the frames of this animation, using the given number of columns. The frames must be acceptable inputs for sage.plot.multigraphics.GraphicsArray.

EXAMPLES:

sage: # needs sage.schemes
sage: E = EllipticCurve('37a')
sage: v = [E.change_ring(GF(p)).plot(pointsize=30)  
.....:     for p in [97, 101, 103]]
sage: a = animate(v, xmin=0, ymin=0, axes=False)
sage: print(a)
Animation with 3 frames
sage: a.show()  
# optional -- ImageMagick

Modify the default arrangement of array:

sage: g = a.graphics_array(); print(g)  
← needs sage.schemes
Graphics Array of size 1 x 3
sage: g.show(figsize=[6, 3])  
← needs sage.schemes

1.4. Animated plots
Specify different arrangement of array and save it with a given file name:

```
sage: g = a.graphics_array(ncols=2); print(g)  # needs sage.schemes
Graphics Array of size 2 x 2
sage: f = tmp_filename(ext='.png'); print(f)  # needs sage.schemes
...png
sage: g.save(f)  # needs sage.schemes
```

Frames can be specified as a generator too; it is internally converted to a list:

```
sage: t = SR.var("t")
sage: b = animate((plot(sin(c*pi*t)) for c in sxrange(1,2,.2)))
sage: g = b.graphics_array()
sage: print(g)  # needs sage.schemes
Graphics Array of size 2 x 3
```

**interactive(**\*\*\kwds\**\)

Create an interactive depiction of the animation.

**INPUT:**

- \*\*\kwds\* – any of the viewing options accepted by show() are valid as keyword arguments to this function and they will behave in the same way. Those that are animation-related and recognized by the Three.js viewer are: animate, animation_controls, auto_play, delay, and loop.

**OUTPUT:**

A 3D graphics object which, by default, will use the Three.js viewer.

**EXAMPLES:**

```
sage: x = SR.var("x")
sage: frames = [point3d((sin(x), cos(x), x))
...: for x in (0, pi/16, .., 2*pi)]
sage: animate(frames).interactive(online=True)
Graphics3d Object
```

Works with frames that are 2D or 3D graphics objects or convertible to 2D or 3D graphics objects via a plot or plot3d method:

```
sage: frames = [dodecahedron(), circle(center=(0, 0), radius=1), x^2]
sage: animate(frames).interactive(online=True, delay=100)
Graphics3d Object
```

**See also:**

Three.js JavaScript WebGL Renderer

**make_image** *(frame, filename, \*\*\kwds)*

Given a frame which has no save_image() method, make a graphics object and save it as an image with the given filename. By default, this is sage.plot.plot.plot(). To make animations of other objects, override this method in a subclass.

**EXAMPLES:**
sage: from sage.plot.animate import Animation
sage: class MyAnimation(Animation):
    ....:     def make_image(self, frame, filename, **kwds):
    ....:         P = parametric_plot(frame[0], frame[1], **frame[2])
    ....:         P.save_image(filename,**kwds)

sage: t = SR.var("t")
sage: x = lambda t: cos(t)
sage: y = lambda n,t: sin(t)/n
sage: B = MyAnimation([[(x(t), y(i+1,t)], (t,0,1),
    ....: 'color':Color((1,0,i/4)), 'aspect_ratio':1, 'ymax':1})
    ....:     for i in range(4)])

sage: d = B.png(); v = os.listdir(d); v.sort(); v
# long time
['00000000.png', '00000001.png', '00000002.png', '00000003.png']
sage: B.show() # not tested

sage: class MyAnimation(Animation):
    ....:     def make_image(self, frame, filename, **kwds):
    ....:         G = frame.plot()
    ....:         G.set_axes_range(floor(G.xmin()), ceil(G.xmax()),
    ....:         floor(G.ymin()), ceil(G.ymax()))
    ....:         G.save_image(filename, **kwds)

sage: B = MyAnimation([graphs.CompleteGraph(n) for n in range(7,11)], figsize=5)

sage: d = B.png()
sage: v = os.listdir(d); v.sort(); v
['00000000.png', '00000001.png', '00000002.png', '00000003.png']
sage: B.show() # not tested

png(dir=None)

Render PNG images of the frames in this animation, saving them in dir. Return the absolute path to that
directory. If the frames have been previously rendered and dir is None, just return the directory in which
they are stored.

When dir is other than None, force re-rendering of frames.

INPUT:

• dir – Directory in which to store frames. Default None; in this case, a temporary directory will be
  created for storing the frames.

OUTPUT:

Absolute path to the directory containing the PNG images

EXAMPLES:

sage: x = SR.var("x")
sage: a = animate([plot(x^2 + n) for n in range(4)], ymin=0, ymax=4)
sage: d = a.png(); v = os.listdir(d); v.sort(); v
# long time
['00000000.png', '00000001.png', '00000002.png', '00000003.png']

save(filename=None, show_path=False, use_ffmpeg=False, **kwds)

Save this animation.

INPUT:
filename - (default: None) name of save file

• show_path - boolean (default: False); if True, print the path to the saved file

• use_ffmpeg - boolean (default: False); if True, use ‘ffmpeg’ by default instead of ‘convert’ when creating GIF files.

If filename is None, then in notebook mode, display the animation; otherwise, save the animation to a GIF file. If filename ends in '.html', save an interactive() version of the animation to an HTML file that uses the Three.js viewer. If filename ends in '.sobj', save to an sobj file. Otherwise, try to determine the format from the filename extension ('.mpg', '.gif', '.avi', etc.). If the format cannot be determined, default to GIF.

For GIF files, either ffmpeg or the ImageMagick suite must be installed. For other movie formats, ffmpeg must be installed. sobj and HTML files can be saved with no extra software installed.

EXAMPLES:

```sage
x = SR.var("x")
a = animate([sin(x + float(k)) for k in srange(0, 2*pi, 0.7)],
            xmin=0, xmax=2*pi, ymin=-1, ymax=1, figsize=[2,1])
```

```sage
td = tmp_dir()
asave()  # not tested
```

```sage
asave(td + 'wave.gif')  # long time # optional -- ImageMagick
```

```sage
asave(td + 'wave.gif', show_path=True)  # long time # optional -- ImageMagick
```

Animation saved to file .../wave.gif.

```sage
asave(td + 'wave.avi', show_path=True)  # long time # optional -- FFmpeg
```

Animation saved to file .../wave.avi.

```sage
asave(td + 'wave0.sobj')
asave(td + 'wave1.sobj', show_path=True)
animation saved to file .../wave1.sobj.
```

```sage
asave(td + 'wave0.html', online=True)
asave(td + 'wave1.html', show_path=True, online=True)
animation saved to file .../wave1.html.
```

show(delay=None, iterations=None, **kwds)

Show this animation immediately.

This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

INPUT:

• delay – (default: 20) delay in hundredths of a second between frames.

• iterations – integer (default: 0); number of iterations of animation. If 0, loop forever.

• format - (default: gif) format to use for output. Currently supported formats are: gif, ogg, webm, mp4, flash, matroska, avi, wmv, quicktime.

OUTPUT:

This method does not return anything. Use save() if you want to save the figure as an image.
Note: Currently this is done using an animated gif, though this could change in the future. This requires that either ffmpeg or the ImageMagick suite (in particular, the convert command) is installed.

See also the ffmpeg() method.

EXAMPLES:

```python
sage: x = SR.var("x")
sage: a = animate([[sin(x + float(k))]
                ....: for k in srange(0,2*pi,0.7)],
                ....: xmin=0, xmax=2*pi, figsize=[2,1])
sage: a.show() # long time # optional -- ImageMagick
```

The preceding will loop the animation forever. If you want to show only three iterations instead:

```python
sage: a.show(iterations=3) # long time # optional -- ImageMagick
```

To put a half-second delay between frames:

```python
sage: a.show(delay=50) # long time # optional -- ImageMagick
```

You can also make use of the HTML5 video element in the Sage Notebook:

```python
sage: # long time, optional -- FFmpeg
sage: a.show(format="ogg")
sage: a.show(format="webm")
sage: a.show(format="mp4")
sage: a.show(format="webm", iterations=1)
```

Other backends may support other file formats as well:

```python
sage: # long time, optional -- FFmpeg
sage: a.show(format="flash")
sage: a.show(format="matroska")
sage: a.show(format="avi")
sage: a.show(format="wmv")
sage: a.show(format="quicktime")
```

Note: If you don’t have ffmpeg or ImageMagick installed, you will get an error message like this:

```
Error: Neither ImageMagick nor ffmpeg appears to be installed. Saving an animation to a GIF file or displaying an animation requires one of these packages, so please install one of them and try again.
```


```
sage.plot.animate.animate(frames, **kwds)
    Animate a list of frames by creating a sage.plot.animate.Animation object.

EXAMPLES:
```

1.4. Animated plots
sage: t = SR.var("t")
sage: a = animate((cos(c*pi*t) for c in sxrange(1, 2, .2)))
sage: a.show()  # long time  # optional -- ImageMagick

See also *sage.plot.animate* for more examples.
2.1 Complex plots

AUTHORS:

• Robert Bradshaw (2009): initial version
• David Lowry-Duda (2022): incorporate matplotlib colormaps

```python
class sage.plot.complex_plot.ComplexPlot(rgb_data, x_range, y_range, options):
    Bases: GraphicPrimitive

    The GraphicsPrimitive to display complex functions in using the domain coloring method
    INPUT:
    • rgb_data – An array of colored points to be plotted.
    • x_range – A minimum and maximum x value for the plot.
    • y_range – A minimum and maximum y value for the plot.

    get_minmax_data()
    Return a dictionary with the bounding box data.
```

```python
sage: p = complex_plot(lambda z: z, (-1, 2), (-3, 4))
sage: sorted(p.get_minmax_data().items())
[('xmax', 2.0), ('xmin', -1.0), ('ymax', 4.0), ('ymin', -3.0)]
sage: p = complex_plot(lambda z: z, (1, 2), (3, 4))
sage: sorted(p.get_minmax_data().items())
[('xmax', 2.0), ('xmin', 1.0), ('ymax', 4.0), ('ymin', 3.0)]
```

```python
sage.plot.complex_plot.add_contours_to_rgb(rgb, delta, dark_rate=0.5)
```

Return an rgb array from given array of \((r, g, b)\) and \((\delta)\).

Each input \((r, g, b)\) is modified by \(\delta\) to be lighter or darker depending on the size of \(\delta\). Negative \(\delta\) values darken the color, while positive \(\delta\) values lighten the pixel.

We assume that the \(\delta\) values come from a function like `sage.plot.complex_plot.mag_to_lightness()`, which maps magnitudes to the range \([-1, +1]\).

INPUT:

• rgb – a grid of length 3 tuples \((r, g, b)\), as an \(N \times M \times 3\) numpy array.
• **delta** – a grid of values as an $N \times M$ numpy array; these represent how much to change the lightness of each $(r, g, b)$. Values should be in $[-1, 1]$.

• **dark_rate** – a positive number (default: 0.5); affects how strongly visible the contours appear.

**OUTPUT:**

An $N \times M \times 3$ floating point Numpy array $X$, where $X[i,j]$ is an $(r, g, b)$ tuple.

**See also:**

- `sage.plot.complex_plot.complex_to_rgb()`
- `sage.plot.complex_plot.add_lightness_smoothing_to_rgb()`

**ALGORITHM:**

Each pixel and lightness-delta is mapped from $(r, g, b, \text{delta}) \mapsto (h, l, s, \text{delta})$ using the standard RGB-to-HLS formula.

Then the lightness is adjusted via $l \mapsto l' = l + 0.5 \cdot \text{delta}$.

Finally map $(h, l', s) \mapsto (r, g, b)$ using the standard HLS-to-RGB formula.

**EXAMPLES:**

```python
sage: # needs numpy
sage: import numpy as np
sage: from sage.plot.complex_plot import add_lightness_smoothing_to_rgb
sage: add_lightness_smoothing_to_rgb(np.array([[0, 0.25, 0.5]]), # abs tol 1e-4
.....: np.array([[0.75]]))
array([[0.25, 0.625, 1.]]])
sage: add_lightness_smoothing_to_rgb(np.array([[0, 0, 0]]), # abs tol 1e-4
.....: np.array([[1]]))
array([[0.5, 0.5, 0.5]])
sage: add_lightness_smoothing_to_rgb(np.array([[1, 1, 1]]), # abs tol 1e-4
.....: np.array([[-0.5]])
array([[0.75, 0.75, 0.75]])]
```

Raising dark_rate leads to bigger adjustments:

```python
sage: add_lightness_smoothing_to_rgb(np.array([[0.5, 0.5, 0.5]]), # abs tol 1e-4
.....: np.array([[[0.5]]], dark_rate=0.1)
array([[0.55, 0.55, 0.55]])
sage: add_lightness_smoothing_to_rgb(np.array([[0.5, 0.5, 0.5]]), # abs tol 1e-4
.....: np.array([[[0.5]]], dark_rate=0.5)
array([[0.75, 0.75, 0.75]])]
```

sage.plot.complex_plot.add_lightness_smoothing_to_rgb(rgb, delta)

Return an rgb array from given array of colors and lightness adjustments.

This smoothly adds lightness from black (when delta is $-1$) to white (when delta is $1$).

Each input $(r, g, b)$ is modified by delta to be lighter or darker depending on the size of delta. When delta is $-1$, the output is black. When delta is $+1$, the output is white. Colors piecewise-linearly vary from black to the initial $(r, g, b)$ to white.
We assume that the \texttt{delta} values come from a function like \texttt{sage.plot.complex_plot.mag_to_lightness()}, which maps magnitudes to the range $[-1,+1]$.

**INPUT:**

- \texttt{rgb} – a grid of length 3 tuples $(r, g, b)$, as an $N \times M \times 3$ numpy array.
- \texttt{delta} – a grid of values as an $N \times M$ numpy array; these represent how much to change the lightness of each $(r, g, b)$. Values should be in $[-1,1]$.

**OUTPUT:**

An $N \times M \times 3$ floating point Numpy array $X$, where $X[i,j]$ is an $(r, g, b)$ tuple.

**See also:**

- \texttt{sage.plot.complex_plot.complex_to_rgb()}
- \texttt{sage.plot.complex_plot.add_contours_to_rgb()}

**EXAMPLES:**

We can call this on grids of values:

```python
sage: # needs numpy
sage: import numpy as np
sage: from sage.plot.complex_plot import add_lightness_smoothing_to_rgb
sage: add_lightness_smoothing_to_rgb(  # abs tol 1e-4
    ....: np.array([[0, 0.25, 0.5]]), np.array([[0.75]]))
array([[0.75 , 0.8125, 0.875 ]])
sage: add_lightness_smoothing_to_rgb(  # abs tol 1e-4
    ....: np.array([[0, 0.25, 0.5]]), np.array([[0.75]]))
array([[0.75 , 0.8125, 0.875 ]])
```

\texttt{sage.plot.complex_plot.complex_plot}(f, x_range, y_range, contoured=False, tiled=False, cmap=None, contour_type='logarithmic', contour_base=None, dark_rate=0.5, nphases=10, plot_points=100, interpolation='catrom', **options)

\texttt{complex_plot} takes a complex function of one variable, $f(z)$ and plots output of the function over the specified $x\_\text{range}$ and $y\_\text{range}$ as demonstrated below. The magnitude of the output is indicated by the brightness and the argument is represented by the hue.

By default, zero magnitude corresponds to black output, infinite magnitude corresponds to white output. The options \texttt{contoured}, \texttt{tiled}, and \texttt{cmap} affect the output.

\texttt{complex_plot}(f, (xmin, xmax), (ymin, ymax), contoured, tiled, cmap, ...)

**INPUT:**

- \texttt{f} – a function of a single complex value $x + iy$
- \texttt{(xmin, xmax)} – 2-tuple, the range of $x$ values
- \texttt{(ymin, ymax)} – 2-tuple, the range of $y$ values
- \texttt{cmap} – None, or the string name of a matplotlib colormap, or an instance of a matplotlib Colormap, or the special string \texttt{'matplotlib'} (default: None); If None, then hues are chosen from a standard color wheel, cycling from red to yellow to blue. If matplotlib, then hues are chosen from a preset matplotlib colormap.

The following named parameter inputs can be used to add contours and adjust their distribution:
• **contoured** – boolean (default: False); causes the magnitude to be indicated by logarithmically spaced 'contours'. The magnitude along one contour is either twice or half the magnitude along adjacent contours.

• **dark_rate** – a positive number (default: 0.5); affects how quickly magnitudes affect how light/dark the image is. When there are contours, this affects how visible each contour is. Large values (near 1.0) have very strong, immediate effects, while small values (near 0.0) have gradual effects.

• **tiled** – boolean (default: False); causes the magnitude to be indicated by logarithmically spaced 'contours' as in **contoured**, and in addition for there to be 10 evenly spaced phase contours.

• **nphases** – a positive integer (default: 10); when **tiled**=True, this is the number of divisions the phase is divided into.

• **contour_type** – either 'logarithmic', or 'linear' (default: 'logarithmic'); causes added contours to be of given type when **contoured**=True.

• **contour_base** – a positive integer; when **contour_type** is 'logarithmic', this sets logarithmic contours at multiples of **contour_base** apart. When **contour_type** is 'linear', this sets contours at distances of **contour_base** apart. If **None**, then a default is chosen depending on **contour_type**.

The following inputs may also be passed in as named parameters:

• **plot_points** – integer (default: 100); number of points to plot in each direction of the grid

• **interpolation** – string (default: 'catrom'); the interpolation method to use: 'bilinear', 'bicubic', 'spline16', 'spline36', 'quadric', 'gaussian', 'sinc', 'bessel', 'mitchell', 'lanczos', 'catrom', 'hermite', 'hanning', 'hamming', 'kaiser'

Any additional parameters will be passed to **show()**, as long as they’re valid.

**Note:** Matplotlib colormaps can be chosen or customized to cater to different types of vision. The colormaps ‘cividis’ and ‘viridis’ in matplotlib are designed to be perceptually uniform to a broader audience. The colormap ‘turbo’ is similar to the default but with more even contrast. See [NAR2018] for more information about colormap choice for scientific visualization.

**EXAMPLES:**

Here we plot a couple of simple functions:

```
sage: complex_plot(sqrt(x), (-5, 5), (-5, 5))      # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

```
sage: complex_plot(sin(x), (-5, 5), (-5, 5))       # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

```
sage: complex_plot(log(x), (-10, 10), (-10, 10))   # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

```
sage: complex_plot(exp(x), (-10, 10), (-10, 10))   # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

A plot with a different choice of colormap:
2.1. Complex plots
2.1. Complex plots
A function with some nice zeros and a pole:

```sage
sage: f(z) = z^5 + z - 1 + 1/z
```

```
Graphics object consisting of 1 graphics primitive
```

The same function as above, but tiled and with the `plasma` colormap:
2.1. Complex plots
When using `tiled=True`, the number of phase subdivisions can be controlled by adjusting `nphases`. We make the same plot with fewer tilings:

```python
sage: f(z) = z^5 + z - 1 + 1/z
    # needs sage.symbolic
sage: complex_plot(f, (-3, 3), (-3, 3), plot_points=300, # needs sage.symbolic
    ....:     tiled=True, cmap='plasma')
Graphics object consisting of 1 graphics primitive
```

It is also possible to use `linear` contours. We plot the same function above on an inset, setting contours to appear 1 apart:

```python
sage: f(z) = z^5 + z - 1 + 1/z
    # needs sage.symbolic
sage: complex_plot(f, (0, 1), (0, 1), plot_points=300, # needs sage.symbolic
    ....:     tiled=True, nphases=5, cmap='plasma')
Graphics object consisting of 1 graphics primitive
```
2.1. Complex plots
Note that tightly spaced contours can lead to Moiré patterns and aliasing problems. For example:

```
sage: f(z) = z^5 + z - 1 + 1/z
sage: complex_plot(f, (-3, 3), (-3, 3), plot_points=300, contoured=True, contour_type='linear', contour_base=1)
```

When choosing colormaps, cyclic colormaps such as `twilight` or `hsv` might be considered more appropriate for showing changes in phase without sharp color contrasts:

```
sage: f(z) = z^5 + z - 1 + 1/z
sage: complex_plot(f, (-3, 3), (-3, 3), plot_points=300, cmap='twilight')
```

Passing `matplotlib` as the colormap gives a special colormap that is similar to the default:
2.1. Complex plots
Chapter 2. Function and Data Plots
sage: \( f(z) = z^5 + z - 1 + 1/z \)  
\arrow needs sage.symbolic

sage: complex_plot(f, (-3, 3), (-3, 3),  
\arrow needs sage.symbolic
....:   plot_points=300, contoured=True, cmap='matplotlib')
Graphics object consisting of 1 graphics primitive

Here is the identity, useful for seeing what values map to what colors:

sage: complex_plot(lambda z: z, (-3, 3), (-3, 3))  
\arrow needs sage.symbolic
Graphics object consisting of 1 graphics primitive

The Riemann Zeta function:

sage: complex_plot(zeta, (-30,30), (-30,30))  
\arrow needs sage.symbolic
Graphics object consisting of 1 graphics primitive

For advanced usage, it is possible to tweak many parameters. Increasing \texttt{dark\_rate} will make regions become darker/lighter faster when there are no contours:
Chapter 2. Function and Data Plots
2.1. Complex plots
Decreasing `dark_rate` has the opposite effect. When there are contours, adjust `dark_rate` affects how visible contours are. Compare:

```sage
complex_plot(zeta, (-1, 9), (10, 20), plot_points=200, contoured=True, cmap='twilight', dark_rate=0.2)
```
Graphics object consisting of 1 graphics primitive

```sage
complex_plot(zeta, (-1, 9), (10, 20), plot_points=200, contoured=True, cmap='twilight', dark_rate=0.75)
```
Graphics object consisting of 1 graphics primitive

In practice, different values of `dark_rate` will work well with different colormaps. Extra options will get passed on to `show()`, as long as they are valid:
2.1. Complex plots

2D Graphics, Release 10.1
REFERENCES:
Plotting complex functions with colormaps follows the strategy from [LD2021] and incorporates contour tech-
niques described in [WegSem2010].

Convert a grid of complex numbers to a grid of rgb values using colors taken from given colormap.

INPUT:
- `z_values` – A grid of complex numbers, as a list of lists
- `cmap` – the string name of a matplotlib colormap, or an instance of a matplotlib Colormap (default: 'turbo').
- `contoured` – boolean (default: False); causes magnitude to be indicated through contour-like adjustments to lightness.
- `tiled` – boolean (default: False); causes magnitude and argument to be indicated through contour-like adjustments to lightness.
- `nphases` – a positive integer (default: 10); when `tiled=True`, this is the number of divisions the phase is divided into.
- `contour_type` – either 'logarithmic', or 'linear' (default: 'logarithmic'); causes added contours to be of given type when `contoured=True`
- `contour_base` – a positive integer; when `contour_type` is 'logarithmic', this sets logarithmic contours at multiples of `contour_base` apart. When `contour_type` is 'linear', this sets contours at distances of `contour_base` apart. If None, then a default is chosen depending on `contour_type`
- `dark_rate` – a positive number (default: 0.5); affects how quickly magnitudes affect how light/dark the image is. When there are contours, this affects how visible each contour is. Large values (near 1.0) have very strong, immediate effects, while small values (near 0.0) have gradual effects.

OUTPUT:
An $N \times M \times 3$ floating point Numpy array $X$, where $X[i,j]$ is an (r, g, b) tuple.

See also:
sage.plot.complex_plot.complex_to_rgb()

EXAMPLES:
We can call this on grids of complex numbers:
We can change contour types and the distances between contours:

```python
sage: complex_to_cmap_rgb([[0, 1 + 1j, 3 + 4j]], contoured=True, contour_type="logarithmic", contour_base=3)
array([[0.64362, 0.98999, 0.23356],
       [0.9323375, 0.8166338, 0.2195099],
       [0.95647342, 0.74861225, 0.14963982]])
```

We see that changing dark_rate affects how visible contours are. In this example, we set contour_base=5 and note that the points 0 and 1 + i are far away from contours, but 2.9 + 4i is near (and just below) a contour. Raising dark_rate should have strong effects on the last coloration and weaker effects on the others:

```python
sage: complex_to_cmap_rgb([[0, 1 + 1j, 2.9 + 4j]], cmap='turb', dark_rate=0.05, contour_base=5)
array([[0.64362, 0.98999, 0.23356],
       [0.9333574, 0.8133052, 0.2305653],
       [0.96357185, 0.75337736, 0.1490913]])
```

We see that changing dark_rate affects how visible contours are. In this example, we set contour_base=5 and note that the points 0 and 1 + i are far away from contours, but 2.9 + 4i is near (and just below) a contour. Raising dark_rate should have strong effects on the last coloration and weaker effects on the others:

```python
sage: complex_to_cmap_rgb([[0, 1 + 1j, 2.9 + 4j]], cmap='turb', dark_rate=0.85, contour_base=5)
array([[0.64362, 0.98999, 0.23356],
       [0.9387468, 0.8284389, 0.2928956],
       [0.5777895, 0.4270328, 0.0261271]])
```

Convert a grid of complex numbers to a grid of rgb values using a default choice of colors.

```
sage.plot.complex_plot.complex_to_rgb(z_values, contoured=False, tiled=False, contour_type='logarithmic', contour_base=None, dark_rate=0.5, nphases=10)
```

Convert a grid of complex numbers to a grid of rgb values using a default choice of colors.

**INPUT:**

- `z_values` – A grid of complex numbers, as a list of lists
- `contoured` – boolean (default: False); causes magnitude to be indicated through contour-like adjustments to lightness.
- `tiled` – boolean (default: False); causes magnitude and argument to be indicated through contour-like adjustments to lightness.
- `nphases` – a positive integer (default: 10); when `tiled=True`, this is the number of divisions the phase is divided into.
- `contour_type` – either 'logarithmic', or 'linear' (default: 'logarithmic'); causes added contours to be of given type when `contoured=True`.
- `contour_base` – a positive integer; when `contour_type` is 'logarithmic', this sets logarithmic contours at multiples of `contour_base` apart. When `contour_type` is 'linear', this sets contours at dis-
tances of \texttt{contour\_base} apart. If \texttt{None}, then a default is chosen depending on \texttt{contour\_type}.

- \texttt{dark\_rate} – a positive number (default: 0.5); affects how quickly magnitudes affect how light/dark the image is. When there are contours, this affects how visible each contour is. Large values (near 1.0) have very strong, immediate effects, while small values (near 0.0) have gradual effects.

**OUTPUT:**

An $N \times M \times 3$ floating point Numpy array $X$, where $X[i,j]$ is an (r,g,b) tuple.

**See also:**

\texttt{sage.plot.complex_plot.complex\_to\_cmap\_rgb()}

**EXAMPLES:**

We can call this on grids of complex numbers:

\begin{verbatim}
\texttt{sage: from sage.plot.complex_plot import complex_to_rgb}
\texttt{sage: complex_to_rgb([[0, 1, 1000]])} # abs tol 1e-4
array([[0. , 0. , 0. ],
       [0.77172568, 0. , 0. ],
       [1. , 0.64421177, 0.64421177]])
\texttt{sage: complex_to_rgb([[0, 1j, 1000j]])} # abs tol 1e-4
array([[0. , 0. , 0. ],
       [0.38586284, 0.77172568, 0. ],
       [0.82210588, 1. , 0.64421177]])
\texttt{sage: complex_to_rgb([[0, 1, 1000]], contoured=True)} # abs tol 1e-4
array([[1. , 0. , 0. ],
       [0.99226756, 0.74420067, 0. ],
       [0.91751324, 0.81245954, 0. ]])
\texttt{sage: complex_to_rgb([[0, 1 + 1j, 3 + 4j]], contoured=True, contour_type="logarithmic", contour_base=3)}
array([[1. , 0.15 , 0.15 ],
       [0.99226756, 0.74420067, 0. ],
       [0.91751324, 0.81245954, 0. ]])
\texttt{sage: complex_to_rgb([[0, 1 + 1j, 3 + 4j]], contoured=True, contour_type="linear", contour_base=3)}
array([[1. , 0.15 , 0.15 ],
       [0.91429774, 0.6857233 , 0. ],
       [0.81666667, 0.72315973, 0. ]])
\end{verbatim}

We can change contour types and the distances between contours:

\begin{verbatim}
\texttt{sage: complex_to_rgb([[0, 1 + 1j, 3 + 4j]], contoured=True, contour_type="logarithmic", contour_base=3)}
array([[1. , 0.15 , 0.15 ],
       [0.91429774, 0.6857233 , 0. ],
       [0.81666667, 0.72315973, 0. ]])
\texttt{sage: complex_to_rgb([[0, 0.5, 1]], dark_rate=0.4)} # abs tol 1e-4
array([[0. , 0. , 0. ],
       [0.65393731, 0. , 0. ],
       [0.77172568, 0. , 0. ]])
\texttt{sage: complex_to_rgb([[0, 0.5, 1]], dark_rate=0.2)} # abs tol 1e-4
array([[0. , 0. , 0. ],
       [0.65393731, 0. , 0. ],
       [0.77172568, 0. , 0. ]])
\end{verbatim}

Lowering \texttt{dark\_rate} causes colors to go to black more slowly near 0:
sage.plot.complex_plot.hls_to_rgb(hls)
Convert array of hls values (each in the range \([0,1]\)) to a numpy array of rgb values (each in the range \([0,1]\))

INPUT:
- hls – an \(N \times 3\) array of floats in the range \([0,1]\); the hls values at each point. (Note that the input can actually be of any dimension, such as \(N \times M \times 3\), as long as the last dimension has length 3).

OUTPUT:
An \(N \times 3\) Numpy array of floats in the range \([0,1]\), with the same dimensions as the input array.

See also:
sage.plot.complex_plot.rgb_to_hls()

EXAMPLES:
We convert a row of floats and verify that we can convert back using rgb_to_hls:

```
sage: from sage.plot.complex_plot import rgb_to_hls, hls_to_rgb
sage: hls = [[0.2, 0.4, 0.5], [0.1, 0.3, 1.0]]
sage: rgb = hls_to_rgb(hls)
sage: rgb # abs tol 1e-4
array([[0.52, 0.6 , 0.2 ],
       [0.6 , 0.36, 0. ]])
sage: rgb_to_hls(rgb) # abs tol 1e-4
array([[0.2, 0.4, 0.5],
       [0.1, 0.3, 1. ]])
```

Multidimensional inputs can be given as well:

```
sage: multidim_arr = [[[0, 0.2, 0.4], [0, 1, 0]], [[0, 0, 0], [0.5, 0.6, 0.9]]]
sage: hls_to_rgb(multidim_arr) # abs tol 1e-4
array([[0.28, 0.12, 0.12],
       [1. , 1. , 1. ],
       [[0. , 0. , 0. ],
       [0.24, 0.96, 0.96]])
```

sage.plot.complex_plot.rgb_to_hls(rgb)
Convert array of rgb values (each in the range \([0,1]\)) to a numpy array of hls values (each in the range \([0,1]\))

INPUT:
- rgb – an \(N \times 3\) array of floats with values in the range \([0,1]\); the rgb values at each point. (Note that the input can actually be of any dimension, such as \(N \times M \times 3\), as long as the last dimension has length 3).

OUTPUT:
An \(N \times 3\) Numpy array of floats in the range \([0,1]\), with the same dimensions as the input array.

See also:
sage.plot.complex_plot.hls_to_rgb()

EXAMPLES:
We convert a row of floats and verify that we can back using hls_to_rgb:
```python
sage: from sage.plot.complex_plot import rgb_to_hls, hls_to_rgb
sage: rgb = [[0.2, 0.4, 0.5], [0.1, 0.3, 1.0]]
sage: hls = rgb_to_hls(rgb)
sage: hls # abs tol 1e-4
array([[0.55555556, 0.35000000, 0.42857143],
       [0.62962963, 0.55000000, 1.00000000]])
sage: hls_to_rgb(hls) # abs tol 1e-4
array([[0.2, 0.4, 0.5],
       [0.1, 0.3, 1.0]])
```

Multidimensional inputs can be given as well:

```python
sage: multidim_arr = [[[0, 0.2, 0.4], [1, 1, 1]], [[0, 0, 0], [0.5, 0.6, 0.9]]]
sage: rgb_to_hls(multidim_arr) # abs tol 1e-4
array([[0.58333333, 0.20000000, 1.00000000],
       [0.00000000, 1.00000000, 0.00000000],
       [0.00000000, 0.00000000, 0.62500000],
       [0.62500000, 0.70000000, 0.66666667]])
```

## 2.2 Contour plots

**class** `sage.plot.contour_plot.ContourPlot(xy_data_array, xrange, yrange, options)`

Bases: `GraphicPrimitive`

Primitive class for the contour plot graphics type.

See `contour_plot?` for help actually doing contour plots.

**INPUT:**

- `xy_data_array` - list of lists giving evaluated values of the function on the grid
- `xrange` - tuple of 2 floats indicating range for horizontal direction
- `yrange` - tuple of 2 floats indicating range for vertical direction
- `options` - dict of valid plot options to pass to constructor

**EXAMPLES:**

Note this should normally be used indirectly via `contour_plot`:

```python
sage: from sage.plot.contour_plot import ContourPlot
sage: C = ContourPlot([[1,3],[2,4]], (1,2), (2,3), options={})
sage: C
ContourPlot defined by a 2 x 2 data grid
sage: C.xrange
(1, 2)
```

**get_minmax_data()**

Return a dictionary with the bounding box data.

**EXAMPLES:**

2.2. Contour plots 203
sage: x,y = var('x,y')
sage: f(x,y) = x^2 + y^2
sage: d = contour_plot(f, (3,6), (3,6))[0].get_minmax_data()
sage: d['xmin']
3.0
sage: d['ymin']
3.0

contour_plot takes a function of two variables, \( f(x, y) \) and plots contour lines of the function over the specified xrange and yrange as demonstrated below.

```
contour_plot(f, (xrange, yrange), plot_points=100, fill=True, contours=None,
            linewidths=None, linestyles=None, labels=False, frame=True,
            axes=False, colorbar=False, legend_label=None, aspect_ratio=1,
            region=None, label_fontsize=9, label_colors='blue',
            label_inline=None, label_inline_spacing=3, label_fmt='%1.2f',
            colorbar_orientation='vertical', colorbar_format=None,
            colorbar_spacing='uniform', **options)
```

The following inputs must all be passed in as named parameters:

- **plot_points** – integer (default: 100); number of points to plot in each direction of the grid. For old computers, 25 is fine, but should not be used to verify specific intersection points.
- **fill** – bool (default: True), whether to color in the area between contour lines
- **cmap** – a colormap (default: 'gray'), the name of a predefined colormap, a list of colors or an instance of a matplotlib Colormap. Type: import matplotlib.cm; matplotlib.cm.datad.keys() for available colormap names.
- **contours** – integer or list of numbers (default: None): If a list of numbers is given, then this specifies the contour levels to use. If an integer is given, then this many contour lines are used, but the exact levels are determined automatically. If None is passed (or the option is not given), then the number of contour lines is determined automatically, and is usually about 5.
- **linewidths** – integer or list of integer (default: None), if a single integer all levels will be of the width given, otherwise the levels will be plotted with the width in the order given. If the list is shorter than the number of contours, then the widths will be repeated cyclically.
- **linestyles** – string or list of strings (default: None), the style of the lines to be plotted, one of: "solid", "dashed", "dashdot", "dotted", respectively "-", "--", "-.", ":" If the list is shorter than the number of contours, then the styles will be repeated cyclically.
- **labels** – boolean (default: False) Show level labels or not.

The following options are to adjust the style and placement of labels, they have no effect if no labels are shown.

- **label_fontsize** – integer (default: 9), the font size of the labels.
– **label_colors** – string or sequence of colors (default: None) If a string, gives the name of a single color with which to draw all labels. If a sequence, gives the colors of the labels. A color is a string giving the name of one or a 3-tuple of floats.

– **label_inline** – boolean (default: False if fill is True, otherwise True), controls whether the underlying contour is removed or not.

– **label_inline_spacing** – integer (default: 3), When inline, this is the amount of contour that is removed from each side, in pixels.

– **label_fmt** – a format string (default: "%.1f"), this is used to get the label text from the level. This can also be a dictionary with the contour levels as keys and corresponding text string labels as values. It can also be any callable which returns a string when called with a numeric contour level.

• **colorbar** – boolean (default: False) Show a colorbar or not.

The following options are to adjust the style and placement of colorbars. They have no effect if a colorbar is not shown.

– **colorbar_orientation** – string (default: ‘vertical’), controls placement of the colorbar, can be either ‘vertical’ or ‘horizontal’

– **colorbar_format** – a format string, this is used to format the colorbar labels.

– **colorbar_spacing** – string (default: ‘proportional’). If ‘proportional’, make the contour divisions proportional to values. If ‘uniform’, space the colorbar divisions uniformly, without regard for numeric values.

• **legend_label** – the label for this item in the legend

• **region** - (default: None) If region is given, it must be a function of two variables. Only segments of the surface where region(x,y) returns a number >0 will be included in the plot.

**Warning:** Due to an implementation detail in matplotlib, single-contour plots whose data all lie on one side of the sole contour may not be plotted correctly. We attempt to detect this situation and to produce something better than an empty plot when it happens; a **UserWarning** is emitted in that case.

**EXAMPLES:**

Here we plot a simple function of two variables. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

```python
sage: x,y = var('x,y')
sage: contour_plot(cos(x^2 + y^2), (x,-4,4), (y,-4,4))
```

Graphics object consisting of 1 graphics primitive

Here we change the ranges and add some options:

```python
sage: x,y = var('x,y')
sage: contour_plot((x^2) * cos(x*y), (x,-10,5), (y,-5,5), fill=False, plot_points=150)
```

Graphics object consisting of 1 graphics primitive

An even more complicated plot:

```python
sage: x,y = var('x,y')
sage: contour_plot(sin(x^2+y^2) * cos(x) * sin(y), (x,-4,4), (y,-4,4), plot_points=150)
```

(continues on next page)
2.2. Contour plots
Some elliptic curves, but with symbolic endpoints. In the first example, the plot is rotated 90 degrees because we switch the variables $x, y$:

```
sage: x,y = var('x,y')
sage: contour_plot(y^2 + 1 - x^3 - x, (y,-pi,pi), (x,-pi,pi))
```

We can play with the contour levels:

```
sage: x,y = var('x,y')
sage: f(x,y) = x^2 + y^2
sage: contour_plot(f, (-2,2), (-2,2))
sage: contour_plot(f, (-2,2), (-2,2), contours=2, cmap=[[1,0,0], [0,1,0], [0,0,1]])
```
2.2. Contour plots
sage: contour_plot(f, (-2,2), (-2,2),
    ....:    contours=(0.1,1.0,1.2,1.4), cmap='hsv')
Graphics object consisting of 1 graphics primitive

2.0
 1.5
 1.0
 0.5
 0.0
0.5
1.0
1.5
2.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
-2.0

sage: contour_plot(f, (-2,2), (-2,2), contours=(1.0,), fill=False)
Graphics object consisting of 1 graphics primitive

sage: contour_plot(x - y^2, (x,-5,5), (y,-3,3), contours=[-4,0,1])
Graphics object consisting of 1 graphics primitive

We can change the style of the lines:

sage: contour_plot(f, (-2,2), (-2,2), fill=False, linewidths=10)
Graphics object consisting of 1 graphics primitive

sage: contour_plot(f, (-2,2), (-2,2), fill=False, linestyles='dashdot')
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(x^2 - y^2, (x,-3,3), (y,-3,3),
    ....:    contours=[0,1,2,3,4], linewidths=[1,5],
    ....:    linestyles=['solid','dashed'], fill=False)

(continues on next page)
2.2. Contour plots
2.2. Contour plots
We can add labels and play with them:

```python
sage: contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
    ....:     fill=False, cmap='hsv', labels=True)
```

Graphics object consisting of 1 graphics primitive
2.2. Contour plots
sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
.....: fill=False, cmap='hsv',
.....: labels=True, label_fmt="%1.0f",
.....: label_colors='black')
sage: P
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
.....: fill=False, cmap='hsv',
.....: labels=True, contours=[-4,0,4],
.....: label_fmt={-4:"low", 0:"medium", 4: "hi"},
.....: label_colors='black')
sage: P
Graphics object consisting of 1 graphics primitive

sage: P = contour_plot(y^2 + 1 - x^3 - x, (x,-pi,pi), (y,-pi,pi),
.....: fill=False, cmap='hsv', labels=True,
.....: contours=[-4,0,4],
.....: label_fmt=lambda x: "z=%s"%x,
.....: label_colors='black', label_inline=True,
.....: label_fontsize=12)
sage: P
Graphics object consisting of 1 graphics primitive
2.2. Contour plots
We can change the color of the labels if so desired:

```python
sage: contour_plot(f, (-2,2), (-2,2), labels=True, label_colors='red')
```

Graphics object consisting of 1 graphics primitive
2.2. Contour plots
We can add a colorbar as well:

```
sage: f(x, y) = x^2 + y^2
sage: contour_plot(f, (x,-3,3), (y,-3,3), colorbar=True)
```

```
sage: contour_plot(f, (x,-3,3), (y,-3,3), colorbar=True, colorbar_orientation='horizontal')
```

```
sage: contour_plot(f, (x,-3,3), (y,-3,3), contours=[-2,-1,4], colorbar=True)
```

```
sage: contour_plot(f, (x,-3,3), (y,-3,3), contours=[0,2,3,6],
....:                          colorbar=True, colorbar_spacing='uniform')
```

```
sage: contour_plot(f, (x,-3,3), (y,-3,3), contours=[0,2,3,6],
....:                          colorbar=True, colorbar_format='%.3f')
```

2.2. Contour plots
2.2. Contour plots
Chapter 2. Function and Data Plots
2.2. Contour plots
sage: contour_plot(f, (x,-3,3), (y,-3,3), labels=True,
....:     label_colors='red', contours=[0,2,3,6],
....:     colorbar=True)
Graphics object consisting of 1 graphics primitive

This should plot concentric circles centered at the origin:

sage: x, y = var('x, y')
sage: contour_plot(x^2 + y^2 - 2,(x,-1,1), (y,-1,1))
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to show(), as long as they are valid:

sage: f(x,y) = cos(x) + sin(y)
sage: contour_plot(f, (0,pi), (0,pi), axes=True)
Graphics object consisting of 1 graphics primitive

sage: contour_plot(f, (0,pi), (0,pi)).show(axes=True) # These are equivalent
2.2. Contour plots
One can also plot over a reduced region:

```python
sage: contour_plot(x**2 - y**2, (x,-2,2), (y,-2,2), region=x - y, plot_points=300)
Graphics object consisting of 1 graphics primitive
```

Note that with `fill=False` and grayscale contours, there is the possibility of confusion between the contours and the axes, so use `fill=False` together with `axes=True` with caution:

```python
sage: contour_plot(f, (-pi,pi), (-pi,pi), fill=False, axes=True)
Graphics object consisting of 1 graphics primitive
```

If you are plotting a sole contour and if all of your data lie on one side of it, then (as part of [github issue #21042](https://github.com/sagemath/sage/issues/21042)) a heuristic may be used to improve the result; in that case, a warning is emitted:

```python
sage: contour_plot(lambda x,y: abs(x^2-y^2), (-1,1), (-1,1),
....: contours=[0], fill=False, cmap=['blue'])
```

UserWarning: pathological contour plot of a function whose values all lie on one side of the sole contour; we are adding more plot points and perturbing your function values.

Graphics object consisting of 1 graphics primitive

Constant functions (with a single contour) can be plotted as well; this was not possible before [github issue #21042](https://github.com/sagemath/sage/issues/21042):
2.2. Contour plots
Chapter 2. Function and Data Plots
sage: contour_plot(lambda x, y: 0, (-1, 1), (-1, 1),
...:             contours=[0], fill=False, cmap=['blue'])
...Graphics object consisting of 1 graphics primitive

sage.plot.contour_plot.equify(f)

Return the equation rewritten as a symbolic function to give negative values when True, positive when False.

EXAMPLES:

sage: from sage.plot.contour_plot import equify
sage: var('x, y')
(x, y)
sage: equify(x^2 < 2)
x^2 - 2
sage: equify(x^2 > 2)
-x^2 + 2
sage: equify(x*y > 1)
-x*y + 1
sage: equify(y > 0)
-y
sage: f = equify(lambda x, y: x > y)
sage: f(1, 2)
1
sage: f(2, 1)
-1

sage.plot.contour_plot.implicit_plot(f, xrange, yrange, plot_points=150, contours=(0,), fill=False, cmap='blue', **options)

implicit_plot takes a function of two variables, \(f(x, y)\) and plots the curve \(f(x, y) = 0\) over the specified \(xrange\) and \(yrange\) as demonstrated below.

\[
\text{implicit_plot}(f, (x,xmin,xmax), (y,ymin,ymax), \ldots)
\]

\[
\text{implicit_plot}(f, (xmin,xmax), (ymin,ymax), \ldots)
\]

**INPUT:**

- \(f\) – a function of two variables or equation in two variables
- \((x,xmin,xmax)\) – 2-tuple, the range of \(x\) values or \((x,xmin,xmax)\)
- \((ymin,ymax)\) – 2-tuple, the range of \(y\) values or \((y,ymin,ymax)\)

The following inputs must all be passed in as named parameters:

- \(\text{plot_points}\) – integer (default: 150); number of points to plot in each direction of the grid
- \(\text{fill}\) – boolean (default: False); if True, fill the region \(f(x, y) < 0\).
- \(\text{fillcolor}\) – string (default: 'blue'), the color of the region where \(f(x, y) < 0\) if \(\text{fill} = \text{True}\). Colors are defined in \sage.plot.colors; try \texttt{colors?} to see them all.
- \(\text{linewidth}\) – integer (default: None), if a single integer all levels will be of the width given, otherwise the levels will be plotted with the widths in the order given.
- \(\text{linestyle}\) – string (default: None), the style of the line to be plotted, one of: "solid", "dashed", "dashdot" or "dotted", respectively "-", "--", "-.", or ":".
- \(\text{color}\) – string (default: 'blue'), the color of the plot. Colors are defined in \sage.plot.colors; try \texttt{colors?} to see them all. If \(\text{fill} = \text{True}\), then this sets only the color of the border of the plot. See \texttt{fillcolor} for setting the color of the fill region.
- \(\text{legend_label}\) – the label for this item in the legend
- \(\text{base}\) – (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple \((\text{basex, basey})\). \text{basex} sets the base of the logarithm along the horizontal axis and \text{basey} sets the base along the vertical axis.
- \(\text{scale}\) – (default: "linear") string. The scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy". The scale can be also be given as single argument that is a list or tuple \((\text{scale, base})\) or \((\text{scale, basex, basey})\).

The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when \texttt{Graphics} is initialized.

**Warning:** Due to an implementation detail in matplotlib, implicit plots whose data are all nonpositive or nonnegative may not be plotted correctly. We attempt to detect this situation and to produce something better than an empty plot when it happens; a \texttt{UserWarning} is emitted in that case.

**EXAMPLES:**
A simple circle with a radius of 2. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

```python
sage: var("x y")
(x, y)
sage: implicit_plot(x^2 + y^2 - 2, (x,-3,3), (y,-3,3))
```

We can do the same thing, but using a callable function so we do not need to explicitly define the variables in the ranges. We also fill the inside:

```python
sage: f(x,y) = x^2 + y^2 - 2
sage: implicit_plot(f, (-3,3), (-3,3), fill=True, plot_points=500) # long time
```

The same circle but with a different line width:

```python
sage: implicit_plot(f, (-3,3), (-3,3), linewidth=6)
```

Again the same circle but this time with a dashdot border:

```python
sage: implicit_plot(f, (-3,3), (-3,3), linestyle='dashdot')
```

2.2. Contour plots
2.2. Contour plots
The same circle with different line and fill colors:

```python
sage: implicit_plot(f, (-3,3), (-3,3), color='red', # long time
           fill=True, fillcolor='green',
           plot_points=500)
Graphics object consisting of 2 graphics primitives
```

You can also plot an equation:

```python
sage: var("x y")
(x, y)
sage: implicit_plot(x^2 + y^2 == 2, (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```

You can even change the color of the plot:

```python
sage: implicit_plot(x^2 + y^2 == 2, (x,-3,3), (y,-3,3), color="red")
Graphics object consisting of 1 graphics primitive
```

The color of the fill region can be changed:

```python
sage: implicit_plot(x^2 + y^2 == 2, (x,-3,3), (y,-3,3), fill=True, fillcolor='red')
Graphics object consisting of 2 graphics primitives
```
2.2. Contour plots
Here is a beautiful (and long) example which also tests that all colors work with this:

```
sage: G = Graphics()
sage: counter = 0
sage: for col in colors.keys():  # long time
....:     G += implicit_plot(x^2 + y^2 == 1 + counter*.1, (x,-4,4),(y,-4,4),→
˓→color=col)
....:     counter += 1
sage: G  # long time
Graphics object consisting of 148 graphics primitives
```

We can define a level-$n$ approximation of the boundary of the Mandelbrot set:

```
sage: def mandel(n):
....:     c = polygen(CDF, 'c')
....:     z = 0
....:     for i in range(n):
....:         z = z*z + c
....:     def f(x,y):
....:         val = z(CDF(x, y))
....:         return val.norm() - 4
....:     return f
```

The first-level approximation is just a circle:

```
2.2. Contour plots
```
A third-level approximation starts to get interesting:

```sage
sage: implicit_plot(mandel(3), (-2, 1), (-1.5, 1.5))
Graphics object consisting of 1 graphics primitive
```

The seventh-level approximation is a degree 64 polynomial, and `implicit_plot` does a pretty good job on this part of the curve. (plot_points=200 looks even better, but it takes over a second.)

```sage
sage: implicit_plot(mandel(7), (-0.3, 0.05), (-1.15, -0.9), plot_points=50)
Graphics object consisting of 1 graphics primitive
```

When making a filled implicit plot using a python function rather than a symbolic expression the user should increase the number of plot points to avoid artifacts:

```sage
sage: implicit_plot(lambda x, y: x^2 + y^2 - 2, (x,-3,3), # long time
                 (y,-3,3), fill=True, plot_points=500)
Graphics object consisting of 2 graphics primitives
```

An example of an implicit plot on ‘loglog’ scale:
2.2. Contour plots
Chapter 2. Function and Data Plots
2.2. Contour plots
region_plot takes a boolean function of two variables, \( f(x, y) \) and plots the region where \( f \) is True over the specified xrange and yrange as demonstrated below.

region_plot\( (f, (x_{\text{min}}, x_{\text{max}}), (y_{\text{min}}, y_{\text{max}}), ...) \)

**INPUT:**

- \( f \) – a boolean function or a list of boolean functions of two variables
- \((x_{\text{min}}, x_{\text{max}})\) – 2-tuple, the range of \( x \) values OR 3-tuple \((x, x_{\text{min}}, x_{\text{max}})\)
- \((y_{\text{min}}, y_{\text{max}})\) – 2-tuple, the range of \( y \) values OR 3-tuple \((y, y_{\text{min}}, y_{\text{max}})\)
- \text{plot_points} – integer (default: 100); number of points to plot in each direction of the grid
- \text{incol} – a color (default: 'blue'), the color inside the region
- \text{outcol} – a color (default: None), the color of the outside of the region
If any of these options are specified, the border will be shown as indicated, otherwise it is only implicit (with color incol) as the border of the inside of the region.

- **bordercol** – a color (default: None), the color of the border
  ('black' if borderwidth or borderstyle is specified but not bordercol)
- **borderstyle** – string (default: 'solid'), one of 'solid', 'dashed', 'dotted', 'dashdot', respectively '-' , '--' , ':' , '-.'.
- **borderwidth** – integer (default: None), the width of the border in pixels
- **alpha** – (default: 1) how transparent the fill is; a number between 0 and 1
- **legend_label** – the label for this item in the legend
- **base** - (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.
- **scale** – (default: "linear") string. The scale of the axes. Possible values are "linear", "loglog", "semilogx", "semilogy".
  The scale can be also be given as single argument that is a list or tuple (scale, base) or (scale, basex, basey).
  The "loglog" scale sets both the horizontal and vertical axes to logarithmic scale. The "semilogx" scale sets the horizontal axis to logarithmic scale. The "semilogy" scale sets the vertical axis to logarithmic scale. The "linear" scale is the default value when Graphics is initialized.

**EXAMPLES:**

Here we plot a simple function of two variables:

```
sage: x,y = var('x,y')
sage: region_plot(cos(x^2 + y^2) <= 0, (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```

Here we play with the colors:

```
sage: region_plot(x^2 + y^3 < 2, (x,-2,2), (y,-2,2), incol='lightblue', bordercol=gray)
Graphics object consisting of 2 graphics primitives
```

An even more complicated plot, with dashed borders:

```
sage: region_plot(sin(x) * sin(y) >= 1/4, (x,-10,10), (y,-10,10),
....:    incol='yellow', bordercol='black',
....:    borderstyle='dashed', plot_points=250)
Graphics object consisting of 2 graphics primitives
```

A disk centered at the origin:

```
sage: region_plot(x^2 + y^2 < 1, (x,-1,1), (y,-1,1))
Graphics object consisting of 1 graphics primitive
```

A plot with more than one condition (all conditions must be true for the statement to be true):

```
sage: region_plot((x^2 + y^2 < 1, x < y], (x,-2,2), (y,-2,2))
Graphics object consisting of 1 graphics primitive
```

2.2. Contour plots
2.2. Contour plots
2.2. Contour plots
Since it does not look very good, let us increase `plot_points`:

```python
sage: region_plot([x^2 + y^2 < 1, x < y], (x,-2,2), (y,-2,2), plot_points=400)
```

Graphics object consisting of 1 graphics primitive

To get plots where only one condition needs to be true, use a function. Using lambda functions, we definitely need the extra `plot_points`:

```python
sage: region_plot(lambda x, y: x^2 + y^2 < 1 or x < y, (x,-2,2), (y,-2,2), plot_points=400)
```

Graphics object consisting of 1 graphics primitive

The first quadrant of the unit circle:

```python
sage: region_plot([y > 0, x > 0, x^2 + y^2 < 1], (x,-1.1,1.1), (y,-1.1,1.1), plot_points=400)
```

Graphics object consisting of 1 graphics primitive

Here is another plot, with a huge border:

```python
sage: region_plot(x*(x-1)*(x+1) + y^2 < 0, (x,-3,2), (y,-3,3),
....: incol='lightblue', bordercol='gray', borderwidth=10,
....: plot_points=50)
```

Graphics object consisting of 2 graphics primitives
Chapter 2. Function and Data Plots
2.2. Contour plots
If we want to keep only the region where x is positive:

```python
sage: region_plot([x*(x-1)*(x+1) + y^2 < 0, x > -1], (x,-3,2), (y,-3,3),
.....:          incol='lightblue', plot_points=50)
Graphics object consisting of 1 graphics primitive
```

Here we have a cut circle:

```python
sage: region_plot([x^2 + y^2 < 4, x > -1], (x,-2,2), (y,-2,2),
.....:          incol='lightblue', bordercol='gray', plot_points=200)
Graphics object consisting of 2 graphics primitives
```

The first variable range corresponds to the horizontal axis and the second variable range corresponds to the vertical axis:

```python
sage: s, t = var('s, t')
sage: region_plot(s > 0, (t,-2,2), (s,-2,2))
Graphics object consisting of 1 graphics primitive
```

An example of a region plot in ‘loglog’ scale:
2.2. Contour plots
Chapter 2. Function and Data Plots
2.3 Density plots

```
class sage.plot.density_plot.DensityPlot(xy_data_array, xrange, yrange, options)
Bases: GraphicPrimitive

Primitive class for the density plot graphics type. See density_plot? for help actually doing density plots.

INPUT:
  • xy_data_array - list of lists giving evaluated values of the function on the grid
  • xrange - tuple of 2 floats indicating range for horizontal direction
  • yrange - tuple of 2 floats indicating range for vertical direction
  • options - dict of valid plot options to pass to constructor

EXAMPLES:
Note this should normally be used indirectly via density_plot:
```
sage: from sage.plot.density_plot import DensityPlot
sage: D = DensityPlot([[1,3],[2,4]], (1,2), (2,3),options={})

sage: D
DensityPlot defined by a 2 x 2 data grid
sage: D.yrange
(2, 3)
sage: D.options()
{}

def get_minmax_data()
    Returns a dictionary with the bounding box data.

    EXAMPLES:
    sage: x,y = var('x,y')
sage: f(x, y) = x^2 + y^2
sage: d = density_plot(f, (3,6), (3,6))[0].get_minmax_data()
sage: d['xmin']
3.0
sage: d['ymin']
3.0

density_plot takes a function of two variables, \( f(x, y) \) and plots the height of the function over the specified xrange and yrange as demonstrated below.

INPUT:
- \( f \) – a function of two variables
- \((xmin,xmax)\) – 2-tuple, the range of \( x \) values OR 3-tuple \((x,xmin,xmax)\)
- \((ymin,ymax)\) – 2-tuple, the range of \( y \) values OR 3-tuple \((y,ymin,ymax)\)

The following inputs must all be passed in as named parameters:
- **plot_points** – integer (default: 25); number of points to plot in each direction of the grid
- **cmap** – a colormap (default: 'gray'), the name of a predefined colormap, a list of colors or an instance of a matplotlib Colormap. Type: import matplotlib.cm; matplotlib.cm.datad.keys() for available colormap names.
- **interpolation** – string (default: 'catrom'), the interpolation method to use: 'bilinear', 'bicubic', 'spline16', 'spline36', 'quadric', 'gaussian', 'sinc', 'bessel', 'mitchell', 'lanczos', 'catrom', 'hermite', 'hanning', 'hamming', 'kaiser'

EXAMPLES:
Here we plot a simple function of two variables. Note that since the input function is an expression, we need to explicitly declare the variables in 3-tuples for the range:

sage: x,y = var('x,y')
sage: density_plot(sin(x) * sin(y), (x,-2,2), (y,-2,2))
Graphics object consisting of 1 graphics primitive
2.3. Density plots
Here we change the ranges and add some options; note that here $f$ is callable (has variables declared), so we can use 2-tuple ranges:

```sage
sage: x,y = var('x,y')
sage: f(x,y) = x^2 * cos(x*y)
sage: density_plot(f, (x,-10,5), (y,-5,5), interpolation='sinc', plot_points=100)
Graphics object consisting of 1 graphics primitive
```

An even more complicated plot:

```sage
sage: x,y = var('x,y')
sage: density_plot(sin(x^2+y^2) * cos(x) * sin(y), (x,-4,4), (y,-4,4), cmap='jet',
plot_points=100)
Graphics object consisting of 1 graphics primitive
```

This should show a “spotlight” right on the origin:

```sage
sage: x,y = var('x,y')
sage: density_plot(1/(x^10 + y^10), (x,-10,10), (y,-10,10))
Graphics object consisting of 1 graphics primitive
```

Some elliptic curves, but with symbolic endpoints. In the first example, the plot is rotated 90 degrees because we switch the variables $x, y$:  

```
```
2.3. Density plots
\texttt{sage: density\_plot(y^2 + 1 - x^3 - x, (y,-\pi,\pi), (x,-\pi,\pi))}
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to \texttt{show()}, as long as they are valid:

\texttt{sage: density\_plot(log(x) + log(y), (x,1,10), (y,1,10), aspect\_ratio=1)}
Graphics object consisting of 1 graphics primitive

\texttt{sage: density\_plot(log(x) + log(y), (x,1,10), (y,1,10)).show(aspect\_ratio=1) \# (These are equivalent)
Chapter 2. Function and Data Plots
2.3. Density plots
2.4 Plotting fields

**class** sage.plot.plot_field.PlotField(xpos_array, ypos_array, xvec_array, yvec_array, options)

Bases: GraphicPrimitive

Primitive class that initializes the PlotField graphics type

**get_minmax_data()**

Returns a dictionary with the bounding box data.

**EXAMPLES:**

```python
sage: x,y = var('x,y')
sage: d = plot_vector_field((x,x+y), (x,10,20), (y,10,20))[0].get_minmax_data()
sage: d['xmin']
10.0
sage: d['ymin']
10.0
```

**sage.plot.plot_field.plot_slope_field(f, xrange, yrange, **kwds)**

`plot_slope_field` takes a function of two variables xvar and yvar (for instance, if the variables are `x` and `y`, take `f(x, y)`), and at representative points `(x_i, y_i)` between `xmin`, `xmax`, and `ymin`, `ymax` respectively, plots a line with slope `f(x_i, y_i)` (see below).

**EXAMPLES:**

A logistic function modeling population growth:

```python
sage: x,y = var('x, y')
sage: capacity = 3 # thousand
sage: growth_rate = 0.7 # population increases by 70% per unit of time
sage: plot_slope_field(growth_rate * (1-y/capacity) * y, (x,0,5), (y,0,capacity*2))
Graphics object consisting of 1 graphics primitive
```

Plot a slope field involving sin and cos:

```python
sage: x,y = var('x, y')
sage: plot_slope_field(sin(x+y) + cos(x+y), (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```

Plot a slope field using a lambda function:

```python
sage: plot_slope_field(lambda x,y: x + y, (-2,2), (-2,2))
Graphics object consisting of 1 graphics primitive
```

**sage.plot.plot_field.plot_vector_field(f_g, xrange, yrange, plot_points=20, frame=True, **options)**

`plot_vector_field` takes two functions of two variables xvar and yvar (for instance, if the variables are `x` and `y`, take `(f(x, y), g(x, y))`) and plots vector arrows of the function over the specified ranges, with `xrange` being of xvar between `xmin` and `xmax`, and `yrange` similarly (see below).

**EXAMPLES:**

Plot some vector fields involving sin and cos:

```python
```
2.4. Plotting fields
2.4. Plotting fields
```
sage: x, y = var('x y')
sage: plot_vector_field((sin(x), cos(y)), (x, -3, 3), (y, -3, 3))
Graphics object consisting of 1 graphics primitive
```

```
sage: x, y = var('x y')
sage: plot_vector_field((y, (cos(x) - 2) * sin(x)), (x, -pi, pi), (y, -pi, pi))
Graphics object consisting of 1 graphics primitive
```

```
Plot a gradient field:
sage: u, v = var('u v')
sage: f = exp(-u^2 + v^2)
sage: plot_vector_field(f.gradient(), (u, -2, 2), (v, -2, 2), color='blue')
Graphics object consisting of 1 graphics primitive
```

```
Plot two orthogonal vector fields:
sage: x, y = var('x,y')
sage: a = plot_vector_field((x, y), (x, -3, 3), (y, -3, 3), color='blue')
sage: b = plot_vector_field((y, -x), (x, -3, 3), (y, -3, 3), color='red')
sage: show(a + b)
```

We ignore function values that are infinite or NaN:
Chapter 2. Function and Data Plots
2.4. Plotting fields
sage: x, y = var('x, y')
sage: plot_vector_field((-x/sqrt(x^2+y^2), -y/sqrt(x^2+y^2)), (x,-10,10), (y,-10,10))
Graphics object consisting of 1 graphics primitive

sage: x, y = var('x, y')
sage: plot_vector_field((-x/sqrt(x+y), -y/sqrt(x+y)), (x,-10, 10), (y,-10,10))
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to show(), as long as they are valid:
sage: plot_vector_field((x, y), (x,-2,2), (y,-2,2), xmax=10)
Graphics object consisting of 1 graphics primitive
sage: plot_vector_field((x, y), (x,-2,2), (y,-2,2)).show(xmax=10)  # These are equivalent
2.4. Plotting fields
2.5 Streamline plots

class sage.plot.streamline_plot.StreamlinePlot(xpos_array, ypos_array, xvec_array, yvec_array, **options)

Bases: GraphicPrimitive

Primitive class that initializes the StreamlinePlot graphics type

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

```
sage: x, y = var('x y')
sage: d = streamline_plot((.01*x, x+y), (x,10,20), (y,10,20))[0].get_minmax_data()
sage: d['xmin']
10.0
sage: d['ymin']
10.0
```

sage.plot.streamline_plot.streamline_plot(f_g, xrange, yrange, plot_points=20, density=1.0, frame=True, **options)

Return a streamline plot in a vector field.

streamline_plot can take either one or two functions. Consider two variables \( x \) and \( y \).

If given two functions \( f(x, y), g(x, y) \), then this function plots streamlines in the vector field over the specified ranges with \( x \text{range} \) being of \( x \), denoted by \( x\text{var} \) below, between \( x\text{min} \) and \( x\text{max} \), and \( y\text{range} \) similarly (see below).

```
streamline_plot((f, g), (xvar, xmin, xmax), (yvar, ymin, ymax))
```

Similarly, if given one function \( f(x, y) \), then this function plots streamlines in the slope field \( \frac{dy}{dx} = f(x, y) \) over the specified ranges as given above.

PLOT OPTIONS:

- **plot_points** – (default: 200) the minimal number of plot points
- **density** – float (default: 1.0); controls the closeness of streamlines
- **start_points** – (optional) list of coordinates of starting points for the streamlines; coordinate pairs can be tuples or lists

EXAMPLES:

Plot some vector fields involving \( \sin \) and \( \cos \):

```
sage: x, y = var('x y')
sage: streamline_plot((sin(x), cos(y)), (x,-3,3), (y,-3,3))
```

Graphics object consisting of 1 graphics primitive

```
sage: streamline_plot((y, (cos(x)-2) * sin(x)), (x,-pi,pi), (y,-pi,pi))
```

Graphics object consisting of 1 graphics primitive

We increase the density of the plot:
Chapter 2. Function and Data Plots
2.5. Streamline plots
We ignore function values that are infinite or NaN:

```python
sage: x, y = var('x y')
sage: streamline_plot((-x/sqrt(x^2+y^2), -y/sqrt(x^2+y^2)),
                   (x,-10,10), (y,-10,10))
Graphics object consisting of 1 graphics primitive
```

Extra options will get passed on to `show()`, as long as they are valid:

```python
sage: streamline_plot((x + y) / sqrt(x^2 + y^2), (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```

We can also construct streamlines in a slope field:

```python
sage: x, y = var('x y')
sage: streamline_plot((x + y) / sqrt(x^2 + y^2), (x,-3,3), (y,-3,3))
Graphics object consisting of 1 graphics primitive
```
2.5. Streamline plots
2.5. Streamline plots
We choose some particular points the streamlines pass through:

```
sage: pts = [[1, 1], [-2, 2], [1, -3/2]]
sage: g = streamline_plot((x + y) / sqrt(x^2 + y^2),
                        (x,-3,3), (y,-3,3), start_points=pts)
sage: g += point(pts, color='red')
sage: g
```

Graphics object consisting of 2 graphics primitives

Note: Streamlines currently pass close to start_points but do not necessarily pass directly through them. That is part of the behavior of matplotlib, not an error on your part.
2.6 Scatter plots

```
class sage.plot.scatter_plot.ScatterPlot(xdata, ydata, options)
    Bases: GraphicPrimitive
    Scatter plot graphics primitive.
    Input consists of two lists/arrays of the same length, whose values give the horizontal and vertical coordinates of each point in the scatter plot. Options may be passed in dictionary format.

    EXAMPLES:
    sage: from sage.plot.scatter_plot import ScatterPlot
    sage: ScatterPlot([0,1,2], [3.5,2,5.1], {'facecolor':'white', 'marker':'s'})
    Scatter plot graphics primitive on 3 data points

    get_minmax_data()
    Returns a dictionary with the bounding box data.

    EXAMPLES:
    sage: s = scatter_plot([[0,1],[2,4],[3.2,6]])
    sage: d = s.get_minmax_data()
    sage: d['xmin']
    0.0
    sage: d['ymin']
    1.0
```

```
sage.plot.scatter_plot.scatter_plot(datalist, alpha=1, markersize=50, marker='o', zorder=5, facecolor='#fec7b8', edgecolor='black', clip=True, aspect_ratio='automatic', **options)
```

Returns a Graphics object of a scatter plot containing all points in the datalist. Type scatter_plot.options to see all available plotting options.

**INPUT:**

- **datalist** – a list of tuples \((x, y)\)
- **alpha** – default: 1
- **markersize** – default: 50
- **marker** - The style of the markers (default "o"). See the documentation of plot() for the full list of markers.
- **facecolor** – default: '#fec7b8'
- **edgecolor** – default: 'black'
- **zorder** – default: 5

**EXAMPLES:**

```
sage: scatter_plot([[0,1],[2,2],[4.3,1.1]], marker='s')
Graphics object consisting of 1 graphics primitive
```

Extra options will get passed on to show(), as long as they are valid:
2.7 Step function plots

`sage.plot.step.plot_step_function(v, vertical_lines=True, **kwds)`

Return the line graphics object that gives the plot of the step function \( f \) defined by the list \( v \) of pairs \((a, b)\). Here if \((a, b)\) is in \( v \), then \( f(a) = b \). The user does not have to worry about sorting the input list \( v \).

INPUT:

- \( v \) – list of pairs \((a, b)\)
- \( \text{vertical_lines} \) – bool (default: \text{True}) if \text{True}, draw vertical risers at each step of this step function. Technically these vertical lines are not part of the graph of this function, but they look very nice in the plot, so we include them by default

EXAMPLES:

We plot the prime counting function:
sage: plot_step_function([(i, prime_pi(i)) for i in range(20)])
Graphics object consisting of 1 graphics primitive

sage: plot_step_function([(i, sin(i)) for i in range(5, 20)])
Graphics object consisting of 1 graphics primitive

We pass in many options and get something that looks like “Space Invaders”:

sage: v = [(i, sin(i)) for i in range(5, 20)]
sage: plot_step_function(v, vertical_lines=False, thickness=30,
....:                     rgbcolor='purple', axes=False)
Graphics object consisting of 14 graphics primitives
2.7. Step function plots
2.8 Histograms

class sage.plot.histogram.Histogram(datalist, options)

Bases: GraphicPrimitive

Graphics primitive that represents a histogram. This takes quite a few options as well.

EXAMPLES:

```python
sage: from sage.plot.histogram import Histogram
sage: g = Histogram([1,3,2,0], {}); g
Histogram defined by a data list of size 4
sage: type(g)
<class 'sage.plot.histogram.Histogram'>
sage: opts = { 'bins':20, 'label':'mydata'}
sage: g = Histogram([random() for _ in range(500)], opts); g
Histogram defined by a data list of size 500
```

We can accept multiple sets of the same length:

```python
sage: g = Histogram([[1,3,2,0], [4,4,3,3]], {}); g
Histogram defined by 2 data lists
```

get_minmax_data()

Get minimum and maximum horizontal and vertical ranges for the Histogram object.

EXAMPLES:

```python
sage: H = histogram([10,3,5], density=True); h = H[0]
sage: h.get_minmax_data()  # rel tol 1e-15
{'xmax': 10.0, 'xmin': 3.0, 'ymax': 0.4761904761904765, 'ymin': 0}
sage: G = histogram([random() for _ in range(500)]); g = G[0]
sage: g.get_minmax_data()  # random output
{'xmax': 0.99729312925213209, 'xmin': 0.00013024562219410285, 'ymax': 61, 'ymin': 0}
sage: Y = histogram([random()*10 for _ in range(500)], range=[2,8]); y = Y[0]
sage: ymm = y.get_minmax_data(); ymm['xmax'], ymm['xmin']
(8.0, 2.0)
sage: Z = histogram([[1,3,2,0], [4,4,3,3]]); z = Z[0]
sage: z.get_minmax_data()
{'xmax': 4.0, 'xmin': 0, 'ymax': 2, 'ymin': 0}
```

sage.plot.histogram.histogram(datalist, aspect_ratio='automatic', align='mid', weights=None, range=None, bins=10, edgecolor='black', **options)

Computes and draws the histogram for list(s) of numerical data. See examples for the many options; even more customization is available using matplotlib directly.

INPUT:

- datalist – A list, or a list of lists, of numerical data
- align – (default: “mid”) How the bars align inside of each bin. Acceptable values are “left”, “right” or “mid”
- alpha – (float in [0,1], default: 1) The transparency of the plot
• **bins** – The number of sections in which to divide the range. Also can be a sequence of points within the range that create the partition

• **color** – The color of the face of the bars or list of colors if multiple data sets are given

• **cumulative** – (boolean - default: False) If True, then a histogram is computed in which each bin gives the counts in that bin plus all bins for smaller values. Negative values give a reversed direction of accumulation

• **edgecolor** – The color of the border of each bar

• **fill** – (boolean - default: True) Whether to fill the bars

• **hatch** – (default: None) symbol to fill the bars with - one of '/', '*', '|', '-', '+', 'x', 'o', 'O', '.', '*' (or None)

• **hue** – The color of the bars given as a hue. See **hue** for more information on the hue

• **label** – A string label for each data list given

• **linewidth** – (float) width of the lines defining the bars

• **linestyle** – (default: ‘solid’) Style of the line. One of ‘solid’ or ‘-’, ‘dashed’ or ‘-’, ‘dotted’ or ‘:’, ‘dashdot’ or ‘-.’

• **density** – (boolean - default: False) If True, the result is the value of the probability density function at the bin, normalized such that the integral over the range is 1.

• **range** – A list [min, max] which define the range of the histogram. Values outside of this range are treated as outliers and omitted from counts

• **rwidth** – (float in [0,1], default: 1) The relative width of the bars as a fraction of the bin width

• **stacked** – (boolean - default: False) If True, multiple data are stacked on top of each other

• **weights** – (list) A sequence of weights the same length as the data list. If supplied, then each value contributes its associated weight to the bin count

• **zorder** – (integer) the layer level at which to draw the histogram

**Note:** The weights option works only with a single list. List of lists representing multiple data are not supported.

**EXAMPLES:**

A very basic histogram for four data points:

```python
sage: histogram([1, 2, 3, 4], bins=2)
Graphics object consisting of 1 graphics primitive
```

We can see how the histogram compares to various distributions. Note the use of the density keyword to guarantee the plot looks like the probability density function:

```python
sage: nv = normalvariate
sage: H = histogram([nv(0, 1) for _ in range(1000)], bins=20, density=True, range=[-5, 5])
sage: P = plot(1/sqrt(2*pi)*e^(-x^2/2), (x, -5, 5), color='red', linestyle='--')
sage: H+P
Graphics object consisting of 2 graphics primitives
```

There are many options one can use with histograms. Some of these control the presentation of the data, even if it is boring:
2.8. Histograms
sage: histogram(list(range(100)), color=(1,0,0), label='mydata', rwidth=.5, align="right")
Graphics object consisting of 1 graphics primitive

This includes many usual matplotlib styling options:

sage: T = RealDistribution('lognormal', [0, 1])
sage: histogram( [T.get_random_element() for _ in range(100)], alpha=0.3, edgecolor='red', fill=False, linestyle='dashed', hatch='O', linewidth=5)
Graphics object consisting of 1 graphics primitive

We can do several data sets at once if desired:

sage: histogram([srange(0, 1, .1)*10, [nv(0, 1) for _ in range(100)]], color=['red', 'green'], bins=5)
Graphics object consisting of 1 graphics primitive

We have the option of stacking the data sets too:
2.8. Histograms
Chapter 2. Function and Data Plots
2.9 Bar charts

```python
sage: histogram([[1, 1, 1, 2, 2, 2, 3, 3, 3], [4, 4, 4, 3, 3, 2, 2, 2]], stacked=True, color=['blue', 'red'])
Graphics object consisting of 1 graphics primitive
```

It is possible to use weights with the histogram as well:

```python
sage: histogram(list(range(10)), bins=3, weights=[1, 2, 3, 4, 5, 5, 4, 3, 2, 1])
Graphics object consisting of 1 graphics primitive
```

---

**2.9 Bar charts**

class `sage.plot.bar_chart.BarChart(ind, datalist, options)`

Bases: `GraphicPrimitive`

Graphics primitive that represents a bar chart.

EXAMPLES:

```python
sage: from sage.plot.bar_chart import BarChart
sage: g = BarChart(list(range(4)), [1,3,2,0], {}); g
BarChart defined by a 4 datalist
```
 sage: type(g)
<class 'sage.plot.bar_chart.BarChart'>

**get_minmax_data()**

Returns a dictionary with the bounding box data.

**EXAMPLES:**

```python
 sage: b = bar_chart([-2.3, 5, -6, 12])
sage: d = b.get_minmax_data()
sage: d['xmin']
0
sage: d['xmax']
4
```

`sage.plot.bar_chart.bar_chart`(datalist, width=0.5, rgbcolor=(0, 0, 1), legend_label=None, aspect_ratio='automatic', **options)

A bar chart of (currently) one list of numerical data. Support for more data lists in progress.

**EXAMPLES:**

A bar_chart with blue bars:

```python
 sage: bar_chart([1, 2, 3, 4])
Graphics object consisting of 1 graphics primitive
```

A bar_chart with thinner bars:

```python
 sage: bar_chart([x^2 for x in range(1, 20)], width=0.2)
Graphics object consisting of 1 graphics primitive
```

A bar_chart with negative values and red bars:

```python
 sage: bar_chart([-3, 5, -6, 11], rgbcolor=(1, 0, 0))
Graphics object consisting of 1 graphics primitive
```

A bar chart with a legend (it’s possible, not necessarily useful):

```python
 sage: bar_chart([-1, 1, -1, 1], legend_label='wave')
Graphics object consisting of 1 graphics primitive
```

Extra options will get passed on to show(), as long as they are valid:

```python
 sage: bar_chart([-2, 8, -7, 3], rgbcolor=(1, 0, 0), axes=False)
Graphics object consisting of 1 graphics primitive
 sage: bar_chart([-2, 8, -7, 3], rgbcolor=(1, 0, 0)).show(axes=False) # These are
equivalent
```

2.9. Bar charts
2.9. Bar charts
2.9. Bar charts

![Bar chart showing two bars at the same value of 1.0 for different categories.]
3.1 Graph plotting

(For LaTeX drawings of graphs, see the graph_latex module.)

All graphs have an associated Sage graphics object, which you can display:

```python
sage: G = graphs.WheelGraph(15)
sage: P = G.plot()
sage: P.show()  # long time
```

When plotting a graph created using Sage’s `Graph` command, node positions are determined using the spring-layout
algorithm. Special graphs available from graphs.* have preset positions. For example, compare the two plots of the Petersen graph, as obtained using Graph or as obtained from that database:

```python
sage: petersen_spring = Graph('I`ES@obGkqegW~')
sage: petersen_spring.show()  # long time
```

All constructors in this database (except some random graphs) prefill the position dictionary, bypassing the spring-layout positioning algorithm.

**Plot options**

Here is the list of options accepted by plot() and the constructor of GraphPlot. Those two functions also accept all options of sage.plot.graphics.Graphics.show().
3.1. Graph plotting
<table>
<thead>
<tr>
<th><strong>layout</strong></th>
<th>A layout algorithm – one of: “acyclic”, “circular” (plots the graph with vertices evenly distributed on a circle), “ranked”, “graphviz”, “planar”, “spring” (traditional spring layout, using the graph's current positions as initial positions), or “tree” (the tree will be plotted in levels, depending on minimum distance for the root).</th>
</tr>
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<tbody>
<tr>
<td><strong>iterations</strong></td>
<td>The number of times to execute the spring layout algorithm.</td>
</tr>
<tr>
<td><strong>heights</strong></td>
<td>A dictionary mapping heights to the list of vertices at this height.</td>
</tr>
<tr>
<td><strong>spring</strong></td>
<td>Use spring layout to finalize the current layout.</td>
</tr>
<tr>
<td><strong>tree_root</strong></td>
<td>A vertex designation for drawing trees. A vertex of the tree to be used as the root for the layout='tree' option. If no root is specified, then one is chosen close to the center of the tree. Ignored unless layout='tree'.</td>
</tr>
<tr>
<td><strong>forest_roots</strong></td>
<td>An iterable specifying which vertices to use as roots for the layout='forest' option. If no root is specified for a tree, then one is chosen close to the center of the tree. Ignored unless layout='forest'.</td>
</tr>
<tr>
<td><strong>tree_orientation</strong></td>
<td>The direction of tree branches – 'up', 'down', 'left' or 'right'.</td>
</tr>
<tr>
<td><strong>save_pos</strong></td>
<td>Whether or not to save the computed position for the graph.</td>
</tr>
<tr>
<td><strong>dim</strong></td>
<td>The dimension of the layout – 2 or 3.</td>
</tr>
<tr>
<td><strong>prog</strong></td>
<td>Which graphviz layout program to use – one of &quot;circo&quot;, &quot;dot&quot;, &quot;fdp&quot;, &quot;neato&quot;, or &quot;twopi&quot;.</td>
</tr>
<tr>
<td><strong>by_component</strong></td>
<td>Whether to do the spring layout by connected component – a boolean.</td>
</tr>
<tr>
<td><strong>pos</strong></td>
<td>The position dictionary of vertices.</td>
</tr>
<tr>
<td><strong>vertex_labels</strong></td>
<td>Vertex labels to draw. This can be True/False to indicate whether to print the vertex string representation of not, a dictionary keyed by vertices and associating to each vertex a label string, or a function taking as input a vertex and returning a label string.</td>
</tr>
<tr>
<td><strong>vertex_color</strong></td>
<td>Default color for vertices not listed in vertex_colors dictionary.</td>
</tr>
<tr>
<td><strong>vertex_colors</strong></td>
<td>A dictionary specifying vertex colors: each key is a color recognizable by matplotlib, and each corresponding value is a list of vertices.</td>
</tr>
<tr>
<td><strong>vertex_size</strong></td>
<td>The size to draw the vertices.</td>
</tr>
<tr>
<td><strong>vertex_shape</strong></td>
<td>The shape to draw the vertices. Currently unavailable for Multi-edged DiGraphs.</td>
</tr>
<tr>
<td><strong>edge_labels</strong></td>
<td>Whether or not to draw edge labels.</td>
</tr>
<tr>
<td><strong>edge_style</strong></td>
<td>The linestyle of the edges. It should be one of “solid”, “dashed”, “dotted”, dash-dot”, or “-”, “-”, “-”, “-”, “-”, respectively.</td>
</tr>
<tr>
<td><strong>edge_thickness</strong></td>
<td>The thickness of the edges.</td>
</tr>
<tr>
<td><strong>edge_color</strong></td>
<td>The default color for edges not listed in edge_colors.</td>
</tr>
<tr>
<td><strong>edge_colors</strong></td>
<td>A dictionary specifying edge colors: each key is a color recognized by matplotlib, and each corresponding value is a list of edges.</td>
</tr>
<tr>
<td><strong>color_by_label</strong></td>
<td>Whether to color the edges according to their labels. This also accepts a function or dictionary mapping labels to colors.</td>
</tr>
<tr>
<td><strong>partition</strong></td>
<td>A partition of the vertex set. If specified, plot will show each cell in a different color; vertex_colors takes precedence.</td>
</tr>
<tr>
<td><strong>loop_size</strong></td>
<td>The radius of the smallest loop.</td>
</tr>
<tr>
<td><strong>dist</strong></td>
<td>The distance between multiedges.</td>
</tr>
<tr>
<td><strong>max_dist</strong></td>
<td>The max distance range to allow multiedges.</td>
</tr>
<tr>
<td><strong>talk</strong></td>
<td>Whether to display the vertices in talk mode (larger and white).</td>
</tr>
<tr>
<td><strong>graph_border</strong></td>
<td>Whether or not to draw a frame around the graph.</td>
</tr>
<tr>
<td><strong>edge_labels_background</strong></td>
<td>The color of the background of the edge labels.</td>
</tr>
</tbody>
</table>

**Default options**

This module defines two dictionaries containing default options for the plot() and show() methods. These two dictionaries are sage.graphs.graph_plot.DEFAULT_PLOT_OPTIONS and sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS, respectively.
Obviously, these values are overruled when arguments are given explicitly.

Here is how to define the default size of a graph drawing to be \((6, 6)\). The first two calls to \texttt{show()} use this option, while the third does not (a value for \texttt{figsize} is explicitly given):

```python
sage: import sage.graphs.graph_plot
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (6, 6)
sage: graphs.PetersenGraph().show()  # long time
sage: graphs.ChvatalGraph().show()   # long time
sage: graphs.PetersenGraph().show(figsize=(4, 4))  # long time
```

We can now reset the default to its initial value, and now display graphs as previously:

```python
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (4, 4)
sage: graphs.PetersenGraph().show()  # long time
sage: graphs.ChvatalGraph().show()   # long time
```

Note:

- While \texttt{DEFAULT_PLOT_OPTIONS} affects both \texttt{G.show()} and \texttt{G.plot()}, settings from \texttt{DEFAULT_SHOW_OPTIONS} only affects \texttt{G.show()}.
- In order to define a default value permanently, you can add a couple of lines to Sage’s startup scripts. Example:

```python
sage: import sage.graphs.graph_plot
sage: sage.graphs.graph_plot.DEFAULT_SHOW_OPTIONS['figsize'] = (4, 4)
```

Index of methods and functions

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<th>Description</th>
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<td>Set the position plotting parameters for this GraphPlot.</td>
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<tr>
<td>\texttt{GraphPlot.set_vertices()}</td>
<td>Set the vertex plotting parameters for this GraphPlot.</td>
</tr>
<tr>
<td>\texttt{GraphPlot.set_edges()}</td>
<td>Set the edge (or arrow) plotting parameters for the GraphPlot object.</td>
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<td>\texttt{GraphPlot.show()}</td>
<td>Show the (Di)Graph associated with this GraphPlot object.</td>
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<td>Compute a nice layout of a tree.</td>
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**class** \texttt{sage.graphs.graph_plot.GraphPlot(graph, options)}

Bases: \texttt{SageObject}

Return a \texttt{GraphPlot} object, which stores all the parameters needed for plotting (Di)Graphs.

A \texttt{GraphPlot} has a plot and show function, as well as some functions to set parameters for vertices and edges. This constructor assumes default options are set. Defaults are shown in the example below.

**EXAMPLES:**

```python
sage: from sage.graphs.graph_plot import GraphPlot
sage: options = {
                'vertex_size': 200,
                'vertex_labels': True,
                'layout': None,
                'edge_style': 'solid',
                'edge_color': 'black',
                'edge_colors': None,
                }
```
layout_tree(root, orientation)

Compute a nice layout of a tree.

INPUT:

• root – the root vertex.
• orientation – whether to place the root at the top or at the bottom:
  – orientation="down" – children are placed below their parent
  – orientation="top" – children are placed above their parent

EXAMPLES:

```python
sage: from sage.graphs.graph_plot import GraphPlot
sage: G = graphs.HoffmanSingletonGraph()
sage: T = Graph()
sage: T.add_edges(G.min_spanning_tree(starting_vertex=0))
sage: T.show(layout='tree', tree_root=0)  # indirect doctest
```

plot(**kwds)

Return a graphics object representing the (di)graph.

INPUT:

The options accepted by this method are to be found in the documentation of the sage.graphs.graph_plot module, and the show() method.

Note: See the module's documentation for information on default values of this method.

We can specify some pretty precise plotting of familiar graphs:

```python
sage: from math import sin, cos, pi
sage: P = graphs.PetersenGraph()
sage: d = {'#FF0000': [0, 5], '#FF9900': [1, 6], '#FFFF00': [2, 7],
      ' #00FF00': [3, 8], '#0000FF': [4,9]}
sage: pos_dict = {}
sage: for i in range(5):
...:    x = float(cos(pi/2 + ((2*pi)/5)*i))
```

(continued on next page)
Here are some more common graphs with typical options:

```python
sage: C = graphs.CubeGraph(8)
sage: P = C.graphplot(vertex_labels=False, vertex_size=0,
.....:                     graph_border=True)
sage: P.show()
```

```python
sage: G = graphs.HeawoodGraph().copy(sparse=True)
sage: for u, v, l in G.edges(sort=True):
.....:     G.set_edge_label(u, v, f'({u},{v})'
sage: G.graphplot(edge_labels=True).show()
```
Chapter 3. Plots of Other Mathematical Objects
3.1. Graph plotting
The options for plotting also work with directed graphs:

```python
sage: D = DiGraph({
    ....:     0: [1, 10, 19], 1: [8, 2], 2: [3, 6], 3: [19, 4],
    ....:     4: [17, 5], 5: [6, 15], 6: [7, 8, 14], 8: [9],
    ....:     9: [10, 13], 10: [11], 11: [12, 18], 12: [16, 13],
    ....:     13: [14, 15], 14: [15], 15: [16], 16: [17], 17: [18],
    ....:     18: [19], 19: []}
)
sage: for u, v, l in D.edges(sort=True):
    ....:     D.set_edge_label(u, v, l'

sage: D.graphplot(edge_labels=True, layout='circular').show()
```

This example shows off the coloring of edges:

```python
sage: from sage.plot.colors import rainbow
sage: C = graphs.CubeGraph(5)
sage: R = rainbow(5)
sage: edge_colors = {}
sage: for i in range(5):
    ....:     edge_colors[R[i]] = []
sage: for u, v, l in C.edges(sort=True):
    ....:     for i in range(5):
    ....:         if u[i] != v[i]:
```

(continues on next page)
With the partition option, we can separate out same-color groups of vertices:

```
sage: D = graphs.DodecahedralGraph()
sage: Pi = [[6, 5, 15, 14, 7], [16, 13, 8, 2, 4],
        [12, 17, 9, 3, 1], [0, 19, 18, 10, 11]]
sage: D.show(partition=Pi)
```

Loops are also plotted correctly:

```
sage: G = graphs.PetersenGraph()
sage: G.allow_loops(True)
sage: G.add_edge(0,0)
sage: G.show()
```

```
sage: D = DiGraph({0:[0,1], 1:[2], 2:[3]}, loops=True)
sage: D.show()
sage: D.show(edge_colors={(0, 1, 0): [(0, 1, None), (1, 2, None)],
                          (0, 0, 0): [(2, 3, None)]})
```
3.1. Graph plotting
More options:

```python
sage: pos = {0: [0.0, 1.5], 1: [-0.8, 0.3], 2: [-0.6, -0.8],
            3: [0.6, -0.8], 4: [0.8, 0.3]}
sage: g = Graph({0: [1], 1: [2], 2: [3], 3: [4], 4: [0]})
sage: g.graphplot(pos=pos, layout='spring', iterations=0).plot()
Graphics object consisting of 11 graphics primitives
```

We can plot multiple graphs:

```python
sage: G = Graph()
sage: P = G.graphplot().plot()
sage: P.axes()
False
sage: G = DiGraph()
sage: P = G.graphplot().plot()
sage: P.axes()
False
```

(continues on next page)
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.graphplot(heights={0: [0], 1: [4, 5, 1], 2: [2], 3: [3, 6]}, edge_labels=True).plot()
3.1. Graph plotting
Graphics object consisting of 20 graphics primitives

```
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.graphplot(layout='tree').show()
```

The tree layout is also useful:

```
sage: t = DiGraph('JCC??@A??G0??C0??G0??')
sage: t.graphplot(layout='tree', tree_root=0,
...:    tree_orientation="up"
...:    ).show()
```

More examples:

```
sage: D = DiGraph({0:[1,2,3], 2:[1,4], 3:[0]})
sage: D.graphplot().show()
```

```
sage: D = DiGraph(multiedges=True, sparse=True)
sage: for i in range(5):
...:    D.add_edge((i, i + 1, 'a'))
...:    D.add_edge((i, i - 1, 'b'))
```

(continues on next page)
3.1. Graph plotting
Chapter 3. Plots of Other Mathematical Objects
3.1. Graph plotting
sage: D.graphplot(edge_labels=True,
.....:    edge_colors=D._color_by_label()
.....: ).plot()
Graphics object consisting of 34 graphics primitives

The `edge_style` option may be provided in the short format too:

```
sage: g.graphplot(edge_labels=True,
.....:    color_by_label=True,
.....:    edge_style='dashed'
.....: ).plot()
```
Graphics object consisting of 22 graphics primitives
**set_edges(**edge_options**)

Set edge plotting parameters for the `GraphPlot` object.

This function is called by the constructor but can also be called to update the vertex options of an existing `GraphPlot` object. Note that the changes are cumulative.

**EXAMPLES:**

```python
sage: g = Graph(loops=True, multiedges=True, sparse=True)
sage: g.add_edges(((0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
......: (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
......: (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h')))
sage: GP = g.graphplot(vertex_size=100, edge_labels=True,
......: color_by_label=True, edge_style='dashed')
sage: GP.set_edges(edge_style='solid')
sage: GP.plot()
```

Graphics object consisting of 22 graphics primitives

```python
sage: GP.set_edges(edge_color='black')
sage: GP.plot()
```

Graphics object consisting of 22 graphics primitives

```python
sage: d = DiGraph(loops=True, multiedges=True, sparse=True)
```

(continues on next page)
3.1. Graph plotting
sage: d.add_edges(((0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
.....: (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
.....: (0, 1, 'f'), (2, 1, 'g'), (2, 2, 'h')))

sage: GP = d.graphplot(vertex_size=100, edge_labels=True,
.....: color_by_label=True, edge_style='dashed')

sage: GP.set_edges(edge_style='solid')

sage: GP.plot()

Graphics object consisting of 24 graphics primitives

sage: GP.set_edges(edge_color='black')

sage: GP.plot()

Graphics object consisting of 24 graphics primitives

set_pos()

Set the position plotting parameters for this GraphPlot.

EXAMPLES:

This function is called implicitly by the code below:

sage: g = Graph({0: [1, 2], 2: [3], 4: [0, 1]})

sage: g.graphplot(save_pos=True, layout='circular') # indirect doctest

GraphPlot object for Graph on 5 vertices
3.1. Graph plotting
The following illustrates the format of a position dictionary, but due to numerical noise we do not check the values themselves:

```
sage: g.get_pos()
{0: (0.0, 1.0),
  1: (-0.951..., 0.309...),
  2: (-0.587..., -0.809...),
  3: (0.587..., -0.809...),
  4: (0.951..., 0.309...)}
```

```
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.plot(heights={0: [0], 1: [4, 5, 1], 2: [2], 3: [3, 6]})
```

Graphics object consisting of 14 graphics primitives

```
sage: T = list(graphs.trees(7))
sage: t = T[3]
sage: t.plot(heights={0: [0], 1: [4, 5, 1], 2: [2], 3: [3, 6]})
```

Graphics object consisting of 14 graphics primitives

### set_vertices(**vertex_options)**

Set the vertex plotting parameters for this GraphPlot.

This function is called by the constructor but can also be called to make updates to the vertex options of an existing GraphPlot object. Note that the changes are cumulative.

**EXAMPLES:**
sage: g = Graph({}, loops=True, multiedges=True, sparse=True)
sage: g.add_edges([(0, 0, 'a'), (0, 0, 'b'), (0, 1, 'c'),
                (0, 1, 'd'), (0, 1, 'e'), (0, 1, 'f'),
                (2, 1, 'g'), (2, 2, 'h')])
sage: GP = g.graphplot(vertex_size=100, edge_labels=True,
                    color_by_label=True, edge_style='dashed')
sage: GP.set_vertices(talk=True)
sage: GP.plot()
Graphics object consisting of 22 graphics primitives
sage: GP.set_vertices(vertex_color='green', vertex_shape='^')
sage: GP.plot()
Graphics object consisting of 22 graphics primitives

Vertex labels are flexible:

sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=False)
Graphics object consisting of 4 graphics primitives
sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=True)
Graphics object consisting of 8 graphics primitives
Chapter 3. Plots of Other Mathematical Objects
```python
sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=dict(zip(g, ['+', '-', '/', '*']))))
```

Graphics object consisting of 8 graphics primitives

```python
sage: g = graphs.PathGraph(4)
sage: g.plot(vertex_labels=lambda x: str(x % 2))
```

Graphics object consisting of 8 graphics primitives

```
show(**kwds)
```

Show the (di)graph associated with this GraphPlot object.

**INPUT:**

This method accepts all parameters of `sage.plot.graphics.Graphics.show()`.

**Note:**

- See the module’s documentation for information on default values of this method.
- Any options not used by plot will be passed on to the `show()` method.

**EXAMPLES:**
3.2 Matrix plots

class sage.plot.matrix_plot.MatrixPlot(xy_data_array, xrange, yrange, options)
    Bases: GraphicPrimitive

Primitive class for the matrix plot graphics type. See matrix_plot? for help actually doing matrix plots.

INPUT:

• xy_data_array - list of lists giving matrix values corresponding to the grid
• xrange - tuple of 2 floats indicating range for horizontal direction (number of columns in the matrix). If None, the defaults are used as indicated in matrix_plot().
• yrange - tuple of 2 floats indicating range for vertical direction (number of rows in the matrix). If None, the defaults are used as indicated in matrix_plot().
• options - dict of valid plot options to pass to constructor
EXAMPLES:

Note this should normally be used indirectly via \texttt{matrix\_plot()}:

```
sage: from sage.plot.matrix_plot import MatrixPlot
sage: M = MatrixPlot([[1,3],[2,4]],(1,2),(2,3),options={'cmap': 'winter'})
sage: M
MatrixPlot defined by a 2 x 2 data grid
sage: M.yrange
(2, 3)
sage: M.xy\_data\_array
[[1, 3], [2, 4]]
sage: M.options()
{'cmap': 'winter'}
```

Extra options will get passed on to \texttt{show()}, as long as they are valid:

```
sage: matrix\_plot([[1, 0], [0, 1]], fontsize=10)
Graphics object consisting of 1 graphics primitive
sage: matrix\_plot([[1, 0], [0, 1]]).show(fontsize=10)  # These are equivalent
```

\texttt{get\_minmax\_data()}  
Returns a dictionary with the bounding box data.

EXAMPLES:

```
sage: m = matrix\_plot(matrix([[1,3,5,1],[2,4,5,6],[1,3,5,7]]))[0]
sage: list(sorted(m.get\_minmax\_data().items()))])
[('xmax', 3.5), ('xmin', -0.5), ('ymax', 2.5), ('ymin', -0.5)]
```

\texttt{sage.plot.matrix\_plot.matrix\_plot(mat, xrange=None, yrange=None, aspect\_ratio=1, axes=False, cmap='Greys', colorbar=False, frame=True, marker=',', norm=None, flip\_y=True, subdivisions=False, ticks\_integer=True, vmin=None, vmax=None, subdivision\_boundaries=None, subdivision\_style=None, colorbar\_orientation='vertical', colorbar\_format=None, **options)}

A plot of a given matrix or 2D array.

If the matrix is sparse, colors only indicate whether an element is nonzero or zero, so the plot represents the sparsity pattern of the matrix.

If the matrix is dense, each matrix element is given a different color value depending on its relative size compared to the other elements in the matrix.

The default is for the lowest number to be black and the highest number to be white in a greyscale pattern; see the information about normalizing below. To reverse this, use \texttt{cmap='Greys'}.

The tick marks drawn on the frame axes denote the row numbers (vertical ticks) and the column numbers (horizontal ticks) of the matrix.

INPUT:

- \texttt{mat} - a 2D matrix or array
- \texttt{xrange} - (default: None) tuple of the horizontal extent \((x_{\text{min}}, x_{\text{max}})\) of the bounding box in which to draw the matrix. The image is stretched individually along x and y to fill the box.

If None, the extent is determined by the following conditions. Matrix entries have unit size in data coordinates. Their centers are on integer coordinates, and their center coordinates range from 0 to columns-1 horizontally and from 0 to rows-1 vertically.
If the matrix is sparse, this keyword is ignored.

- **yrange** - (default: None) tuple of the vertical extent (ymin, ymax) of the bounding box in which to draw the matrix. See `xrange` for details.

The following input must all be passed in as named parameters, if default not used:

- **cmap** - a colormap (default: ‘Greys’), the name of a predefined colormap, a list of colors, or an instance of a matplotlib Colormap.

  The list of predefined color maps can be visualized in matplotlib’s documentation. You can also type `import matplotlib.cm; matplotlib.cm.datad.keys()` to list their names.

- **colorbar** – boolean (default: False) Show a colorbar or not (dense matrices only).

  The following options are used to adjust the style and placement of colorbars. They have no effect if a colorbar is not shown.

  - **colorbar_orientation** – string (default: ‘vertical’), controls placement of the colorbar, can be either ‘vertical’ or ‘horizontal’
  - **colorbar_format** – a format string, this is used to format the colorbar labels.
  - **colorbar_options** – a dictionary of options for the matplotlib.colorbar API. Documentation for the matplotlib.colorbar module has details.

- **norm** - If None (default), the value range is scaled to the interval [0,1]. If ‘value’, then the actual value is used with no scaling. A matplotlib.colors.Normalize instance may also passed.

- **vmin** - The minimum value (values below this are set to this value)

- **vmax** - The maximum value (values above this are set to this value)

- **flip_y** - (default: True) boolean. If False, the first row of the matrix is on the bottom of the graph. Otherwise, the first row is on the top of the graph.

- **subdivisions** - If True, plot the subdivisions of the matrix as lines.

- **subdivision_boundaries** - a list of lists in the form `[row_subdivisions, column_subdivisions]`, which specifies the row and column subdivisions to use. If not specified, defaults to the matrix subdivisions

- **subdivision_style** - a dictionary of properties passed on to the `line2d()` command for plotting subdivisions. If this is a two-element list or tuple, then it specifies the styles of row and column divisions, respectively.

**EXAMPLES:**

A matrix over $\mathbb{Z}$ colored with different grey levels:

```sage
sage: matrix_plot(matrix([[1,3,5,1],[2,4,5,6],[1,3,5,7]]))
Graphics object consisting of 1 graphics primitive
```

Here we make a random matrix over $\mathbb{R}$ and use cmap='hsv' to color the matrix elements different RGB colors:

```sage
sage: matrix_plot(random_matrix(RDF, 50), cmap='hsv')
Graphics object consisting of 1 graphics primitive
```

By default, entries are scaled to the interval [0,1] before determining colors from the color map. That means the two plots below are the same:

```sage
sage: P = matrix_plot(matrix(2,[1,1,3,3]))
sage: Q = matrix_plot(matrix(2,[2,2,3,3]))
sage: P; Q
```

(continues on next page)
However, we can specify which values scale to 0 or 1 with the `vmin` and `vmax` parameters (values outside the range are clipped). The two plots below are now distinguished:

```plaintext
sage: P = matrix_plot(matrix(2,[1,1,3,3]), vmin=0, vmax=3, colorbar=True)
sage: Q = matrix_plot(matrix(2,[2,2,3,3]), vmin=0, vmax=3, colorbar=True)
sage: P; Q
Graphics object consisting of 1 graphics primitive
```

We can also specify a norm function of 'value', which means that there is no scaling performed:

```plaintext
sage: matrix_plot(random_matrix(ZZ,10)*.05, norm='value', colorbar=True)
Graphics object consisting of 1 graphics primitive
```

Matrix subdivisions can be plotted as well:

```plaintext
sage: m=random_matrix(RR,10)
sage: m.subdivide([2,4],[6,8])
sage: matrix_plot(m, subdivisions=True, subdivision_style=dict(color='red',thickness=3))
Graphics object consisting of 1 graphics primitive
```

You can also specify your own subdivisions and separate styles for row or column subdivisions:

```plaintext
sage: m=random_matrix(RR,10)
sage: matrix_plot(m, subdivisions=True, subdivision_boundaries=[[2,4],[6,8]], subdivision_style=[dict(color='red',thickness=3), dict(linestyle='--',thickness=6)])
Graphics object consisting of 1 graphics primitive
```

Generally matrices are plotted with the (0,0) entry in the upper left. However, sometimes if we are plotting an image, we'd like the (0,0) entry to be in the lower left. We can do that with the `flip_y` argument:

```plaintext
sage: matrix_plot(identity_matrix(100), flip_y=False)
```

A custom bounding box in which to draw the matrix can be specified using the `xrange` and `yrange` arguments:

```plaintext
sage: P = matrix_plot(identity_matrix(100), xrange=(0, pi), yrange=(-pi, 0)); P
```

```plaintext
{\'xmax\': 3.14159..., \'xmin\': 0.0, \'ymax\': 0.0, \'ymin\': -3.14159...}
```

If the horizontal and vertical dimension of the image are very different, the default `aspect_ratio=1` may be unsuitable and can be changed to `automatic`:

```plaintext
sage: matrix_plot(random_matrix(RDF, 2, 2), (-100, 100), (0, 1), aspect_ratio='automatic')
```
Another random plot, but over $\mathbb{F}_{389}$:

```
sage: m = random_matrix(GF(389), 10)  # needs sage.rings.finite_rings
sage: matrix_plot(m, cmap='Oranges')  # needs sage.rings.finite_rings
```

Graphics object consisting of 1 graphics primitive

It also works if you lift it to the polynomial ring:

```
sage: matrix_plot(m.change_ring(GF(389)['x']), cmap='Oranges')  # needs sage.rings.finite_rings
```

Graphics object consisting of 1 graphics primitive

We have several options for colorbars:

```
sage: matrix_plot(random_matrix(RDF, 50), colorbar=True, colorbar_orientation='horizontal')
```

Graphics object consisting of 1 graphics primitive

```
sage: matrix_plot(random_matrix(RDF, 50), colorbar=True, colorbar_format='%.3f')
```

Graphics object consisting of 1 graphics primitive

The length of a color bar and the length of the adjacent matrix plot dimension may be quite different. This example shows how to adjust the length of the colorbar by passing a dictionary of options to the matplotlib colorbar routines.

```
sage: m = random_matrix(ZZ, 40, 80, x=-10, y=10)
sage: m.plot(colorbar=True, colorbar_orientation='vertical', colorbar_options={'shrink':0.50})
```

Graphics object consisting of 1 graphics primitive

Here we plot a random sparse matrix:

```
sage: sparse = matrix(dict(((randint(0, 10), randint(0, 10)), 1) for i in range(100)))
sage: matrix_plot(sparse)
```

Graphics object consisting of 1 graphics primitive

```
sage: A = random_matrix(ZZ, 100000, density=.00001, sparse=True)
sage: matrix_plot(A, marker=',')
```

Graphics object consisting of 1 graphics primitive

As with dense matrices, sparse matrix entries are automatically converted to floating point numbers before plotting. Thus the following works:

```
sage: b = random_matrix(GF(2), 200, sparse=True, density=0.01)  # needs sage.rings.finite_rings
sage: matrix_plot(b)  # needs sage.rings.finite_rings
```

Graphics object consisting of 1 graphics primitive

While this returns an error:
```python
sage: b = random_matrix(CDF, 200, sparse=True, density=0.01)
sage: matrix_plot(b)
Traceback (most recent call last):
  ...  
ValueError: cannot convert entries to floating point numbers
```

To plot the absolute value of a complex matrix, use the apply_map method:

```python
sage: b = random_matrix(CDF, 200, sparse=True, density=0.01)
sage: matrix_plot(b.apply_map(abs))
```

Plotting lists of lists also works:

```python
sage: matrix_plot([[1, 3, 5, 1], [2, 4, 5, 6], [1, 3, 5, 7]])
```

As does plotting of NumPy arrays:

```python
sage: import numpy
sage: matrix_plot(numpy.random.rand(10, 10))
```

A plot title can be added to the matrix plot:

```python
sage: matrix_plot(identity_matrix(50), flip_y=False, title='not identity')
```

The title position is adjusted upwards if the flip_y keyword is set to True (this is the default):

```python
sage: matrix_plot(identity_matrix(50), title='identity')
```
4.1 Arcs of circles and ellipses

class sage.plot.arc.Arc(x, y, r1, r2, angle, s1, s2, options)

Bases: GraphicPrimitive

Primal class for the Arc graphics type. See arc? for information about actually plotting an arc of a circle or an ellipse.

INPUT:
• \(x, y\) - coordinates of the center of the arc
• \(r1, r2\) - lengths of the two radii
• \(angle\) - angle of the horizontal with width
• \(sector\) - sector of angle
• \(options\) - dict of valid plot options to pass to constructor

EXAMPLES:

Note that the construction should be done using arc:

```
sage: from math import pi
sage: from sage.plot.arc import Arc
sage: print(Arc(0,0,1,1,pi/4,pi/4,pi/2,{}))
Arc with center (0.0,0.0) radii (1.0,1.0) angle 0.78539816339... inside the sector˓→(0.78539816339...,1.5707963267...)
```

bezier_path()

Return self as a Bezier path.

This is needed to concatenate arcs, in order to create hyperbolic polygons.

EXAMPLES:

```
sage: from sage.plot.arc import Arc
sage: op = {'alpha':1,'thickness':1,'rgbcolor':'blue','zorder':0,˓→'linestyle':'--'}
sage: Arc(2,3,2,2,0,2,3,op).bezier_path()
Graphics object consisting of 1 graphics primitive
```

(continues on next page)
sage: b = a[0].bezier_path()
sage: b[0]
Bezier path from (1.133..., 0.8237...) to (-0.2655..., 0.9911...)

get_minmax_data()

Return a dictionary with the bounding box data.

The bounding box is computed as minimal as possible.

EXAMPLES:

An example without angle:

sage: p = arc((-2, 3), 1, 2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-3.0
sage: d['xmax']
-1.0
sage: d['ymin']
1.0
sage: d['ymax']
5.0

The same example with a rotation of angle π/2:

sage: from math import pi
sage: p = arc((-2, 3), 1, 2, pi/2)
sage: d = p.get_minmax_data()
sage: d['xmin']
-4.0
sage: d['xmax']
0.0
sage: d['ymin']
2.0
sage: d['ymax']
4.0

plot3d()

sage.plot.arc.arc(center, r1, r2=None, angle=0.0, sector=(0.0, 6.283185307179586), alpha=1, thickness=1, linestyle='solid', zorder=5, rgbcolor='blue', aspect_ratio=1.0, **options)

An arc (that is a portion of a circle or an ellipse)

Type arc.options to see all options.

INPUT:

- center - 2-tuple of real numbers - position of the center.
- r1, r2 - positive real numbers - radii of the ellipse. If only r1 is set, then the two radii are supposed to be equal and this function returns an arc of circle.
- angle - real number - angle between the horizontal and the axis that corresponds to r1.
- sector - 2-tuple (default: (0,2*pi))- angles sector in which the arc will be drawn.

OPTIONS:
• **alpha** - float (default: 1) - transparency
• **thickness** - float (default: 1) - thickness of the arc
• **color**, **rgbcolor** - string or 2-tuple (default: ‘blue’) - the color of the arc
• **linestyle** - string (default: 'solid') - The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '--', respectively.

**EXAMPLES:**
Plot an arc of circle centered at (0,0) with radius 1 in the sector \((\pi/4, 3\pi/4)\):

```python
sage: from math import pi
sage: arc((0,0), 1, sector=(pi/4,3*pi/4))
Graphics object consisting of 1 graphics primitive
```

Plot an arc of an ellipse between the angles 0 and \(\pi/2\):

```python
sage: arc((2,3), 2, 1, sector=(0,pi/2))
Graphics object consisting of 1 graphics primitive
```

Plot an arc of a rotated ellipse between the angles 0 and \(\pi/2\):

```python
sage: arc((2,3), 2, 1, angle=pi/5, sector=(0,pi/2))
Graphics object consisting of 1 graphics primitive
```
Chapter 4. Basic Shapes
4.1. Arcs of circles and ellipses
Plot an arc of an ellipse in red with a dashed linestyle:

\[
\text{sage: } \text{arc}((0,0), 2, 1, 0, (0,\pi/2), \text{linestyle="dashed"}, \text{color="red"})
\]

Graphics object consisting of 1 graphics primitive

\[
\text{sage: } \text{arc}((0,0), 2, 1, 0, (0,\pi/2), \text{linestyle="--"}, \text{color="red"})
\]

Graphics object consisting of 1 graphics primitive

The default aspect ratio for arcs is 1.0:

\[
\text{sage: } \text{arc}((0,0), 1, \text{sector=(pi/4,3*pi/4))}.\text{aspect_ratio()}
\]

1.0

It is not possible to draw arcs in 3D:

\[
\text{sage: } A = \text{arc}((0,0,0), 1)
\]

Traceback (most recent call last):
...
NotImplementedError
4.2 Arrows

class sage.plot.arrow.Arrow(xtail, ytail, xhead, yhead, options)
    Bases: GraphicPrimitive

Primitive class that initializes the (line) arrow graphics type

EXAMPLES:

We create an arrow graphics object, then take the 0th entry in it to get the actual Arrow graphics primitive:

sage: P = arrow((0,1), (2,3))[0]
sage: type(P)
<class 'sage.plot.arrow.Arrow'>
sage: P
Arrow from (0.0,1.0) to (2.0,3.0)

get_minmax_data()
    Returns a bounding box for this arrow.

EXAMPLES:

sage: d = arrow((1,1), (5,5)).get_minmax_data()
sage: d['xmin']
1.0
sage: d['xmax']
5.0

plot3d(ztail=0, zhead=0, **kwds)
    Takes 2D plot and places it in 3D.

EXAMPLES:

sage: A = arrow((0,0),(1,1))[0].plot3d()
sage: A.jmol_repr(A.testing_render_params())[0]
'draw line_1 diameter 2 arrow {0.0 0.0 0.0} {1.0 1.0 0.0} '

Note that we had to index the arrow to get the Arrow graphics primitive. We can also change the height via the Graphics.plot3d() method, but only as a whole:

sage: A = arrow((0,0),(1,1)).plot3d(3)
sage: A.jmol_repr(A.testing_render_params())[0][0]
'draw line_1 diameter 2 arrow {0.0 0.0 3.0} {1.0 1.0 3.0} '

Optional arguments place both the head and tail outside the xy-plane, but at different heights. This must be done on the graphics primitive obtained by indexing:

sage: A=arrow((0,0),(1,1))[0].plot3d(3,4)
sage: A.jmol_repr(A.testing_render_params())[0]
'draw line_1 diameter 2 arrow {0.0 0.0 3.0} {1.0 1.0 4.0} '

class sage.plot.arrow.CurveArrow(path, options)
    Bases: GraphicPrimitive

Returns an arrow graphics primitive along the provided path (bezier curve).

EXAMPLES:
sage: from sage.plot.arrow import CurveArrow
sage: b = CurveArrow(path=[[0,0],(.5,.5),(1,0)],[(.5,1),(0,0)],
....: options={})
sage: b
CurveArrow from (0, 0) to (0, 0)

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

sage: from sage.plot.arrow import CurveArrow
sage: b = CurveArrow(path=[[0,0],(.5,.5),(1,0)],[(.5,1),(0,0)],
....: options={})
sage: d = b.get_minmax_data()
sage: d['xmin']
0.0
sage: d['xmax']
1.0

sage.plot.arrow.arrow(tailpoint=None, headpoint=None, **kwds)

Returns either a 2-dimensional or 3-dimensional arrow depending on value of points.

For information regarding additional arguments, see either arrow2d? or arrow3d?.

EXAMPLES:

sage: arrow((0,0), (1,1))
Graphics object consisting of 1 graphics primitive

sage: arrow((0,0,1), (1,1,1))
Graphics3d Object

sage.plot.arrow.arrow2d(tailpoint=None, headpoint=None, path=None, width=2, rgbcolor=(0, 0, 1),
....: zorder=2, head=1, linestyle='solid', legend_label=None, **options)

If tailpoint and headpoint are provided, returns an arrow from (xtail, ytail) to (xhead, yhead). If tailpoint or headpoint is None and path is not None, returns an arrow along the path. (See further info on paths in bezier_path).

INPUT:

• tailpoint - the starting point of the arrow
• headpoint - where the arrow is pointing to
• path - the list of points and control points (see bezier_path for detail) that the arrow will follow from source to destination
• head - 0, 1 or 2, whether to draw the head at the start (0), end (1) or both (2) of the path (using 0 will swap headpoint and tailpoint). This is ignored in 3D plotting.
• linestyle - (default: 'solid') The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '-', '.', respectively.
• width - (default: 2) the width of the arrow shaft, in points
• color - (default: (0,0,1)) the color of the arrow (as an RGB tuple or a string)
• hue - the color of the arrow (as a number)
4.2. Arrows
• **arrowsize** - the size of the arrowhead

• **arrowshorten** - the length in points to shorten the arrow (ignored if using path parameter)

• **legend_label** - the label for this item in the legend

• **legend_color** - the color for the legend label

• **zorder** - the layer level to draw the arrow—note that this is ignored in 3D plotting.

**EXAMPLES:**

A straight, blue arrow:

```
sage: arrow2d((1,1), (3,3))
Graphics object consisting of 1 graphics primitive
```

![Image of a blue arrow from (1,1) to (3,3)](image)

Make a red arrow:

```
sage: arrow2d((-1,-1), (2,3), color=(1,0,0))
Graphics object consisting of 1 graphics primitive
```

```
sage: arrow2d((-1,-1), (2,3), color='red')
Graphics object consisting of 1 graphics primitive
```

You can change the width of an arrow:
Use a dashed line instead of a solid one for the arrow:

```python
sage: arrow2d((1,1), (3,3), linestyle='dashed')
Graphics object consisting of 1 graphics primitive
```

A pretty circle of arrows:

```python
sage: sum(arrow2d((0,0), (cos(x),sin(x)), hue=x/(2*pi))
# needs sage.symbolic
˓
....: for x in [0..2*pi, step=0.1])
Graphics object consisting of 63 graphics primitives
```

If we want to draw the arrow between objects, for example, the boundaries of two lines, we can use the `arrowshorten` option to make the arrow shorter by a certain number of points:

```python
sage: L1 = line([(0,0), (1,0)], thickness=10)
sage: L2 = line([(0,1), (1,1)], thickness=10)
sage: A = arrow2d((0.5,0), (0.5,1), arrowshorten=10, rgbcolor=(1,0,0))
```
4.2. Arrows

4.2. Arrows
If BOTH headpoint and tailpoint are None, then an empty plot is returned:

```
sage: arrow2d(headpoint=None, tailpoint=None)
Graphics object consisting of 0 graphics primitives
```

We can also draw an arrow with a legend:

```
sage: arrow((0,0), (0,2), legend_label='up', legend_color='purple')
Graphics object consisting of 1 graphics primitive
```

Extra options will get passed on to `Graphics.show()`, as long as they are valid:

```
sage: arrow2d((-2,2), (7,1), frame=True)
Graphics object consisting of 1 graphics primitive
```

```
sage: arrow2d((-2,2), (7,1)).show(frame=True)
```
4.2. Arrows
4.3 Bezier paths

class sage.plot.bezier_path.BezierPath(path, options)

Bases: GraphicPrimitive_xydata

Path of Bezier Curves graphics primitive.

The input to this constructor is a list of curves, each a list of points, along which to create the curves, along with a dict of any options passed.

EXAMPLES:

```python
sage: from sage.plot.bezier_path import BezierPath
sage: BezierPath([[(0,0), (.5,.5),(1,0)],[(.5,1),(0,0)]]), {'linestyle': 'dashed'})
Bezier path from (0.0, 0.0) to (0.0, 0.0)
```

We use `bezier_path()` to actually plot Bezier curves:

```python
sage: bezier_path([[0,0], (.5,.5),(1,0)],[(.5,1),(0,0)]]), linestyle="dashed")
Graphics object consisting of 1 graphics primitive
```

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:
```
sage: b = bezier_path([[0,0],(.5,.5),(1,0),(.5,1),(0,0)])
sage: d = b.get_minmax_data()
sage: d['xmin']
0.0
sage: d['xmax']
1.0
```

**plot3d(z=0, **kwds)**

Returns a 3D plot (Jmol) of the Bezier path. Since a BezierPath primitive contains only $x, y$ coordinates, the path will be drawn in some plane (default is $z = 0$). To create a Bezier path with nonzero (and nonidentical) $z$ coordinates in the path and control points, use the function `bezier3d()` instead of `bezier_path()`.

EXAMPLES:

```
sage: b = bezier_path([[0,0],(0,1),(1,0)])
sage: A = b.plot3d()  # needs sage.symbolic
sage: B = b.plot3d(z=2)  # needs sage.symbolic
sage: A + B  # needs sage.symbolic
Graphics3d Object
```
Returns a Graphics object of a Bezier path corresponding to the path parameter. The path is a list of curves, and each curve is a list of points. Each point is a tuple \((x, y)\).

The first curve contains the endpoints as the first and last point in the list. All other curves assume a starting point given by the last entry in the preceding list, and take the last point in the list as their opposite endpoint. A curve can have 0, 1 or 2 control points listed between the endpoints. In the input example for path below, the first and second curves have 2 control points, the third has one, and the fourth has no control points:

```python
path = [[p1, c1, c2, p2], [c3, c4, p3], [c5, p4], [p5], ...]
```

In the case of no control points, a straight line will be drawn between the two endpoints. If one control point is supplied, then the curve at each of the endpoints will be tangent to the line from that endpoint to the control point. Similarly, in the case of two control points, at each endpoint the curve will be tangent to the line connecting that endpoint with the control point immediately after or immediately preceding it in the list.

So in our example above, the curve between p1 and p2 is tangent to the line through p1 and c1 at p1, and tangent to the line through p2 and c2 at p2. Similarly, the curve between p2 and p3 is tangent to line(p2,c3) at p2 and tangent to line(p3,c4) at p3. Curve(p3,p4) is tangent to line(p3,c5) at p3 and tangent to line(p4,c5) at p4. Curve(p4,p5) is a straight line.
4.3. Bezier paths
INPUT:

- **path** – a list of lists of tuples (see above)
- **alpha** – default: 1
- **fill** – default: False
- **thickness** – default: 1
- **linestyle** – default: 'solid', The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '.', respectively.
- **rgbcolor** – default: (0,0,0)
- **zorder** – the layer in which to draw

EXAMPLES:

```python
sage: path = [[[0,0], (.5,.1), (.75,3), (1,0)],[(.5,1),(.5,0)],[(.2,.5)]]
sage: b = bezier_path(path, linestyle='dashed', rgbcolor='green')
sage: b
Graphics object consisting of 1 graphics primitive
```

To construct a simple curve, create a list containing a single list:
sage: path = [[(0,0),(.5,1),(1,0)]]
sage: curve = bezier_path(path, linestyle='dashed', rgbcolor='green')
sage: curve
Graphics object consisting of 1 graphics primitive

Extra options will get passed on to `show()`, as long as they are valid:

sage: bezier_path([[0,1),(.5,0),(1,1)]), fontsize=50)
Graphics object consisting of 1 graphics primitive
sage: bezier_path([[0,1),(.5,0),(1,1)]).show(fontsize=50) # These are equivalent

4.4 Circles

class sage.plot.circle.Circle(x, y, r, options)

Bases: GraphicPrimitive

Primitive class for the `Circle` graphics type. See `circle?` for information about actually plotting circles.

INPUT:

- `x` - $x$-coordinate of center of Circle
- `y` - $y$-coordinate of center of Circle
• \( r \) – radius of Circle object
• \text{options} – dict of valid plot options to pass to constructor

\textbf{EXAMPLES:}

Note this should normally be used indirectly via \texttt{circle}:

\begin{verbatim}
sage: from sage.plot.circle import Circle
sage: C = Circle(2,3,5,{'zorder':2})
sage: C
Circle defined by (2.0,3.0) with r=5.0
sage: C.options()['zorder']
2
sage: C.r
5.0
\end{verbatim}

\texttt{get\_minmax\_data()}  
Return a dictionary with the bounding box data.

\textbf{EXAMPLES:}

\begin{verbatim}
sage: p = circle((3, 3), 1)
sage: d = p.get_minmax_data()
sage: d['xmin']
2.0
sage: d['ymin']
2.0
\end{verbatim}

\texttt{plot3d(z=0, **kwds)}  
Plots a 2D circle (actually a 50-gon) in 3D, with default height zero.

\textbf{INPUT:}

• \( z \) - optional 3D height above \( xy \)-plane.

\textbf{EXAMPLES:}

\begin{verbatim}
sage: circle((0,0), 1).plot3d()  
Graphics3d Object
This example uses this method implicitly, but does not pass the optional parameter \( z \) to this method:

sage: sum(circle((random(),random()), random()).plot3d(z=random())  
....:      for _ in range(20))
Graphics3d Object
These examples are explicit, and pass \( z \) to this method:

sage: from math import pi
sage: C = circle((2,pi), 2, hue=.8, alpha=.3, fill=True)
sage: c = C[0]
sage: d = c.plot3d(z=2)
sage: d.texture.opacity
0.3
\end{verbatim}
```
sage: C = circle((2,pi), 2, hue=.8, alpha=.3, linestyle='dotted')  
sage: c = C[0]  
sage: d = c.plot3d(z=2)  
sage: d.jmol_repr(d.testing_render_params())[0][-1]  
'color $line_1$ translucent 0.7 $[204,0,255]$'
```

```
sage.plot.circle.circle(center, radius, alpha=1, fill=False, thickness=1, edgecolor='blue', facecolor='blue', linestyle='solid', zorder=5, legend_label=None, legend_color=None, clip=True, aspect_ratio=1.0, **options)
```

Return a circle at a point center = (x, y) (or (x, y, z) and parallel to the xy-plane) with radius = r. Type circle.

OPTIONS:

- **alpha** - default: 1
- **fill** - default: False
- **thickness** - default: 1
- **linestyle** - default: 'solid' (2D plotting only) The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '-.-', ':', '-.', respectively.
- **edgecolor** - default: 'blue' (2D plotting only)
- **facecolor** - default: 'blue' (2D plotting only, useful only if fill=True)
- **rgbcolor** - 2D or 3D plotting. This option overrides edgecolor and facecolor for 2D plotting.
- **legend_label** - the label for this item in the legend
- **legend_color** - the color for the legend label

EXAMPLES:

The default color is blue, the default linestyle is solid, but this is easy to change:

```
sage: c = circle((1,1), 1)  
sage: c  
Graphics object consisting of 1 graphics primitive
```

```
sage: c = circle((1,1), 1, rgbcolor=(1,0,0), linestyle='-.')  
sage: c  
Graphics object consisting of 1 graphics primitive
```

We can also use this command to plot three-dimensional circles parallel to the xy-plane:

```
sage: c = circle((1,1,3), 1, rgbcolor=(1,0,0))  
sage: c  
Graphics3d Object
```

```
sage: type(c)  
<class 'sage.plot.plot3d.base.TransformGroup'>
```

To correct the aspect ratio of certain graphics, it is necessary to show with a figsize of square dimensions:

```
sage: c.show(figsize=[5,5],xmin=-1,xmax=3,ymin=-1,ymax=3)
```

Here we make a more complicated plot, with many circles of different colors:
4.4. Circles
sage: g = Graphics()
sage: step = 6; ocur = 1/5; paths = 16
sage: PI = math.pi  # numerical for speed -- fine for graphics
sage: for r in range(1, paths+1):
    ....:     for x, y in [[(r+ocur)*math.cos(n), (r+ocur)*math.sin(n)]
    ....:         for n in srange(0, 2*PI+PI/step, PI/step)]:
    ....:         g += circle((x, y), ocur, rgbcolor=hue(r/paths))
    ....:     rnext = (r+1)^2
    ....:     ocur = (rnext-r)-ocur
sage: g.show(xmin=-(paths+1)^2, xmax=(paths+1)^2,
    ....:         ymin=-(paths+1)^2, ymax=(paths+1)^2, figsize=[6,6])

Note that the rgbcolor option overrides the other coloring options. This produces red fill in a blue circle:

sage: circle((2,3), 1, fill=True, edgecolor='blue', facecolor='red')
Graphics object consisting of 1 graphics primitive

This produces an all-green filled circle:

sage: circle((2,3), 1, fill=True, edgecolor='blue', rgbcolor='green')
Graphics object consisting of 1 graphics primitive

The option hue overrides all other options, so be careful with its use. This produces a purplish filled circle:
Chapter 4. Basic Shapes
4.4. Circles
sage: circle((2,3), 1, fill=True, edgecolor='blue', rgbcolor='green', hue=.8)
Graphics object consisting of 1 graphics primitive

And circles with legends:

sage: circle((4,5), 1, rgbcolor='yellow', fill=True, legend_label='the sun').show(xmin=0, ymin=0)
sage: circle((4,5), 1, legend_label='the sun', legend_color='yellow').show(xmin=0, ymin=0)

Extra options will get passed on to show(), as long as they are valid:

sage: circle((0,0), 2, figsize=[10,10])  # That circle is huge!
Graphics object consisting of 1 graphics primitive

sage: circle((0,0), 2).show(figsize=[10,10])  # These are equivalent
4.4. Circles
Chapter 4. Basic Shapes
4.5 Disks

class sage.plot.disk.Disk(point, r, angle, options)

Bases: GraphicPrimitive

Primitive class for the Disk graphics type. See disk? for information about actually plotting a disk (the Sage term for a sector or wedge of a circle).

INPUT:

- point - coordinates of center of disk
- r - radius of disk
- angle - beginning and ending angles of disk (i.e. angle extent of sector/wedge)
- options - dict of valid plot options to pass to constructor

EXAMPLES:

Note this should normally be used indirectly via disk:

```
sage: from math import pi
sage: from sage.plot.disk import Disk
sage: D = Disk((1,2), 2, (pi/2,pi), {'zorder':3})
sage: D
Disk defined by (1.0,2.0) with r=2.0
spanning (1.5707963267..., 3.1415926535...) radians
sage: D.options()['zorder']
3
sage: D.x
1.0
```

get_minmax_data()

Returns a dictionary with the bounding box data.

EXAMPLES:

```
sage: from math import pi
sage: D = disk((5,4), 1, (pi/2, pi))
sage: d = D.get_minmax_data()
sage: d['xmin']
4.0
sage: d['ymin']
3.0
sage: d['xmax']
6.0
sage: d['ymax']
5.0
```

plot3d(z=0, **kwds)

Plots a 2D disk (actually a 52-gon) in 3D, with default height zero.

INPUT:

- z - optional 3D height above xy-plane.

AUTHORS:
• Karl-Dieter Crisman (05-09)

EXAMPLES:

```
sage: from math import pi
dsage: disk((0,0), 1, (0, pi/2)).plot3d()
Graphics3d Object
dsage: disk((0,0), 1, (0, pi/2)).plot3d(z=2)
Graphics3d Object
dsage: disk((0,0), 1, (pi/2, 0), fill=False).plot3d(3)
Graphics3d Object
```

These examples show that the appropriate options are passed:

```
sage: D = disk((2,3), 1, (pi/4,pi/3), hue=.8, alpha=.3, fill=True)
sage: d = D[0]
sage: d.plot3d(z=2).texture.opacity
0.3
```

```
sage: D = disk((2,3), 1, (pi/4,pi/3), hue=.8, alpha=.3, fill=False)
sage: d = D[0]
sage: dd = d.plot3d(z=2)
sage: dd.jmol_repr(dd.testing_render_params())[0][-1]
'color $line_4 translucent 0.7 [204,0,255]'
```

sage.plot.disk.disk(point, radius, angle, alpha=1, fill=True, rgbcolor=(0, 0, 1), thickness=0, legend_label=None, aspect_ratio=1.0, **options)

A disk (that is, a sector or wedge of a circle) with center at a point = \((x, y)\) (or \((x, y, z)\) and parallel to the \(xy\)-plane) with radius = \(r\) spanning (in radians) angle = \((\text{rad1}, \text{rad2})\).

Type disk.options to see all options.

EXAMPLES:

Make some dangerous disks:

```
sage: from math import pi
sage: bl = disk((0.0,0.0), 1, (pi, 3*pi/2), color='yellow')
sage: tr = disk((0.0,0.0), 1, (0, pi/2), color='yellow')
sage: tl = disk((0.0,0.0), 1, (pi/2, pi), color='black')
sage: br = disk((0.0,0.0), 1, (3*pi/2, 2*pi), color='black')
sage: P = tl + tr + bl + br
sage: P.show(xmin=-2, xmax=2, ymin=-2, ymax=2)
```

The default aspect ratio is 1.0:

```
sage: disk((0.0,0.0), 1, (pi, 3*pi/2)).aspect_ratio()
1.0
```

Another example of a disk:

```
sage: bl = disk((0.0,0.0), 1, (pi, 3*pi/2), rgbcolor=(1,1,0))
sage: bl.show(figsize=[5,5])
```

Note that since thickness defaults to zero, it is best to change that option when using fill=False:
The previous two examples also illustrate using \texttt{hue} and \texttt{rgbcolor} as ways of specifying the color of the graphic.

We can also use this command to plot three-dimensional disks parallel to the $xy$-plane:

\begin{verbatim}
sage: d = disk((1,1,3), 1, (pi,3*pi/2), rgbcolor=(1,0,0))
sage: d
Graphics3d Object
sage: type(d)
<... 'sage.plot.plot3d.index_face_set.IndexFaceSet'>
\end{verbatim}

Extra options will get passed on to \texttt{show()}, as long as they are valid:

\begin{verbatim}
sage: disk((0, 0), 5, (0, pi/2), rgbcolor=(1, 0, 1),
       xmin=0, xmax=5, ymin=0, ymax=5, figsize=(2,2))
Graphics object consisting of 1 graphics primitive
sage: disk((0, 0), 5, (0, pi/2), rgbcolor=(1, 0, 1)).show(
       # These are equivalent
       xmin=0, xmax=5, ymin=0, ymax=5, figsize=(2,2))
\end{verbatim}
4.6 Ellipses

```python
class sage.plot.ellipse.Ellipse(x, y, r1, r2, angle, options):
    Bases: GraphicPrimitive

    Primitive class for the Ellipse graphics type. See ellipse? for information about actually plotting ellipses.

    INPUT:
    - x, y - coordinates of the center of the ellipse
    - r1, r2 - radii of the ellipse
    - angle - angle
    - options - dictionary of options

    EXAMPLES:

    Note that this construction should be done using ellipse:

    ```
    sage: from math import pi
    sage: from sage.plot.ellipse import Ellipse
    sage: Ellipse(0, 0, 2, 1, pi/4, {})
    ```

    Ellipse centered at (0.0, 0.0) with radii (2.0, 1.0) and angle 0.78539816339...

    get_minmax_data()

    Return a dictionary with the bounding box data.

    The bounding box is computed to be as minimal as possible.

    EXAMPLES:

    An example without an angle:

    ```
    sage: p = ellipse((-2, 3), 1, 2)
    sage: d = p.get_minmax_data()
    sage: d['xmin']
    -3.0
    sage: d['xmax']
    -1.0
    sage: d['ymin']
    1.0
    sage: d['ymax']
    5.0
    ```

    The same example with a rotation of angle π/2:

    ```
    sage: from math import pi
    sage: p = ellipse((-2, 3), 1, 2, pi/2)
    sage: d = p.get_minmax_data()
    sage: d['xmin']
    -4.0
    sage: d['xmax']
    0.0
    sage: d['ymin']
    2.0
    ```

    (continues on next page)
plot3d()

Plotting in 3D is not implemented.

sage.plot.ellipse.ellipse(center, r1, r2, angle=0, alpha=1, fill=False, thickness=1, edgecolor='blue', facecolor='blue', linestyle='solid', zorder=5, aspect_ratio=1.0, legend_label=None, legend_color=None, **options)

Return an ellipse centered at a point center = (x, y) with radii = r1, r2 and angle angle. Type ellipse. options to see all options.

INPUT:
• center - 2-tuple of real numbers - coordinates of the center
• r1, r2 - positive real numbers - the radii of the ellipse
• angle - real number (default: 0) - the angle between the first axis and the horizontal

OPTIONS:
• alpha - default: 1 - transparency
• fill - default: False - whether to fill the ellipse or not
• thickness - default: 1 - thickness of the line
• linestyle - default: 'solid' - The style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '-', '.', respectively.
• edgecolor - default: ‘black’ - color of the contour
• facecolor - default: ‘red’ - color of the filling
• rgbcolor - 2D or 3D plotting. This option overrides edgecolor and facecolor for 2D plotting.
• legend_label - the label for this item in the legend
• legend_color - the color for the legend label

EXAMPLES:
An ellipse centered at (0,0) with major and minor axes of lengths 2 and 1. Note that the default color is blue:

sage: ellipse((0,0),2,1)
Graphics object consisting of 1 graphics primitive

More complicated examples with tilted axes and drawing options:

sage: from math import pi
sage: ellipse((0,0), 3, 1, pi/6, fill=True, alpha=0.3, linestyle="dashed")
Graphics object consisting of 1 graphics primitive

other way to indicate dashed linestyle:

sage: ellipse((0,0),3,1,pi/6,fill=True,alpha=0.3,linestyle="--")
Graphics object consisting of 1 graphics primitive

with colors
4.6. Ellipses
Chapter 4. Basic Shapes
4.6. Ellipses
We see that rgbcolor overrides these other options, as this plot is green:

```
sage: ellipse((0,0),3,1,pi/6,fill=\texttt{True},edgecolor='black',facecolor='red',rgbcolor='green')
```

The default aspect ratio for ellipses is 1.0:

```
sage: ellipse((0,0),2,1).aspect_ratio()
```

One cannot yet plot ellipses in 3D:

```
sage: ellipse((0,0,0),2,1)
Traceback (most recent call last):
...
NotImplementedError: plotting ellipse in 3D is not implemented
```

We can also give ellipses a legend:
4.7 Line plots

```python
sage: from sage.plot.line import Line
sage: Line([[1,2,7], [1,5,-1], {}])
```
Line defined by 3 points

```python
plot3d(z=0, **kwds)
```
Plots a 2D line in 3D, with default height zero.

EXAMPLES:
sage: E = EllipticCurve('37a').plot(thickness=5).plot3d()  # needs sage.schemes
sage: F = EllipticCurve('37a').plot(thickness=5).plot3d(z=2)  # needs sage.schemes
sage: E + F  # long time (5s on sage.math, 2012), needs sage.schemes
Graphics3d Object

sage.plot.line.line(points, **kwds)

Returns either a 2-dimensional or 3-dimensional line depending on value of points.

INPUT:

- points - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either line2d? or line3d?.

EXAMPLES:

sage: line([(0,0), (1,1)])
Graphics object consisting of 1 graphics primitive
sage: line([(0,0,1), (1,1,1)])
Graphics3d Object

Create the line through the given list of points.

**INPUT:**

- `points` - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

Type `line2d.options` for a dictionary of the default options for lines. You can change this to change the defaults for all future lines. Use `line2d.reset()` to reset to the default options.

**INPUT:**

- `alpha` – How transparent the line is
- `thickness` – How thick the line is
- `rgbcolor` – The color as an RGB tuple
- `hue` – The color given as a hue
- `legend_color` – The color of the text in the legend
- `legend_label` – the label for this item in the legend
Any MATPLOTLIB line option may also be passed in. E.g.,

- **linestyle** - (default: “-”) The style of the line, which is one of
  - ":" or "solid"
  - "--" or "dashed"
  - ":." or "dash dot"
  - ":" or "dotted"
  - "None" or " " or "" (nothing)

The linestyle can also be prefixed with a drawing style (e.g., "steps--")
- "default" (connect the points with straight lines)
- "steps" or "steps-pre" (step function; horizontal line is to the left of point)
- "steps-mid" (step function; points are in the middle of horizontal lines)
- "steps-post" (step function; horizontal line is to the right of point)

- **marker** - The style of the markers, which is one of
  - "None" or " " or "" (nothing) – default
  - ",", (pixel), "." (point)
  - "-" (horizontal line), "|" (vertical line)
  - "o" (circle), "p" (pentagon), "s" (square), "x" (x), "+" (plus), "*" (star)
  - "D" (diamond), "d" (thin diamond)
  - "H" (hexagon), "h" (alternative hexagon)
  - "<" (triangle left), ">" (triangle right), ">" (triangle up), "v" (triangle down)
  - "1" (tri down), "2" (tri up), "3" (tri left), "4" (tri right)
  - Ø (tick left), 1 (tick right), 2 (tick up), 3 (tick down)
  - 4 (caret left), 5 (caret right), 6 (caret up), 7 (caret down)
  - "$\ldots\$" (math TeX string)

- **markersize** – the size of the marker in points
- **markeredgecolor** – the color of the marker edge
- **markerfacecolor** – the color of the marker face
- **markeredgewidth** – the size of the marker edge in points

**EXAMPLES:**

A line with no points or one point:

```python
sage: line([])       # returns an empty plot
Graphics object consisting of 0 graphics primitives
```

```python
sage: import numpy; line(numpy.array([])) # needs numpy
Graphics object consisting of 0 graphics primitives
```

```python
sage: line([(1,1)])
Graphics object consisting of 1 graphics primitive
```

A line with numpy arrays:
A line with a legend:

```
sage: line([[0,0],(1,1)], legend_label='line')
```

Graphics object consisting of 1 graphics primitive

Lines with different colors in the legend text:

```
sage: p1 = line([(0,0),(1,1)], legend_label='line')
sage: p2 = line([(1,1),(2,4)], legend_label='squared', legend_color='red')
sage: p1 + p2
```

Graphics object consisting of 2 graphics primitives

Extra options will get passed on to show(), as long as they are valid:

```
sage: line([(0,1), (3,4)], figsize=[10, 2])
```

Graphics object consisting of 1 graphics primitive

```
sage: line([(0,1), (3,4)]).show(figsize=[10, 2]) # These are equivalent
```

We can also use a logarithmic scale if the data will support it:
```python
sage: line([(1,2),(2,4),(3,4),(4,8),(4.5,32)], scale='loglog', base=2)
Graphics object consisting of 1 graphics primitive
```

Many more examples below!

A blue conchoid of Nicomedes:

```python
sage: from math import pi
sage: L = [[1 + 5*cos(pi/2+pi*i/100),
        tan(pi/2+pi*i/100) * (1+5*cos(pi/2+pi*i/100))]
        for i in range(1,100)]
sage: line(L, rgbcolor=(1/4,1/8,3/4))
Graphics object consisting of 1 graphics primitive
```

A line with 2 complex points:

```python
sage: i = CC(0,1)
sage: line([1 + i, 2 + 3*i])
Graphics object consisting of 1 graphics primitive
```

A blue hypotrochoid (3 leaves):

```python
sage: n = 4; h = 3; b = 2
sage: L = [[n*cos(pi*i/100) + h*cos((n/b)*pi*i/100),
        ....]
        for i in range(1,100)]
sage: line(L, rgbcolor=(1/4,1/8,3/4))
Graphics object consisting of 1 graphics primitive
```

(continues on next page)
4.7. Line plots
A blue hypotrochoid (4 leaves):

```python
sage: n = 6; h = 5; b = 2
sage: L = [[n*cos(pi*i/100) + h*cos((n/b)*pi*i/100),
      n*sin(pi*i/100) - h*sin((n/b)*pi*i/100)] for i in range(200)]
sage: line(L, rgbcolor=(1/4,1/4,3/4))
Graphics object consisting of 1 graphics primitive
```

A red limacon of Pascal:

```python
sage: L = [[sin(pi*i/100) + sin(pi*i/50),
       -(1 + cos(pi*i/100) + cos(pi*i/50))] for i in range(-100,101)]
sage: line(L, rgbcolor=(1,1/4,1/2))
Graphics object consisting of 1 graphics primitive
```

A light green trisectrix of Maclaurin:

```python
sage: L = [[2 * (1-4*cos(-pi/2+pi*i/100)^2),
       (1-4*cos(-pi/2+pi*i/100)^2)]]
```

(continues on next page)
Chapter 4. Basic Shapes
\[
10 \cdot \tan(-\pi/2 + \pi \cdot i/100) \cdot (1 - 4 \cdot \cos(-\pi/2 + \pi \cdot i/100)^2)\]
for \(i \in \) \(\range(1,100)\)

\[
\text{sage: line(L, rgbcolor=(1/4,1,1/8))}
\]
Graphics object consisting of 1 graphics primitive

A green lemniscate of Bernoulli:

\[
\text{sage: cosines = [cos(-\pi/2 + \pi \cdot i/100) for i in range(201)]}
\]
\[
\text{sage: v = [(1/c, tan(-\pi/2 + \pi \cdot i/100)) for i,c in enumerate(cosines) if c != 0]}
\]
\[
\text{sage: L = [(a/(a^2+b^2), b/(a^2+b^2)) for a,b in v]}
\]
\[
\text{sage: line(L, rgbcolor=(1/4,3/4,1/8))}
\]
Graphics object consisting of 1 graphics primitive

A red plot of the Jacobi elliptic function \(\text{sn}(x, 2), -3 < x < 3\):

\[
\text{sage: L = [(i/100.0, real_part(jacobi('sn', i/100.0, 2.0))) for i in range(-300, 300, 30)]}
\]
\[
\text{sage: line(L, rgbcolor=(3/4, 1/4, 1/8))}
\]
Graphics object consisting of 1 graphics primitive

A red plot of \(J\)-Bessel function \(J_2(x), 0 < x < 10\):
Chapter 4. Basic Shapes
4.7. Line plots
A purple plot of the Riemann zeta function $\zeta(1/2 + it)$, $0 < t < 30$:

```
sage: i = CDF.gen()
sage: v = [zeta(0.5 + n/10 * i) for n in range(300)]
sage: L = [(z.real(), z.imag()) for z in v]
sage: line(L, rgbcolor=(3/4, 1/2, 5/8))
Graphics object consisting of 1 graphics primitive
```

A purple plot of the Hasse-Weil $L$-function $L(E, 1 + it)$, $-1 < t < 10$:

```
sage: # needs sage.schemes
sage: E = EllipticCurve('37a')
sage: vals = E.lseries().values_along_line(1-I, 1+10*I, 100) # critical line
sage: L = [(z[1].real(), z[1].imag()) for z in vals]
sage: line(L, rgbcolor=(3/4, 1/2, 5/8))
Graphics object consisting of 1 graphics primitive
```

A red, blue, and green “cool cat”:...
4.7. Line plots
```python
sage: G = plot(-cos(x), -2, 2, thickness=5, rgbcolor=(0.5,1,0.5))
sage: P = polygon([[1,2], [5,6], [5,0]], rgbcolor=(1,0,0))
sage: Q = polygon([(-x,y) for x,y in P[0]], rgbcolor=(0,0,1))
sage: G + P + Q  # show the plot
```

Graphics object consisting of 3 graphics primitives

4.8 Points

class sage.plot.point.Point(xdata, ydata, options)

Bases: GraphicPrimitive_xydata

Primitive class for the point graphics type. See point?, point2d? or point3d? for information about actually plotting points.

INPUT:

- `xdata` – list of x values for points in Point object
- `ydata` – list of y values for points in Point object
- `options` – dict of valid plot options to pass to constructor

EXAMPLES:
Note this should normally be used indirectly via `point()` and friends:

```python
sage: from sage.plot.point import Point
sage: P = Point([1,2],[2,3],{'alpha':.5})
sage: P
Point set defined by 2 point(s)
sage: P.options()['alpha']
0.5
sage: P.xdata
[1, 2]
```

**plot3d(z=0, **kwds)**

Plots a two-dimensional point in 3-D, with default height zero.

**INPUT:**

- `z` - optional 3D height above $xy$-plane. May be a list if self is a list of points.

**EXAMPLES:**

One point:

```python
sage: A = point((1, 1))
sage: a = A[0]; a
Point set defined by 1 point(s)
sage: b = a.plot3d()
```

![Plot 3D Point](image-url)
One point with a height:

```sage
sage: A = point((1, 1))
sage: a = A[0]; a
Point set defined by 1 point(s)
sage: b = a.plot3d(z=3)
sage: b.loc[2]
3.0
```

Multiple points:

```sage
sage: P = point([(0, 0), (1, 1)])
sage: p = P[0]; p
Point set defined by 2 point(s)
sage: q = p.plot3d(size=22)
```

Multiple points with different heights:

```sage
sage: P = point([(0, 0), (1, 1)])
sage: p = P[0]
sage: q = p.plot3d(z=[2, 3])
sage: q.all[0].loc[2]
2.0
```
Chapter 4. Basic Shapes
sage: q.all[1].loc[2]
3.0

Note that keywords passed must be valid point3d options:

```python
sage: A = point((1, 1), size=22)
sage: a = A[0]; a
Point set defined by 1 point(s)
sage: b = a.plot3d()
sage: b.size
22
sage: b = a.plot3d(pointsize=23)  # only 2D valid option
sage: b.size
22
sage: b = a.plot3d(size=23)  # correct keyword
sage: b.size
23
```

`sage.plot.point.point(points, **kwds)`

Return either a 2-dimensional or 3-dimensional point or sum of points.

**INPUT:**
• **points** - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either point2d? or point3d?.

**See also:**

`sage.plot.point.point2d()`, `sage.plot.plot3d.shapes2.point3d()`

**EXAMPLES:**

```
sage: point((1, 2))
Graphics object consisting of 1 graphics primitive
```

```
sage: point((1, 2, 3))
Graphics3d Object
```

```
sage: point([(0, 0), (1, 1)])
Graphics object consisting of 1 graphics primitive
```

```
sage: point([(0, 0, 1), (1, 1, 1)])
Graphics3d Object
```

Extra options will get passed on to show(), as long as they are valid:
4.8. Points
A point of size size defined by point $(x, y)$.

**INPUT:**

- **points** – either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers
- **alpha** – how transparent the point is
- **faceted** – if True, color the edge of the point (only for 2D plots)
- **hue** – the color given as a hue
- **legend_color** – the color of the legend text
- **legend_label** – the label for this item in the legend
- **marker** – the marker symbol for 2D plots only (see documentation of plot() for details)
- **markeredgecolor** – the color of the marker edge (only for 2D plots)
- **rgbcolor** – the color as an RGB tuple
- **size** – how big the point is (i.e., area in points^2=(1/72 inch)^2)
- **zorder** – the layer level in which to draw

**EXAMPLES:**

A purple point from a single tuple of coordinates:

```
sage: point((0.5, 0.5), rgbcolor=hue(0.75))
```

Points with customized markers and edge colors:

```
sage: r = [(random(), random()) for _ in range(10)]
sage: point(r, marker='d', markeredgecolor='red', size=20)
```

Passing an empty list returns an empty plot:

```
sage: point([])
sage: import numpy; point(numpy.array([]))
```

If you need a 2D point to live in 3-space later, this is possible:
4.8. Points
```sage
A = point((1, 1))
a = A[0]; a
Point set defined by 1 point(s)
b = a.plot3d(z=3)
```

This is also true with multiple points:

```sage
P = point(((0, 0), (1, 1)))
p = P[0]
q = p.plot3d(z=[2, 3])
```

Here are some random larger red points, given as a list of tuples:

```sage
point(((0.5, 0.5), (1, 2), (0.5, 0.9), (-1, -1)), rgbcolor=hue(1), size=30)
```

And an example with a legend:

```sage
point((0, 0), rgbcolor='black', pointsize=40, legend_label='origin')
```

The legend can be colored:
Chapter 4. Basic Shapes
sage: P = points([(0, 0), (1, 0)], pointsize=40, 
....:    legend_label='origin', legend_color='red')
sage: P + plot(x^2, (x, 0, 1), legend_label='plot', legend_color='green')
#needs sage.symbolic
Graphics object consisting of 2 graphics primitives

Extra options will get passed on to show(), as long as they are valid:

sage: point([[cos(theta), sin(theta))
....:    for theta in srange(0, 2*pi, pi/8)], frame=True)
Graphics object consisting of 1 graphics primitive
sage: point([[cos(theta), sin(theta))
....:    for theta in srange(0, 2*pi, pi/8)]).show(frame=True)

For plotting data, we can use a logarithmic scale, as long as we are sure not to include any nonpositive points in the logarithmic direction:

sage: point([[1, 2),(2, 4),(3, 4),(4, 8),(4.5, 32]], scale='semilogy', base=2)
Graphics object consisting of 1 graphics primitive

Since Sage Version 4.4 (github issue #8599), the size of a 2d point can be given by the argument size instead of pointsize. The argument pointsize is still supported:
4.8. Points

Points 451
We can plot a single complex number:

```
sage: point(1 + I, pointsize=100)  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

```
sage: point(sqrt(2) + I, pointsize=100)  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

We can also plot a list of complex numbers:

```
sage: point([I, 1 + I, 2 + 2*I], pointsize=100)  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

```
sage.plot.point.points(points, **kwds)
Return either a 2-dimensional or 3-dimensional point or sum of points.
```
4.8. Points
INPUT:

- **points** - either a single point (as a tuple), a list of points, a single complex number, or a list of complex numbers.

For information regarding additional arguments, see either `point2d?` or `point3d?`.

See also:

`sage.plot.point.point2d()`, `sage.plot.plot3d.shapes2.point3d()`

EXAMPLES:

```python
sage: point((1, 2))
Graphics object consisting of 1 graphics primitive
```

![Graphical representation of point((1, 2))](image)

```python
sage: point((1, 2, 3))
Graphics3d Object
```

```python
sage: point([(0, 0), (1, 1)])
Graphics object consisting of 1 graphics primitive
```

```python
sage: point([(0, 0, 1), (1, 1, 1)])
Graphics3d Object
```

Extra options will get passed on to `show()`, as long as they are valid:
4.8. Points
2D Graphics, Release 10.1

```python
sage: point([(cos(theta), sin(theta))])  # needs sage.symbolic
˓→
# These are equivalent
˓→

for theta in srange(0, 2*pi, pi/8)).show(frame=True)

Graphics object consisting of 1 graphics primitive

sage: point([(cos(theta), sin(theta))])
˓→
# These are equivalent
˓→

for theta in srange(0, 2*pi, pi/8)).show(frame=True)
```

4.9 Polygons

```python
class sage.plot.polygon.Polygon(xdata, ydata, options)

Bases: GraphicPrimitive_xydata

Primitive class for the Polygon graphics type. For information on actual plotting, please see polygon(), polygon2d(), or polygon3d().

INPUT:

- xdata – list of x-coordinates of points defining Polygon
- ydata – list of y-coordinates of points defining Polygon
- options – dict of valid plot options to pass to constructor

EXAMPLES:

Note this should normally be used indirectly via polygon():

```python
sage: from sage.plot.polygon import Polygon
sage: P = Polygon([1,2,3],[2,3,2],{'alpha':.5})
sage: P
Polygon defined by 3 points
sage: P.options()['alpha']
0.500000000000000
sage: P.ydata
[2, 3, 2]
```

plot3d(z=0, **kwds)

Plots a 2D polygon in 3D, with default height zero.

INPUT:

- z - optional 3D height above xy-plane, or a list of heights corresponding to the list of 2D polygon points.

EXAMPLES:

A pentagon:

```python
sage: polygon([(cos(t), sin(t))])  # needs sage.symbolic
˓→
˓→

for t in srange(0, 2*pi, 2*pi/5)).plot3d()

Graphics3d Object
```

Showing behavior of the optional parameter z:
```python
sage: P = polygon([(0,0), (1,2), (0,1), (-1,2)])
```
```
sage: p = P[0]; p
Polygon defined by 4 points
```
```
sage: q = p.plot3d()
```
```
sage: q.obj_repr(q.testing_render_params())
['v 0 0 0', 'v 1 2 0', 'v 0 1 0', 'v -1 2 0']
```
```
sage: r = p.plot3d(z=3)
```
```
sage: r.obj_repr(r.testing_render_params())
['v 0 0 3', 'v 1 2 3', 'v 0 1 3', 'v -1 2 3']
```
```
sage: s = p.plot3d(z=[0,1,2,3])
```
```
sage: s.obj_repr(s.testing_render_params())
['v 0 0 0', 'v 1 2 1', 'v 0 1 2', 'v -1 2 3']
```

```python
sage.plot.polygon.polygon(points, **options)
```

Return either a 2-dimensional or 3-dimensional polygon depending on value of points.

For information regarding additional arguments, see either `polygon2d()` or `polygon3d()`. Options may be found and set using the dictionaries `polygon2d.options` and `polygon3d.options`.

**EXAMPLES:**

```python
sage: polygon([(0,0), (1,1), (0,1)])
Graphics object consisting of 1 graphics primitive
```

4.9. Polygons
sage: polygon([(0,0,1), (1,1,1), (2,0,1)])
Graphics3d Object

Extra options will get passed on to show(), as long as they are valid:

sage: polygon([(0,0), (1,1), (0,1)], axes=False)
sage: polygon([(0,0), (1,1), (0,1)]).show(axes=False)  # These are equivalent

sage.plot.polygon.polygon2d(points, alpha=1, rgbcolor=(0, 0, 1), edgecolor=None, thickness=None, legend_label=None, legend_color=None, aspect_ratio=1.0, fill=True, **options)

Return a 2-dimensional polygon defined by points.

Type polygon2d.options for a dictionary of the default options for polygons. You can change this to change the defaults for all future polygons. Use polygon2d.reset() to reset to the default options.

EXAMPLES:

We create a purple-ish polygon:

sage: polygon2d([[1,2], [5,6], [5,0]], rgbcolor=(1,0,1))
Graphics object consisting of 1 graphics primitive

By default, polygons are filled in, but we can make them without a fill as well:
In either case, the thickness of the border can be controlled:

```python
sage: polygon2d([[1,2], [5,6], [5,0]], fill=False, thickness=4, color='orange')
```

For filled polygons, one can use different colors for the border and the interior as follows:

```python
sage: L = [[0,0]]+[[i/100, 1.1+cos(i/20)] for i in range(100)]+[[1,0]]
# needs sage.symbolic
sage: polygon2d(L, color="limegreen", edgecolor="black", axes=False)  
```

Some modern art – a random polygon, with legend:

```python
sage: v = [(randrange(-5,5), randrange(-5,5)) for _ in range(10)]
sage: polygon2d(v, legend_label='some form')
```

An aperiodic monotile, [Smi2023]:

---

4.9. Polygons
Chapter 4. Basic Shapes
A purple hexagon:

```
sage: L = [[cos(pi*i/3),sin(pi*i/3)] for i in range(6)]  # needs sage.symbolic
sage: polygon2d(L, rgbcolor=(1,0,1))  # needs sage.symbolic
```

Graphics object consisting of 1 graphics primitive

A green deltoid:

```
sage: L = [[-1+cos(pi*i/100)*(1+cos(pi*i/100)),
        2*sin(pi*i/100)*(1-cos(pi*i/100))] for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,3/4,1/2))  # needs sage.symbolic
```

(continues on next page)
Chapter 4. Basic Shapes
A blue hypotrochoid:

```
sage: L = [[6*cos(pi*i/100)+5*cos((6/2)*pi*i/100),
        # needs sage.symbolic
        6*sin(pi*i/100)-5*sin((6/2)*pi*i/100)]
      for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,1/4,1/2))
```

Graphics object consisting of 1 graphics primitive

Another one:

```
sage: n = 4; h = 5; b = 2
sage: L = [[n*cos(pi*i/100)+h*cos((n/b)*pi*i/100),
        # needs sage.symbolic
        n*sin(pi*i/100)-h*sin((n/b)*pi*i/100)]
      for i in range(200)]
sage: polygon2d(L, rgbcolor=(1/8,1/4,3/4))
```

Graphics object consisting of 1 graphics primitive

A purple epicycloid:
4.9. Polygons
sage: m = 9; b = 1
sage: L = [[m*cos(pi*i/100)+b*cos((m/b)*pi*i/100), #
       m*sin(pi*i/100)-b*sin((m/b)*pi*i/100)] for i in range(200)]

sage: polygon2d(L, rgbcolor=(7/8,1/4,3/4)) #

Graphics object consisting of 1 graphics primitive

A brown astroid:

sage: L = [[cos(pi*i/100)^3, sin(pi*i/100)^3] for i in range(200)] #

sage: polygon2d(L, rgbcolor=(3/4,1/4,1/4)) #

Graphics object consisting of 1 graphics primitive

And, my favorite, a greenish blob:

sage: L = [[cos(pi*i/100)*(1+cos(pi*i/50)), #
        sin(pi*i/100)*(1+sin(pi*i/50))]
    for i in range(200)]

sage: polygon2d(L, rgbcolor=(1/8,3/4,1/2)) #

(Graphics object consisting of 1 graphics primitive)

(continues on next page)
4.9. Polygons
This one is for my wife:

```sage
def L = [[sin(pi*i/100)+sin(pi*i/50),
         -(1+cos(pi*i/100)+cos(pi*i/50))] for i in range(-100,100)]
sage: polygon2d(L, rgbcolor=(1,1/4,1/2))  # needs sage.symbolic
```

One can do the same one with a colored legend label:

```sage:
sage: polygon2d(L, color='red', legend_label='For you!', legend_color='red')  # needs sage.symbolic
```

Polygons have a default aspect ratio of 1.0:

```sage:
sage: polygon2d([[1,2], [5,6], [5,0]]).aspect_ratio()  # needs sage.symbolic
```

AUTHORS:
4.9. Polygons
For you!
Three models of the hyperbolic plane are implemented: Upper Half Plane, Poincaré Disk, and Klein Disk, each with its different domain and metric tensor.

**Upper half plane (UHP)**

In this model, hyperbolic points are described by two coordinates, which we will represent by a complex number in the domain

\[ H = \{ z \in \mathbb{C} \mid \Im(z) > 0 \} \]

with the corresponding metric tensor

\[ ds^2 = \frac{dzd\bar{z}}{\Im(z)^2}. \]

**Poincaré disk (PD)**

In this model, hyperbolic points are described by two coordinates, which we will represent by a complex number within the unit circle, having therefore the following domain

\[ D = \{ z \in \mathbb{C} \mid |z| < 1 \} \]

with the corresponding metric tensor

\[ ds^2 = 4 \frac{dzd\bar{z}}{(1 - |z|^2)^2}. \]

**Klein disk (KM)**

In this model, the domain is again complex numbers within the unit circle as in the Poincaré disk model, but the corresponding metric tensor is different:

\[ ds^2 = \frac{dzd\bar{z}}{1 - |z|^2} + \frac{(z \cdot dz)^2}{(1 - |z|^2)^2}. \]

See also:

`sage.geometry.hyperbolic_space.hyperbolic_geodesic`

REFERENCES:

For additional models of the hyperbolic plane and its relationship see [CFKP1997]. For a more detailed explanation on hyperbolic arcs see [Sta1993].
**class sage.plot.hyperbolic_arc.HyperbolicArc(A, B, model, options)**

Bases: `HyperbolicArcCore`

Primitive class for hyperbolic arc type.

See `hyperbolic_arc?` for information about plotting a hyperbolic arc in the complex plane.

**INPUT:**

- `A`, `B` – end points of the hyperbolic arc
- `model` – the hyperbolic model used, which is one of the following:
  - `'UHP'` - upper half plane
  - `'PD'` - Poincaré disk
  - `'KM'` - Klein disk

**class sage.plot.hyperbolic_arc.HyperbolicArcCore(path, options)**

Bases: `BezierPath`

Base class for Hyperbolic arcs and hyperbolic polygons in the hyperbolic plane.

The Upper Half Model, Poincaré Disk Model, and Klein Disk model are supported.

**sage.plot.hyperbolic_arc.hyperbolic_arc(a, b, model='UHP', alpha=1, fill=False, thickness=1, rgbcolor='blue', zorder=2, linestyle='solid', **options)**

Plot an arc from `a` to `b` in hyperbolic plane.

**INPUT:**

- `a`, `b` - complex numbers connected by a hyperbolic arc
- `model` – (default: `'UHP'`) hyperbolic model used, which is one of the following:
  - `'UHP'` - upper half plane
  - `'PD'` - Poincaré disk
  - `'KM'` - Klein disk
  - `'HM'` - hyperboloid model

**OPTIONS:**

- `alpha` – default: 1
- `thickness` – default: 1
- `rgbcolor` – default: `blue`
- `linestyle` – (default: 'solid') the style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '---', ':', '-', '.', respectively

**EXAMPLES:**

Show a hyperbolic arc from 0 to 1:

```
sage: hyperbolic_arc(0, 1)
Graphics object consisting of 1 graphics primitive
```

Show a hyperbolic arc from 1/2 to `i` with a red thick line:

```
sage: hyperbolic_arc(0.5, 1, color='red', thickness=2)
Graphics object consisting of 1 graphics primitive
```
4.10. Arcs in hyperbolic geometry
Chapter 4. Basic Shapes
Show a hyperbolic arc from $1 + i$ to $1 + 2i$ with dashed line:

```
sage: hyperbolic_arc(1+I, 1+2*I, linestyle='dashed', color='green')
```

Graphics object consisting of 1 graphics primitive

![Graph showing a hyperbolic arc from $1 + i$ to $1 + 2i$ with dashed line.]

Show a hyperbolic arc from $a$ to infinity:

```
sage: hyperbolic_arc(1 + I, infinity, color='brown')
```

Graphics object consisting of 1 graphics primitive

![Graph showing a hyperbolic arc from $1 + i$ to infinity.]

We can also plot hyperbolic arcs in other models.

We show a hyperbolic arc from $i$ to $-1$ in red, another hyperbolic arc from $e^{i\pi/3}$ to $0.6 \cdot e^{i3\pi/4}$ with dashed style in green, and finally a hyperbolic arc from $-0.5 + 0.5i$ to $0.5 - 0.5i$ together with the disk frontier in the Poincaré disk model:

```
sage: z1 = CC(0,1)
sage: z2 = CC(-1,0)
sage: z3 = CC((cos(pi/3),sin(pi/3)))
sage: z4 = CC((0.6*cos(3*pi/4),0.6*sin(3*pi/4)))
sage: z5 = CC(-0.5,0.5)
```

(continues on next page)
4.10. Arcs in hyperbolic geometry
sage: z6 = CC(0.5, -0.5)
sage: a1 = hyperbolic_arc(z1, z2, model="PD", color="red")

We show the arcs defined by the same endpoints in the Klein disk model (note that these are *not* the image of those arcs when changing between the models):

sage: a1 = hyperbolic_arc(z1, z2, model="KM", color="red")

Show a hyperbolic arc from \((1, 2, \sqrt{6})\) to \((-2, -3, \sqrt{14})\) in the hyperboloid model:

sage: a = (1,2,sqrt(6))
sage: b = (-2,-3,sqrt(14))

sage: hyperbolic_arc(a, b, model="HM")
Graphics3d Object
4.11 Polygons and triangles in hyperbolic geometry

AUTHORS:

• Hartmut Monien (2011-08)
• Vincent Delecroix (2014-11)

class sage.plot.hyperbolic_polygon.HyperbolicPolygon(pts, model, options)

Bases: HyperbolicArcCore

Primitive class for hyperbolic polygon type.

See hyperbolic_polygon? for information about plotting a hyperbolic polygon in the complex plane.

INPUT:

• pts – coordinates of the polygon (as complex numbers)
• options – dict of valid plot options to pass to constructor

EXAMPLES:

Note that constructions should use hyperbolic_polygon() or hyperbolic_triangle():

```
sage: from sage.plot.hyperbolic_polygon import HyperbolicPolygon
sage: print(HyperbolicPolygon([0, 1/2, I], "UHP", {}))
Hyperbolic polygon (0.000000000000000, 0.500000000000000, 1.000000000000000*I)
```

Return a hyperbolic polygon in the hyperbolic plane with vertices pts.

Type ?hyperbolic_polygon to see all options.

INPUT:

• pts – a list or tuple of complex numbers

OPTIONS:

• model – default: UHP Model used for hyperbolic plane
• alpha – default: 1
• fill – default: False
• thickness – default: 1
• rgbcolor – default: 'blue'
• linestyle – (default: 'solid') the style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '---', ':' , '-', '--', respectively

EXAMPLES:

Show a hyperbolic polygon with coordinates $-1, 3i, 2+2i, 1+i$:

```
sage: hyperbolic_polygon([-1,3*I,2+2*I,1+I])
Graphics object consisting of 1 graphics primitive
```

With more options:
2D Graphics, Release 10.1

```
sage: hyperbolic_polygon([-1,3*I,2+2*I,1+I], fill=True, color='red')
Graphics object consisting of 1 graphics primitive
```

With a vertex at $\infty$:
```
sage: hyperbolic_polygon([-1,0,1,Infinity], color='green')
Graphics object consisting of 1 graphics primitive
```

Poincare disc model is supported via the parameter `model`. Show a hyperbolic polygon in the Poincare disc model with coordinates $1, i, -1, -i$:
```
sage: hyperbolic_polygon([1,I,-1,-I], model="PD", color='green')
Graphics object consisting of 2 graphics primitives
```

With more options:
```
sage: hyperbolic_polygon([1,I,-1,-I], model="PD", color='green', fill=True, linestyle="-" )
Graphics object consisting of 2 graphics primitives
```

Klein model is also supported via the parameter `model`. Show a hyperbolic polygon in the Klein model with coordinates $1, e^{i\pi/3}, e^{2i\pi/3}, -1, e^{4i\pi/3}, e^{5i\pi/3}$:

4.11. Polygons and triangles in hyperbolic geometry
4.11. Polygons and triangles in hyperbolic geometry
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Hyperbolic model is supported partially, via the parameter model. Show a hyperbolic polygon in the hyperboloid model with coordinates $(3, 3, \sqrt{19}), (3, -3, \sqrt{19}), (-3, -3, \sqrt{19}), (-3, 3, \sqrt{19})$:

```
sage: pts = [(3,3,sqrt(19)),(3,-3,sqrt(19)),(-3,-3,sqrt(19)),(-3,3,sqrt(19))]
sage: hyperbolic_polygon(pts, model="HM")
```

Graphics3d Object

Filling a hyperbolic_polygon in hyperboloid model is possible although jaggy. We show a filled hyperbolic polygon in the hyperboloid model with coordinates $(1, 1, \sqrt{3}), (0, 2, \sqrt{5}), (2, 0, \sqrt{5})$. (The doctest is done at lower resolution than the picture below to give a faster result.)

```
sage: pts = [(1,1,sqrt(3)), (0,2,sqrt(5)), (2,0,sqrt(5))]
sage: hyperbolic_polygon(pts, model="HM", resolution=50, color='yellow', fill=True)
```

(continues on next page)
sage.plot.hyperbolic_polygon.hyperbolic_triangle(a, b, c, model='UHP', **options)

Return a hyperbolic triangle in the hyperbolic plane with vertices (a, b, c).

Type ?hyperbolic_polygon to see all options.

INPUT:
• a, b, c – complex numbers in the upper half complex plane

OPTIONS:
• alpha – default: 1
• fill – default: False
• thickness – default: 1
• rgbcolor – default: 'blue'
• linestyle – (default: 'solid') the style of the line, which is one of 'dashed', 'dotted', 'solid', 'dashdot', or '--', ':', '-', '.', respectively.

EXAMPLES:
Show a hyperbolic triangle with coordinates 0, 1/2 + i√3/2 and -1/2 + i√3/2:
A hyperbolic triangle with coordinates 0, 1 and 2 + i and a dashed line:

```
sage: hyperbolic_triangle(0, 1, 2+i, fill=true, rgbcolor='red', linestyle='--')
```
Graphics object consisting of 1 graphics primitive

A hyperbolic triangle with a vertex at $\infty$:

```
sage: hyperbolic_triangle(-5,Infinity,5)
```
Graphics object consisting of 1 graphics primitive

It can also plot a hyperbolic triangle in the Poincaré disk model:

```
sage: z1 = CC((cos(pi/3),sin(pi/3)))
sage: z2 = CC((0.6*cos(3*pi/4),0.6*sin(3*pi/4)))
sage: z3 = 1
sage: hyperbolic_triangle(z1, z2, z3, model="PD", color="red")
```
Graphics object consisting of 2 graphics primitives

```
sage: hyperbolic_triangle(0.3+0.3*i, 0.8*i, -0.5-0.5*i, model="PD", color='magenta')
```
Graphics object consisting of 2 graphics primitives
4.11. Polygons and triangles in hyperbolic geometry
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4.11. Polygons and triangles in hyperbolic geometry
4.12 Regular polygons in the upper half model for hyperbolic plane

AUTHORS:

- Javier Honrubia (2016-01)

class sage.plot.hyperbolic_regular_polygon.HyperbolicRegularPolygon(sides, i_angle, center, options)

Bases: HyperbolicPolygon

Primitive class for regular hyperbolic polygon type.

See hyperbolic_regular_polygon? for information about plotting a hyperbolic regular polygon in the upper complex halfplane.

INPUT:

- sides – number of sides of the polygon
- i_angle – interior angle of the polygon
- center – center point as a complex number of the polygon

EXAMPLES:

Note that constructions should use hyperbolic_regular_polygon():

```sage
from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
print(HyperbolicRegularPolygon(5, pi/2, I, {}))
```

Hyperbolic regular polygon (sides=5, i_angle=1/2*pi, center=1.00000000000000*I)

The code verifies if there exists a compact hyperbolic regular polygon with the given data, checking

\[ A(P) = \pi(s - 2) - s \cdot \alpha > 0, \]

where \( s \) is \texttt{sides} and \( \alpha \) is \texttt{i_angle}``. This raises an error if the \``i_angle\ is less than the minimum to generate a compact polygon:

```sage
from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
P = HyperbolicRegularPolygon(4, pi/2, I, {})
```

Traceback (most recent call last):
...
ValueError: there exists no hyperbolic regular compact polygon, for sides=4 the interior angle must be less than 1/2*pi

It is an error to give a center outside the upper half plane in this model

```sage
from sage.plot.hyperbolic_regular_polygon import HyperbolicRegularPolygon
P = HyperbolicRegularPolygon(4, pi/4, I, {})
```

Traceback (most recent call last):
...
ValueError: center: 1.00000000000000 - 1.00000000000000*I is not a valid point in the upper half plane model of the hyperbolic plane

sage.plot.hyperbolic_regular_polygon.hyperbolic_regular_polygon(sides, i_angle, center=1.00000000000000*I, alpha=1, fill=False, thickness=1, rgbcolor='blue', zorder=2, linestyle='solid', **options)
Return a hyperbolic regular polygon in the upper half model of Hyperbolic plane given the number of sides, interior angle and possibly a center.

Type `?hyperbolic_regular_polygon` to see all options.

**INPUT:**
- `sides` – number of sides of the polygon
- `i_angle` – interior angle of the polygon
- `center` – (default: `i`) hyperbolic center point (complex number) of the polygon

**OPTIONS:**
- `alpha` – default: 1
- `fill` – default: `False`
- `thickness` – default: 1
- `rgbcolor` – default: `'blue'`
- `linestyle` – (default: `'solid'`) the style of the line, which can be one of the following:
  - `'dashed'` or `'-.'`
  - `'dotted'` or `':'
  - `'solid'` or `'-'
  - `'dashdot'` or `'-.'`

**EXAMPLES:**
Show a hyperbolic regular polygon with 6 sides and square angles:

```sage
g = hyperbolic_regular_polygon(6, pi/2)
g.plot()
```

Graphics object consisting of 1 graphics primitive

With more options:

```sage
g = hyperbolic_regular_polygon(6, pi/2, center=3+2*I, fill=True, color='red')
g.plot()
```

Graphics object consisting of 1 graphics primitive

The code verifies is there exists a hyperbolic regular polygon with the given data, checking

\[
A(P) = \pi(s - 2) - s \cdot \alpha > 0,
\]

where `s` is `sides` and `\alpha` is `i_angle`. This raises an error if the `\`i_angle` is less than the minimum to generate a compact polygon:

```sage
hyperbolic_regular_polygon(4, pi/2)
```

Traceback (most recent call last):

```
ValueError: there exists no hyperbolic regular compact polygon, for sides=4 the interior angle must be less than 1/2*pi
```

It is an error to give a center outside the upper half plane in this model:
4.12. Regular polygons in the upper half model for hyperbolic plane
Chapter 4. Basic Shapes
sage: from sage.plot.hyperbolic_regular_polygon import hyperbolic_regular_polygon
sage: hyperbolic_regular_polygon(4, pi/4, 1-I)
Traceback (most recent call last):
...
ValueError: center: 1.00000000000000 - 1.00000000000000*I is not
a valid point in the upper half plane model of the hyperbolic plane
5.1 Graphics objects

This file contains the definition of the class \texttt{Graphics}. Usually, you don’t call the constructor of this class directly (although you can do it), you would use \texttt{plot()} instead.

**AUTHORS:**

- Jeroen Demeyer (2012-04-19): split off this file from plot.py (github issue \#12857)
- Punarbasu Purkayastha (2012-05-20): Add logarithmic scale (github issue \#4529)
- Emily Chen (2013-01-05): Add documentation for \texttt{show()} figsize parameter (github issue \#5956)
- Eric Gourgoulhon (2015-03-19): Add parameter axes_labels_size (github issue \#18004)

**class** \texttt{sage.plot.graphics.Graphics}

**Bases:** \texttt{WithEqualityById, SageObject}

The \texttt{Graphics} object is an empty list of graphics objects. It is useful to use this object when initializing a for loop where different graphics object will be added to the empty object.

**EXAMPLES:**

```python
sage: G = Graphics(); print(G)
Graphics object consisting of 0 graphics primitives
sage: c = circle((1,1), 1)
sage: G += c; print(G)
Graphics object consisting of 1 graphics primitive
```

Here we make a graphic of embedded isosceles triangles, coloring each one with a different color as we go:

```python
sage: h = 10; c = 0.4; p = 0.5
sage: G = Graphics()
sage: for x in srange(1, h+1):  # needs sage.symbolic
    l = [[0,x*sqrt(3)],[x/2,-x*sqrt(3)/2],[-x/2,-x*sqrt(3)/2],[0,x*sqrt(3)]]
    G += line(l, color=hue(c + p*(x/h)))
sage: G.show(figsize=[5,5])  # needs sage.symbolic
```

We can change the scale of the axes in the graphics before displaying.
```python
sage: G = plot(exp, 1, 10)  # long time
˓→needs sage.symbolic
sage: G.show(scale='semilogy')  # long time
˓→needs sage.symbolic

_rich_repr_(display_manager, **kwds)

Rich Output Magic Method

See sage.repl.rich_output for details.

EXAMPLES:

```python
sage: from sage.repl.rich_output import get_display_manager
sage: dm = get_display_manager()
sage: g = Graphics()
sage: g._rich_repr_(dm)
```

LEGEND_OPTIONS = {'back_color': 'white', 'borderaxespad': None, 'borderpad': 0.6, 'columnspacing': None, 'fancybox': False, 'font family': 'sans-serif', 'font size': 'medium', 'font style': 'normal', 'font variant': 'normal', 'font weight': 'medium', 'handlelength': 0.05, 'handletextpad': 0.5, 'labelspacing': 0.02, 'loc': 'best', 'markerscale': 0.6, 'ncol': 1, 'numpoints': 2, 'shadow': True, 'title': None}

SHOW_OPTIONS = {'aspect_ratio': None, 'axes': None, 'axes_labels': None, 'axes_labels_size': None, 'base': None, 'dpi': 100, 'fig_tight': True, 'figsize': None, 'flip x': False, 'flip y': False, 'fontsize': None, 'frame': False, 'gridlines': None, 'gridlinesstyle': None, 'hgridlinesstyle': None, 'legend_options': {}, 'scale': None, 'show legend': False, 'tick_formatter': None, 'ticks': None, 'ticks integer': False, 'title': None, 'title pos': None, 'transparent': False, 'typeset': 'default', 'vgridlinesstyle': None, 'xmax': None, 'xmin': None, 'yamax': None, 'ymin': None}

add_primitive(primitive)

Adding a primitive to this graphics object.

EXAMPLES:

We give a very explicit example:

```python
sage: G = Graphics()
sage: from math import e
sage: from sage.plot.line import Line
sage: from sage.plot.arrow import Arrow
sage: L = Line([3,4,2,7,-2], [1,2,e,4,5.],
            { 'alpha': 1, 'thickness': 2, 'rgbcolor': (0,1,1),
              'legend_label': '' })
sage: A = Arrow(2, -5, .1, .2,
            { 'width': 3, 'head': 0, 'rgbcolor': (1,0,0),
              'linestyle': 'dashed', 'zorder': 8, 'legend_label': '' })
sage: G.add_primitive(L)
sage: G.add_primitive(A)
```

(continues on next page)
Graphics object consisting of 2 graphics primitives

aspect_ratio()
Get the current aspect ratio, which is the ratio of height to width of a unit square, or 'automatic'.

OUTPUT: a positive float (height/width of a unit square), or 'automatic' (expand to fill the figure).

EXAMPLES:
The default aspect ratio for a new blank Graphics object is 'automatic':

```
sage: P = Graphics()
sage: P.aspect_ratio()
'automatic'
```

The aspect ratio can be explicitly set different from the object's default:

```
sage: P = circle((1,1), 1)
sage: P.aspect_ratio()
1.0
sage: P.set_aspect_ratio(2)
sage: P.aspect_ratio()
2.0
sage: P.set_aspect_ratio('automatic')
sage: P.aspect_ratio()
'automatic'
```

axes(show=None)
Set whether or not the x and y axes are shown by default.

INPUT:
  * show - bool
If called with no input, return the current axes setting.

EXAMPLES:

```
sage: L = line([(1,2), (3,-4), (2, 5), (1,2)])
```

By default the axes are displayed.

```
sage: L.axes()
True
```

But we turn them off, and verify that they are off

```
sage: L.axes(False)
sage: L.axes()
False
```

Displaying L now shows a triangle but no axes.

```
sage: L
Graphics object consisting of 1 graphics primitive
```
**axes_color**(*c=None*)

Set the axes color.

If called with no input, return the current axes_color setting.

**INPUT:**

• *c* - an RGB color 3-tuple, where each tuple entry is a float between 0 and 1

**EXAMPLES:** We create a line, which has like everything a default axes color of black.

```
sage: L = line([[1,2], (3,-4), (2, 5), (1,2)])
sage: L.axes_color()
(0, 0, 0)
```

We change the axes color to red and verify the change.

```
sage: L.axes_color((1,0,0))
sage: L.axes_color()
(1.0, 0.0, 0.0)
```

When we display the plot, we’ll see a blue triangle and bright red axes.

```
sage: L
Graphics object consisting of 1 graphics primitive
```

**axes_label_color**(*c=None*)

Set the color of the axes labels.

The axes labels are placed at the edge of the x and y axes, and are not on by default (use the **axes_labels()** command to set them; see the example below). This function just changes their color.

**INPUT:**

• *c* - an RGB 3-tuple of numbers between 0 and 1

If called with no input, return the current axes_label_color setting.

**EXAMPLES:** We create a plot, which by default has axes label color black.

```
sage: p = plot(sin, (-1,1)) # needs sage.symbolic
sage: p.axes_label_color()
# needs sage.symbolic
(0, 0, 0)
```

We change the labels to be red, and confirm this:

```
sage: p.axes_label_color((1,0,0)) # needs sage.symbolic
sage: p.axes_label_color()
# needs sage.symbolic
(1.0, 0.0, 0.0)
```

We set labels, since otherwise we won’t see anything.

```
sage: p.axes_labels(['$x$ axis', '$y$ axis']) # needs sage.symbolic
```
In the plot below, notice that the labels are red:

```
sage: p
    # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

**axes_labels**

Set the axes labels.

**INPUT:**

- `l` - (default: `None`) a list of two strings or `None`

**OUTPUT:** a 2-tuple of strings

If `l` is `None`, returns the current `axes_labels`, which is itself by default `None`. The default labels are both empty.

**EXAMPLES:** We create a plot and put x and y axes labels on it.

```
sage: p = plot(sin(x), (x, 0, 10))  # needs sage.symbolic
sage: p.axes_labels(['$x$', '$y$'])  # needs sage.symbolic
sage: p.axes_labels()  # needs sage.symbolic
('$x$', '$y$')
```

Now when you plot `p`, you see x and y axes labels:

```
sage: p
    # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

Notice that some may prefer axes labels which are not typeset:

```
sage: plot(sin(x), (x, -3, 3), axes_labels=['$x$','$y$'])  # needs sage.symbolic
```

**axes_labels_size**

Set the relative size of axes labels w.r.t. the axes tick marks.

**INPUT:**

- `s` - float, relative size of axes labels w.r.t. to the tick marks, the size of the tick marks being set by `fontsize()`.

If called with no input, return the current relative size.

**EXAMPLES:**

```
sage: # needs sage.symbolic
sage: p = plot(sin(x^2), (x, -3, 3), axes_labels=['$x$','$y$'])
sage: p.axes_labels_size()  # default value
1.6
sage: p.axes_labels_size(2.5)
sage: p.axes_labels_size()
2.5
```
Now the axes labels are large w.r.t. the tick marks:

```python
sage: p  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

**axes_range**(xmin=None, xmax=None, ymin=None, ymax=None)

Set the ranges of the $x$ and $y$ axes.

INPUT:

- `xmin`, `xmax`, `ymin`, `ymax` - floats

**EXAMPLES:**

```python
sage: L = line([(1,2), (3,-4), (2, 5), (1,2)])
sage: L.set_axes_range(-1, 20, 0, 2)
sage: d = L.get_axes_range()
sage: d['xmin'], d['xmax'], d['ymin'], d['ymax']
(-1.0, 20.0, 0.0, 2.0)
```

**axes_width**(w=None)

Set the axes width. Use this to draw a plot with really fat or really thin axes.

INPUT:

- `w` - a float

If called with no input, return the current `axes_width` setting.

**EXAMPLES:** We create a plot, see the default axes width (with funny Python float rounding), then reset the width to 10 (very fat).

```python
sage: # needs sage.symbolic
sage: p = plot(cos, (-3,3))
sage: p.axes_width()
0.8
sage: p.axes_width(10)
sage: p.axes_width()
10.0
```

Finally we plot the result, which is a graph with very fat axes.

```python
sage: p  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

description()

Print a textual description to stdout.

This method is mostly used for doctests.

**EXAMPLES:**

```python
sage: print(polytopes.hypercube(2).plot().description())  # needs sage.geometry.polyhedron
Polygon defined by 4 points: [(-1.0, -1.0), (1.0, -1.0), (1.0, 1.0), (-1.0, 1.0)]
```

(continues on next page)
Line defined by 2 points: [(-1.0, 1.0), (-1.0, -1.0)]
Line defined by 2 points: [(1.0, -1.0), (-1.0, -1.0)]
Line defined by 2 points: [(1.0, -1.0), (1.0, 1.0)]
Line defined by 2 points: [(1.0, 1.0), (-1.0, 1.0)]
Point set defined by 4 point(s): [(1.0, -1.0), (1.0, 1.0), (-1.0, 1.0), (-1.0, -1.0)]

**flip**(flip\_x=False, flip\_y=False)

Get the flip options and optionally mirror this graphics object.

INPUT:

* flip\_x – boolean (default: False); if True, replace the current flip\_x option by its opposite
* flip\_y – boolean (default: False); if True, replace the current flip\_y option by its opposite

OUTPUT: a tuple containing the new flip options

EXAMPLES:

When called without arguments, this just returns the current flip options:

```python
sage: L = line([(1, 0), (2, 3)])
sage: L.flip()
(False, False)
```

Otherwise, the specified options are changed and the new options are returned:

```python
sage: L.flip(flip\_y=True)
(False, True)
sage: L.flip(True, True)
(True, False)
```

**fontsize**(s=None)

Set the font size of axes labels and tick marks.

Note that the relative size of the axes labels font w.r.t. the tick marks font can be adjusted via `axes\_labels\_size()`.

INPUT:

* s - integer, a font size in points.

If called with no input, return the current fontsize.

EXAMPLES:

```python
sage: L = line([(1,2), (3,-4), (2, 5), (1,2)])
sage: L.fontsize()
10
sage: L.fontsize(20)
sage: L.fontsize()
20
```

All the numbers on the axes will be very large in this plot:

```python
sage: L
Graphics object consisting of 1 graphics primitive
```
get_axes_range()

Returns a dictionary of the range of the axes for this graphics object. This is fall back to the ranges in get_minmax_data() for any value which the user has not explicitly set.

**Warning:** Changing the dictionary returned by this function does not change the axes range for this object. To do that, use the set_axes_range() method.

**EXAMPLES:**

```sage
sage: L = line(((1,2), (3,-4), (2, 5), (1,2)))
sage: list(sorted(L.get_axes_range().items()))
[('xmax', 3.0), ('xmin', 1.0), ('ymax', 5.0), ('ymin', -4.0)]
sage: L.set_axes_range(xmin=-1)
sage: list(sorted(L.get_axes_range().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 5.0), ('ymin', -4.0)]
```

get_minmax_data()

Return the x and y coordinate minimum and maximum

**Warning:** The returned dictionary is mutable, but changing it does not change the xmin/xmax/ymin/ymax data. The minmax data is a function of the primitives which make up this Graphics object. To change the range of the axes, call methods xmin(), xmax(), ymin(), ymax(), or set_axes_range().

**OUTPUT:**

A dictionary whose keys give the xmin, xmax, ymin, and ymax data for this graphic.

**EXAMPLES:**

```sage
sage: g = line(((1,1), (3,2)))
sage: list(sorted(g.get_minmax_data().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 2.0), ('ymin', 1.0)]
```

Note that changing ymax doesn’t change the output of get_minmax_data:

```sage
sage: g.ymax(10)
sage: list(sorted(g.get_minmax_data().items()))
[('xmax', 3.0), ('xmin', -1.0), ('ymax', 2.0), ('ymin', 1.0)]
```

The width/height ratio (in output units, after factoring in the chosen aspect ratio) of the plot is limited to $10^{-15}$ ... $10^{15}$, otherwise floating point errors cause problems in matplotlib:

```sage
sage: l = line(((1e-19,-1), (-1e-19,+1)), aspect_ratio=1.0)
sage: l.get_minmax_data()
{'xmax': 1.000000000000000e-15, 'xmin': -9.999000000000000e-16, 'ymax': 1.0, 'ymin': -1.0}
sage: l = line(((0,0), (1,1)), aspect_ratio=1e19)
sage: l.get_minmax_data()
(continues on next page)
```
inset(graphics, pos=None, fontsize=None)

Add a graphics object as an inset.

INPUT:

- graphics – the graphics object (instance of Graphics) to be added as an inset to the current graphics
- pos – (default: None) 4-tuple (left, bottom, width, height) specifying the location and size of the inset on the final figure, all quantities being in fractions of the figure width and height; if None, the value (0.7, 0.7, 0.2, 0.2) is used
- fontsize – (default: None) integer, font size (in points) for the inset; if None, the value of 6 points is used, unless fontsize has been explicitly set in the construction of graphics (in this case, it is not overwritten here)

OUTPUT:

- instance of MultiGraphics

EXAMPLES:

```
sage: # needs sage.symbolic
sage: f(x) = x^2*sin(1/x)
sage: g1 = plot(f(x), (x, -2, 2), axes_labels=['$x$', '$y$'])
sage: g2 = plot(f(x), (x, -0.3, 0.3), axes_labels=['$x$', '$y$'],
           ....: frame=True)
sage: g1.inset(g2)
Multigraphics with 2 elements
```

Using non-default values for the position/size and the font size:

```
sage: g1.inset(g2, pos=(0.15, 0.7, 0.25, 0.25), fontsize=8)  # needs sage.symbolic
Multigraphics with 2 elements
```

We can add another inset by invoking inset on the last output:

```
sage: g1g2 = _  # needs sage.symbolic
sage: g3 = plot(f(x), (x, -0.05, 0.05), axes_labels=['$x$', '$y$'],
           ....: frame=True)
```

```
sage: g1g2.inset(g3, pos=(0.65, 0.12, 0.25, 0.25))  # needs sage.symbolic
Multigraphics with 3 elements
```

legend(show=None)

Set whether or not the legend is shown by default.

INPUT:

- show – (default: None) a boolean

If called with no input, return the current legend setting.
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Chapter 5. Infrastructure and Low-Level Functions
EXAMPLES:

By default no legend is displayed:

```
sage: P = plot(sin)                     # needs sage.symbolic
sage: P.legend()                       # needs sage.symbolic
False
```

But if we put a label then the legend is shown:

```
sage: P = plot(sin, legend_label='sin') # needs sage.symbolic
sage: P.legend()                        # needs sage.symbolic
True
```

We can turn it on or off:

```
sage: # needs sage.symbolic
sage: P.legend(False)
False
sage: P.legend(True)                    # show with the legend
Graphics object consisting of 1 graphics primitive
```

Construct or modify a Matplotlib figure by drawing `self` on it.

INPUT (partial description, involving only Matplotlib objects; see `show()` for the other arguments):

- `figure` – (default: None) Matplotlib figure (class `matplotlib.figure.Figure`) on which `self` is to be displayed; if `None`, the figure will be created from the parameter `figsize`
- `figsize` – (default: `None`) width or `[width, height]` in inches of the Matplotlib figure in case `figure` is `None`; if `figsize` is `None`, Matplotlib’s default (6.4 x 4.8 inches) is used
- `sub` – (default: `None`) subpart of the figure, as an instance of Matplotlib “axes” (class `matplotlib.axes.Axes`) on which `self` is to be drawn; if `None`, the subpart will be created so as to cover the whole figure

OUTPUT:

- a `matplotlib.figure.Figure` object; if the argument `figure` is provided, this is the same object as `figure`.

EXAMPLES:

```
sage: c = circle((1,1),1)
sage: print(c.matplotlib())
Figure(640x480)
```
To obtain the first Matplotlib Axes object inside of the figure, you can do something like the following.

```
sage: p = plot(sin(x), (x, -2*pi, 2*pi))  # needs sage.symbolic
sage: figure = p.matplotlib()  # needs sage.symbolic
sage: axes = figure.axes[0]  # needs sage.symbolic
```

**plot()**

Draw a 2D plot of this graphics object, which just returns this object since this is already a 2D graphics object.

**EXAMPLES:**

```
sage: S = circle((0,0), 2)
sage: S.plot() is S
True
```

It does not accept any argument (github issue #19539):

```
sage: S.plot(1)
Traceback (most recent call last):
  ... plot() takes 1 positional argument but 2 were given
sage: S.plot(hey="hou")
Traceback (most recent call last):
  ... plot() got an unexpected keyword argument 'hey'
```

**plot3d(z=0, **kwds)**

Return an embedding of this 2D plot into the xy-plane of 3D space, as a 3D plot object. An optional parameter \( z \) can be given to specify the z-coordinate.

**EXAMPLES:**

```
sage: sum.plot(z*sin(x), 0, 10).plot3d(z)  # long time  # needs sage.symbolic
....: for z in range(6))
Graphics3d Object
```

**save(filename, legend_back_color='white', legend_borderpad=0.6, legend_borderaxespad=None, legend_columnspacing=None, legend_fancybox=False, legend_font_family='sans-serif', legend_font_size='medium', legend_font_style='normal', legend_font_variant='normal', legend_font_weight='medium', legend_handlelength=0.05, legend_handletextpad=0.5, legend_labelspacing=0.02, legend_loc='best', legend_markerscale=0.6, legend_ncol=1, legend_numpoints=2, legend_shadow=True, legend_title=None, **kwds)**

Save the graphics to an image file.

**INPUT:**

- `filename` – string. The filename and the image format given by the extension, which can be one of the following:
  - `.eps`,
- .pdf,
- .pgf,
- .png,
- .ps,
- .sobj (for a Sage object you can load later),
- .svg,
- empty extension will be treated as .sobj.

All other keyword arguments will be passed to the plotter.

OUTPUT:

* none.

EXAMPLES:

```python
sage: c = circle((1,1), 1, color='red')
sage: from tempfile import NamedTemporaryFile
sage: with NamedTemporaryFile(suffix='.png') as f:
....:     c.save(f.name, xmin=-1, xmax=3, ymin=-1, ymax=3)
```

To make a figure bigger or smaller, use figsize:

```python
sage: c.save(f.name, figsize=5, xmin=-1, xmax=3, ymin=-1, ymax=3)
```

By default, the figure grows to include all of the graphics and text, so the final image may not be exactly the figure size you specified. If you want a figure to be exactly a certain size, specify the keyword fig_tight=False:

```python
sage: c.save(f.name, figsize=[8,4], fig_tight=False, 
....:     xmin=-1, xmax=3, ymin=-1, ymax=3)
```

You can also pass extra options to the plot command instead of this method, e.g.

```python
sage: plot(x^2 - 5, (x, 0, 5), ymin=0).save(tmp_filename(ext='.png'))
```

will save the same plot as the one shown by this command:

```python
sage: plot(x^2 - 5, (x, 0, 5), ymin=0)
```

(Graphics object consisting of 1 graphics primitive)

(This test verifies that github issue #8632 is fixed.)

```python
save_image(filename=None, *args, **kwds)
```

Save an image representation of self.

The image type is determined by the extension of the filename. For example, this could be .png, .jpg, .gif, .pdf, .svg. Currently this is implemented by calling the `save()` method of self, passing along all arguments and keywords.

**Note:** Not all image types are necessarily implemented for all graphics types. See `save()` for more details.
EXAMPLES:

```python
sage: import tempfile
c sage: c = circle((1,1), 1, color='red')
sage: with tempfile.NamedTemporaryFile(suffix='_*.png') as f:
    ..... c.save_image(f.name, xmin=-1, xmax=3,
    ..... ym in=-1, ym ax=3)
```

**set_aspect_ratio(ratio)**

Set the aspect ratio, which is the ratio of height and width of a unit square (i.e., height/width of a unit square), or ‘automatic’ (expand to fill the figure).

**INPUT:**

* ratio - a positive real number or ‘automatic’

**EXAMPLES:** We create a plot of the upper half of a circle, but it doesn’t look round because the aspect ratio is off:

```python
sage: P = plot(sqrt(1-x^2),(x,-1,1)); P
# needs sage.symbolic
```

Graphics object consisting of 1 graphics primitive

So we set the aspect ratio and now it is round:

```python
sage: P.set_aspect_ratio(1)  # needs sage.symbolic
sage: P.aspect_ratio()  # needs sage.symbolic
1.0
sage: P
# needs sage.symbolic
```

Graphics object consisting of 1 graphics primitive

Note that the aspect ratio is inherited upon addition (which takes the max of aspect ratios of objects whose aspect ratio has been set):

```python
sage: P + plot(sqrt(4-x^2),(x,-2,2))  # needs sage.symbolic
```

Graphics object consisting of 2 graphics primitives

In the following example, both plots produce a circle that looks twice as tall as wide:

```python
sage: Q = circle((0,0), 0.5); Q.set_aspect_ratio(2)
sage: (P + Q).aspect_ratio(); P + Q  # needs sage.symbolic
```

2.0

Graphics object consisting of 2 graphics primitives

**set_axes_range(xmin=None, xmax=None, ymin=None, ymax=None)**

Set the ranges of the $x$ and $y$ axes.

**INPUT:**
• \textit{xmin, xmax, ymin, ymax} - floats

**EXAMPLES:**

```python
sage: L = line([(1,2), (3,-4), (2, 5), (1,2)])
sage: L.set_axes_range(-1, 20, 0, 2)
sage: d = L.get_axes_range()
sage: d['xmin'], d['xmax'], d['ymin'], d['ymax']
(-1.0, 20.0, 0.0, 2.0)
```

\textbf{set\_flip}(\textit{flip\_x=\textit{None}, flip\_y=\textit{None}})

Set the flip options for this graphics object.

**INPUT:**

- \textit{flip\_x} – boolean (default: \textit{None}); if not \textit{None}, set the \textit{flip\_x} option to this value
- \textit{flip\_y} – boolean (default: \textit{None}); if not \textit{None}, set the \textit{flip\_y} option to this value

**EXAMPLES:**

```python
sage: L = line([(1, 0), (2, 3)])
sage: L.set_flip(flip_y=True)
sage: L.flip()
(False, True)
sage: L.set_flip(True, False)
sage: L.flip()
(True, False)
```

\textbf{set\_legend\_options}(\textit{**kwds})

Set various legend options.

**INPUT:**

- \textit{title} - (default: \textit{None}) string, the legend title
- \textit{ncol} - (default: 1) positive integer, the number of columns
- \textit{columnspacing} - (default: \textit{None}) the spacing between columns
- \textit{borderaxespad} - (default: \textit{None}) float, length between the axes and the legend
- \textit{back\_color} - (default: ‘white’) This parameter can be a string denoting a color or an RGB tuple. The string can be a color name as in (‘red’, ‘green’, ‘yellow’, ...) or a floating point number like ‘0.8’ which gets expanded to (0.8, 0.8, 0.8). The tuple form is just a floating point RGB tuple with all values ranging from 0 to 1.
- \textit{handle\_length} - (default: 0.05) float, the length of the legend handles
- \textit{handletext\_pad} - (default: 0.5) float, the pad between the legend handle and text
- \textit{label\_spacing} - (default: 0.02) float, vertical space between legend entries
- \textit{loc} - (default: ‘best’) May be a string, an integer or a tuple. String or
  integer inputs must be one of the following:

  - 0, ‘best’
  - 1, ‘upper right’
  - 2, ‘upper left’
  - 3, ‘lower left’

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- 4, 'lower right'
- 5, 'right'
- 6, 'center left'
- 7, 'center right'
- 8, 'lower center'
- 9, 'upper center'
- 10, 'center'

Tuple arguments represent an absolute (x, y) position on the plot in axes coordinates (meaning from 0 to 1 in each direction).

- **markerscale** - (default: 0.6) float, how much to scale the markers in the legend.
- **numpoints** - (default: 2) integer, the number of points in the legend for line
- **borderpad** - (default: 0.6) float, the fractional whitespace inside the legend border (between 0 and 1)
- **font_family** - (default: 'sans-serif') string, one of 'serif', 'sans-serif', 'cursive', 'fantasy', 'monospace'
- **font_style** - (default: 'normal') string, one of 'normal', 'italic', 'oblique'
- **font_variant** - (default: 'normal') string, one of 'normal', 'small-caps'
- **font_weight** - (default: 'medium') string, one of 'black', 'extra bold', 'bold', 'semibold', 'medium', 'normal', 'light'
- **font_size** - (default: 'medium') string, one of 'xx-small', 'x-small', 'small', 'medium', 'large', 'x-large', 'xx-large' or an absolute font size (e.g. 12)
- **shadow** - (default: True) boolean - draw a shadow behind the legend
- **fancybox** - (default: False) a boolean. If True, draws a frame with a round fancybox.

These are all keyword arguments.

OUTPUT: a dictionary of all current legend options

EXAMPLES:

By default, no options are set:

```
sage: p = plot(tan, legend_label='tan') # needs sage.symbolic
sage: p.set_legend_options() # needs sage.symbolic
{}
```

We build a legend without a shadow:

```
sage: p.set_legend_options(shadow=False) # needs sage.symbolic
sage: p.set_legend_options()['shadow'] # needs sage.symbolic
False
```

To set the legend position to the center of the plot, all these methods are roughly equivalent:
sage: p.set_legend_options(loc='center'); p # needs sage.symbolic

Graphics object consisting of 1 graphics primitive

sage: p.set_legend_options(loc=10); p # needs sage.symbolic

Graphics object consisting of 1 graphics primitive

sage: p.set_legend_options(loc=(0.5, 0.5)); p # aligns the bottom of the box to the center # needs sage.symbolic

Graphics object consisting of 1 graphics primitive

show(legend_back_color='white', legend_borderpad=0.6, legend_borderaxespad=None, legend_columnspacing=None, legend_fancybox=False, legend_font_family='sans-serif', legend_font_size='medium', legend_font_style='normal', legend_font_variant='normal', legend_font_weight='medium', legend_handlelength=0.05, legend_handletextpad=0.5, legend_labelspacing=0.02, legend_loc='best', legend_markerscale=0.6, legend_ncol=1, legend_numpoints=2, legend_shadow=True, legend_title=None, **kwds)

Show this graphics image immediately.

This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

OPTIONAL INPUT:

- **dpi** - (default: 100) dots per inch
- **figsize** - (default: [6.4, 4.8]) [width, height] inches. The maximum value of each of the width and the height can be 327 inches, at the default dpi of 100 dpi, which is just shy of the maximum allowed value of 32768 dots (pixels).
- **fig_tight** - (default: True) whether to clip the drawing tightly around drawn objects. If True, then the resulting image will usually not have dimensions corresponding to figsize. If False, the resulting image will have dimensions corresponding to figsize.
- **aspect_ratio** - the perceived height divided by the perceived width. For example, if the aspect ratio is set to 1, circles will look round and a unit square will appear to have sides of equal length, and if the aspect ratio is set 2, vertical units will be twice as long as horizontal units, so a unit square will be twice as high as it is wide. If set to 'automatic', the aspect ratio is determined by figsize and the picture fills the figure.
- **axes** - (default: True)
- **axes_labels** - (default: None) list (or tuple) of two strings; the first is used as the label for the horizontal axis, and the second for the vertical axis.
- **axes_labels_size** - (default: current setting – 1.6) scale factor relating the size of the axes labels with respect to the size of the tick marks.
- **fontsize** - (default: current setting – 10) positive integer; used for axes labels; if you make this very large, you may have to increase figsize to see all labels.
- **frame** - (default: False) draw a frame around the image
- **gridlines** - (default: None) can be any of the following:
  - None, False: do not add grid lines.
- True, “automatic”, “major”: add grid lines at major ticks of the axes.
- “minor”: add grid at major and minor ticks.
- [xlist,ylist]: a tuple or list containing two elements, where xlist (or ylist) can be any of the following.
  * None, False: don’t add horizontal (or vertical) lines.
  * True, “automatic”, “major”: add horizontal (or vertical) grid lines at the major ticks of the axes.
  * “minor”: add horizontal (or vertical) grid lines at major and minor ticks of axes.
  * an iterable yielding numbers n or pairs (n,opts), where n is the coordinate of the line and opt is a dictionary of MATPLOTLIB options for rendering the line.

- gridlinesstyle, hgridlinesstyle, vgridlinesstyle - (default: None) a dictionary of MATPLOTLIB options for the rendering of the grid lines, the horizontal grid lines or the vertical grid lines, respectively.
- transparent - (default: False) If True, make the background transparent.
- axes_pad - (default: 0.02 on "linear" scale, 1 on "log" scale).
  - In the "linear" scale, it determines the percentage of the axis range that is added to each end of each axis. This helps avoid problems like clipping lines because of line-width, etc. To get axes that are exactly the specified limits, set axes_pad to zero.
  - On the "log" scale, it determines the exponent of the fraction of the minimum (resp. maximum) that is subtracted from the minimum (resp. added to the maximum) value of the axis. For instance if the minimum is \( m \) and the base of the axis is \( b \) then the new minimum after padding the axis will be \( m - m/b^{\text{axes\_pad}} \).

- ticks_integer - (default: False) guarantee that the ticks are integers (the ticks option, if specified, will override this)

- ticks - A matplotlib locator for the major ticks, or a number. There are several options. For more information about locators, type from matplotlib import ticker and then ticker?.
  - If this is a locator object, then it is the locator for the horizontal axis. A value of None means use the default locator.
  - If it is a list of two locators, then the first is for the horizontal axis and one for the vertical axis. A value of None means use the default locator (so a value of [None, my_locator] uses my_locator for the vertical axis and the default for the horizontal axis).
  - If in either case above one of the entries is a number \( m \) (something which can be coerced to a float), it will be replaced by a MultipleLocator which places major ticks at integer multiples of \( m \). See examples.
  - If in either case above one of the entries is a list of numbers, it will be replaced by a FixedLocator which places ticks at the locations specified. This includes the case of of the empty list, which will give no ticks. See examples.

- tick_formatter - A matplotlib formatter for the major ticks. There are several options. For more information about formatters, type from matplotlib import ticker and then ticker?.
  If the value of this keyword is a single item, then this will give the formatting for the horizontal axis only (except for the "latex" option). If it is a list or tuple, the first is for the horizontal axis, the second for the vertical axis. The options are below:
  - If one of the entries is a formatter object, then it used. A value of None means to use the default locator (so using tick_formatter=[None, my_formatter] uses my_formatter for the vertical axis and the default for the horizontal axis).
If one of the entries is a symbolic constant such as $\pi$, $e$, or $\sqrt{2}$, ticks will be formatted nicely at rational multiples of this constant.

**Warning:** This should only be used with the **ticks** option using nice rational multiples of that constant!

If one of the entries is the string "latex", then the formatting will be nice typesetting of the ticks. This is intended to be used when the tick locator for at least one of the axes is a list including some symbolic elements. This uses matplotlib's internal LaTeX rendering engine. If you want to use an external LaTeX compiler, then set the keyword option **typeset**. See examples.

- **title** - (default: None) The title for the plot
- **title_pos** - (default: None) **The position of the title for the plot.** It must be a tuple or a list of two real numbers ($x_{\text{pos}}$, $y_{\text{pos}}$) which indicate the relative position of the title within the plot. The plot itself can be considered to occupy, in relative terms, the region within a unit square $[0, 1] \times [0, 1]$. The title text is centered around the horizontal factor $x_{\text{pos}}$ of the plot. The baseline of the title text is present at the vertical factor $y_{\text{pos}}$ of the plot. Hence, title_pos=(0.5, 0.5) will center the title in the plot, whereas title_pos=(0.5, 1.1) will center the title along the horizontal direction, but will place the title a fraction 0.1 times above the plot.

- If the first entry is a list of strings (or numbers), then the formatting for the horizontal axis will be typeset with the strings present in the list. Each entry of the list of strings must be provided with a corresponding number in the first entry of **ticks** to indicate its position on the axis. To typeset the strings with "latex" enclose them within "$" symbols. To have similar custom formatting of the labels along the vertical axis, the second entry must be a list of strings and the second entry of **ticks** must also be a list of numbers which give the positions of the labels. See the examples below.

- **show_legend** - (default: None) If True, show the legend
- **legend_*** - all the options valid for **set_legend_options()** prefixed with legend_
- **base** - (default: 10) the base of the logarithm if a logarithmic scale is set. This must be greater than 1. The base can be also given as a list or tuple (basex, basey). basex sets the base of the logarithm along the horizontal axis and basey sets the base along the vertical axis.
- **scale** – (default: "linear") string. The scale of the axes. Possible values are
  - "linear" – linear scaling of both the axes
  - "loglog" – sets both the horizontal and vertical axes to logarithmic scale
  - "semilogx" – sets only the horizontal axis to logarithmic scale.
  - "semilogy" – sets only the vertical axis to logarithmic scale.

The scale can be also be given as single argument that is a list or tuple (scale, base) or (scale, base, base).  

**Note:**

- If the scale is "linear", then irrespective of what base is set to, it will default to 10 and will remain unused.

---

5.1. Graphics objects 527
• xmin – starting x value in the rendered figure.
• xmax – ending x value in the rendered figure.
• ymin – starting y value in the rendered figure.
• ymax – ending y value in the rendered figure.
• flip_x – (default: False) boolean. If True, flip the horizontal axis.
• flip_y – (default: False) boolean. If True, flip the vertical axis.
• typeset – (default: "default") string. The type of font rendering that should be used for the text. The possible values are
  – "default" – Uses matplotlib’s internal text rendering engine called Mathtext (see https://matplotlib.org/users/mathtext.html). If you have modified the default matplotlib settings, for instance via a matplotlibrc file, then this option will not change any of those settings.
  – "latex" – LaTeX is used for rendering the fonts. This requires LaTeX, dvipng and Ghostscript to be installed.
  – "type1" – Type 1 fonts are used by matplotlib in the text in the figure. This requires LaTeX, dvipng and Ghostscript to be installed.

OUTPUT:
This method does not return anything. Use save() if you want to save the figure as an image.

EXAMPLES:

```
sage: c = circle((1,1), 1, color='red')
sage: c.show(xmin=-1, xmax=3, ymin=-1, ymax=3)
```

You can make the picture larger by changing figsize with width, height each having a maximum value of 327 inches at default dpi:

```
sage: p = ellipse((0,0),4,1)
sage: p.show(figsize=[327,10], dpi=100)
sage: p.show(figsize=[328,10], dpi=80)
```

You can turn off the drawing of the axes:

```
sage: show(plot(sin,-4,4), axes=False)   # needs sage.symbolic
```

You can also label the axes. Putting something in dollar signs formats it as a mathematical expression:

```
sage: show(plot(sin,-4,4), axes_labels=('x','y'))   # needs sage.symbolic
```

You can add a title to a plot:

```
sage: show(plot(sin,-4,4), title=r'A plot of $\sin(x)$')   # needs sage.symbolic
```

You can also provide the position for the title to the plot. In the plot below the title is placed on the bottom left of the figure.
sage: plot(sin, -4, 4, title='Plot \( \sin(x) \)', title_pos=(0.05,-0.05))  # needs sage.symbolic
Graphics object consisting of 1 graphics primitive

If you want all the text to be rendered by using an external LaTeX installation then set the typeset to "latex". This requires that LaTeX, dvipng and Ghostscript be installed:

sage: plot(x, typeset='latex')  # optional - latex, needs sage.symbolic
Graphics object consisting of 1 graphics primitive

If you want all the text in your plot to use Type 1 fonts, then set the typeset option to "type1". This requires that LaTeX, dvipng and Ghostscript be installed:

sage: plot(x, typeset='type1')  # optional - latex, needs sage.symbolic

You can turn on the drawing of a frame around the plots:

sage: show(plot(sin,-4,4), frame=True)  # needs sage.symbolic

You can make the background transparent:

sage: plot(sin(x), (x, -4, 4), transparent=True)  # needs sage.symbolic

Prior to github issue #19485, legends by default had a shadowless gray background. This behavior can be recovered by passing in certain legend_options:

sage: p = plot(sin(x), legend_label=r'\( \sin(x) \)')  # needs sage.symbolic
sage: p.show(legend_options={'back_color': (0.9,0.9,0.9),  
                           'shadow': False})

We can change the scale of the axes in the graphics before displaying:

sage: G = plot(exp, 1, 10)  # needs sage.symbolic
sage: G.show(scale='semilogy')  # needs sage.symbolic

We can change the base of the logarithm too. The following changes the vertical axis to be on log scale, and with base 2. Note that the base argument will ignore any changes to the axis which is in linear scale:

sage: G.show(scale='semilogy', base=2)  # y axis as powers of 2  # long
sage: G.show(scale='semilogy', base=(3,2))  # base ignored for x-axis  # short

The scale can be also given as a 2-tuple or a 3-tuple.
The base need not be an integer, though it does have to be made a float:

```
sage: G.show(scale='semilogx', base=float(e))  # base is e  
```

Logarithmic scale can be used for various kinds of plots. Here are some examples:

```
sage: G = list_plot([10**i for i in range(10)])  # long  

sage: G.show(scale='semilogy')  # long  
```

```
sage: G = parametric_plot((x, x**2), (x, 1, 10))  

sage: G.show(scale='loglog')  
```

```
sage: disk((5,5), 4, (0, 3*pi/2)).show(scale='loglog', base=2)  
```

```
sage: x, y = var('x, y')  

sage: G = plot_vector_field((2^x,y^2), (x,1,10), (y,1,100))  

sage: G.show(scale='semilogx',base=2)  
```

```
sage: G = plot(x^3, -2, 3)  

sage: G.show(flip_x=True)  

sage: G.show(flip_y=True)  
```

```
sage: c = circle((0,0), 1)  
sage: c.show(gridlines=True)  
sage: c.show(gridlines="automatic")  
sage: c.show(gridlines="major")  
```

```
sage: x, y = var('u v')  
```

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(continued from previous page)

```python
sage: f = exp(-(u^2+v^2))
sage: p = plot_vector_field(f.gradient(), (u,-2,2), (v,-2,2))
sage: p.show(gridlines="minor")
```

Add only horizontal or vertical grid lines.

```python
sage: p = plot(sin, -10, 20)  # needs sage.symbolic
sage: p.show(gridlines=[None, "automatic"])  # needs sage.symbolic
sage: p.show(gridlines=["minor", False])  # needs sage.symbolic
```

Add grid lines at specific positions (using lists/tuples).

```python
sage: x, y = var('x, y')  # needs sage.symbolic
sage: p = implicit_plot((y^2-x^2)*(x-1)*(2*x-3) - 4*(x^2+y^2-2*x)^2, ...
                      (x,-2,2), (y,-2,2), plot_points=1000)
sage: p.show(gridlines=[[1,0],[-1,0,1]])  # long time, needs sage.symbolic
```

Add grid lines at specific positions (using iterators).

```python
sage: def maple_leaf(t):
....:     return (100/(100+(t-pi/2)^8))*(2-sin(7*t)-cos(30*t)/2)  # long time
sage: p = polar_plot(maple_leaf, -pi/4, 3*pi/2, color="red",plot_points=1000)
```

Add grid lines at specific positions (using functions).

```python
sage: # needs sage.symbolic
sage: y = x^5 + 4*x^4 - 10*x^3 - 40*x^2 + 9*x + 36
sage: p = plot(y, -4.1, 1.1)
sage: xlines = lambda a, b: [z for z, m in y.roots()]
sage: p.show(gridlines=[xlines, [0]], frame=True, axes=False)
```

Change the style of all the grid lines.

```python
sage: b = bar_chart([-3,5,-6,11], color='red')
sage: b.show(gridlines=[[-1,-0.5,..,4], True],
         gridlinesstyle=dict(color="blue", linestyle="-"))
```

Change the style of the horizontal or vertical grid lines separately.

```python
sage: p = polar_plot(2 + 2*cos(x), 0, 2*pi, color=hue(0.3))  # needs sage.symbolic
sage: p.show(gridlines=\[True, \]
```
Change the style of each grid line individually.

```python
sage: x, y = var('x, y')
# needs sage.symbolic
sage: p = implicit_plot((y^2-x^2)*(x-1)*(2*x-3) - 4*(x^2+y^2-2*x)^2, # needs sage.symbolic
    (x,-2,2), (y,-2,2), plot_points=1000)
sage: p.show(gridlines=(hgridlinesstyle=dict(color="red", linewidth=1.0),
    vgridlinesstyle=dict(color="blue", linestyle=""))
```

Grid lines can be added to contour plots.

```python
sage: f = sin(x^2 + y^2)*cos(x)*sin(y)  # needs sage.symbolic
sage: c = contour_plot(f, (x, -4, 4), (y, -4, 4), plot_points=100)  # needs sage.symbolic
sage: c.show(gridlines=True, gridlinesstyle={'linestyle': ':', 'linewidth': 1, 'color': 'red'})
```

Grid lines can be added to matrix plots.

```python
sage: M = MatrixSpace(QQ,10).random_element()
sage: matrix_plot(M).show(gridlines=True)
```

By default, Sage increases the horizontal and vertical axes limits by a certain percentage in all directions. This is controlled by the `axes_pad` parameter. Increasing the range of the axes helps avoid problems with lines and dots being clipped because the linewidth extends beyond the axes. To get axes limits that are exactly what is specified, set `axes_pad` to zero. Compare the following two examples

```python
sage: (plot(sin(x), (x, -pi, pi), thickness=2)
    + point((pi, -1), pointsize=15))
Graphics object consisting of 2 graphics primitives
sage: (plot(sin(x), (x, -pi, pi), thickness=2, axes_pad=0)
    + point((pi, -1), pointsize=15))
Graphics object consisting of 2 graphics primitives
```

The behavior of the `axes_pad` parameter is different if the axis is in the "log" scale. If \( b \) is the base of
the axis, the minimum value of the axis, is decreased by the factor $1/b_{\text{axes_pad}}$ of the minimum and the maximum value of the axis is increased by the same factor of the maximum value. Compare the axes in the following two plots to see the difference.

```python
sage: plot_loglog(x, (1.1*10**-2, 9990)) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive

sage: plot_loglog(x, (1.1*10**-2, 9990), axes_pad=0) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

Via matplotlib, Sage allows setting of custom ticks. See above for more details.

Here the labels are not so useful:

```python
sage: plot(sin(pi*x), (x, -8, 8)) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

Now put ticks at multiples of 2:

```python
sage: plot(sin(pi*x), (x, -8, 8), ticks=2) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

Or just choose where you want the ticks:

```python
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[[-7,-3,0,3,7], [-1/2,0,1/2]]) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

Or no ticks at all:

```python
sage: plot(sin(pi*x), (x, -8, 8), ticks=[[[]], []]) #
˓→needs sage.symbolic
Graphics object consisting of 1 graphics primitive
```

This can be very helpful in showing certain features of plots.

```python
sage: plot(1.5/(1+e^(-x)), (x, -10, 10)) # doesn't quite show value of inflection point # needs sage.symbolic
˓→Graphics object consisting of 1 graphics primitive
```

```python
sage: plot(1.5/(1+e^(-x)), (x, -10, 10), # It's right at f(x)=0.75!
˓→ticks=[None, 1.5/4])
˓→needs sage.symbolic
˓→Graphics object consisting of 1 graphics primitive
```

But be careful to leave enough room for at least two major ticks, so that the user can tell what the scale is:

```python
sage: plot(x^2, (x,1,8), ticks=6).show() #
˓→needs sage.symbolic
Traceback (most recent call last):
...
```

(continues on next page)
ValueError: Expand the range of the independent variable to allow two multiples of your tick locator (option `ticks`).

We can also do custom formatting if you need it. See above for full details:

```
sage: plot(2*x + 1, (x,0,5), # not tested (broken with matplotlib 3.6),
           ticks=[[0,1,e,pi,sqrt(20)], 2],
           tick_formatter="latex")
```

Graphics object consisting of 1 graphics primitive

This is particularly useful when setting custom ticks in multiples of $\pi$.

```
sage: plot(sin(x), (x,0,2*pi), ticks=pi/3, tick_formatter=pi)
```

Graphics object consisting of 1 graphics primitive

But keep in mind that you will get exactly the formatting you asked for if you specify both formatters. The first syntax is recommended for best style in that case.

```
sage: plot(arcsin(x), (x,-1,1), ticks=[None, pi/6], # Nice-looking!
           tick_formatter=["latex", pi])
```

Graphics object consisting of 1 graphics primitive

```
sage: plot(arcsin(x), (x,-1,1), ticks=[None, pi/6], # Not so nice-looking
           tick_formatter=[None, pi])
```

Graphics object consisting of 1 graphics primitive

Custom tick labels can be provided by providing the keyword `tick_formatter` with the list of labels, and simultaneously providing the keyword `ticks` with the positions of the labels.

```
sage: plot(x, (x,0,3), ticks=[[1,2.5], None],
           tick_formatter=[["$x_1$","$x_2$"], None])
```

Graphics object consisting of 1 graphics primitive

The following sets the custom tick labels only along the horizontal axis.

```
sage: plot(x**2, (x,0,2), ticks=[[1,2], None],
           tick_formatter=[["$x_1$","$x_2$"], None])
```

Graphics object consisting of 1 graphics primitive

If the number of tick labels do not match the number of positions of tick labels, then it results in an error:

```
sage: plot(x**2, (x,0,2), ticks=[[2], None],
           tick_formatter=[["$x_1$","$x_2$"], None]).show()
```

Traceback (most recent call last):
...
ValueError: If the first component of the list `tick_formatter` is a list then the first component of `ticks` must also be a list of equal length.

When using logarithmic scale along the axis, make sure to have enough room for two ticks so that the user can tell what the scale is. This can be effected by increasing the range of the independent variable, or by changing the base, or by providing enough tick locations by using the `ticks` parameter.

By default, Sage will expand the variable range so that at least two ticks are included along the logarithmic axis. However, if you specify `ticks` manually, this safety measure can be defeated:

```
sage: list_plot_loglog([(1,2),(2,3)], plotjoined=True, ticks=[[1],[1]])
doctest:...: UserWarning: The x-axis contains fewer than 2 ticks; the logarithmic scale of the plot may not be apparent to the reader.
doctest:...: UserWarning: The y-axis contains fewer than 2 ticks; the logarithmic scale of the plot may not be apparent to the reader.
Graphics object consisting of 1 graphics primitive
```

This one works, since the horizontal axis is automatically expanded to contain two ticks and the vertical axis is provided with two ticks:

```
sage: list_plot_loglog([(1,2),(2,3)], plotjoined=True, ticks=[None,[1,10]])
Graphics object consisting of 1 graphics primitive
```

Another example in the log scale where both the axes are automatically expanded to show two major ticks:

```
sage: list_plot_loglog([(2,0.5), (3, 4)], plotjoined=True)
Graphics object consisting of 1 graphics primitive
```

When using `title_pos`, it must be ensured that a list or a tuple of length two is used. Otherwise, a warning is raised:

```
sage: plot(x, -4, 4, title='Plot x', title_pos=0.05)  # needs sage.symbolic
doctest:...: ...RichReprWarning: Exception in _rich_repr_ while displaying object: 'title_pos' must be a list or tuple of two real numbers.
Graphics object consisting of 1 graphics primitive
```

**tick_label_color**

Set the color of the axes tick labels.

**INPUT:**

- `c` - an RGB 3-tuple of numbers between 0 and 1

If called with no input, return the current tick_label_color setting.

**EXAMPLES:**

```
sage: # needs sage.symbolic
sage: p = plot(cos, (-3,3))
sage: p.tick_label_color()
(0, 0, 0)
sage: p.tick_label_color((1,0,0))
sage: p.tick_label_color((1.0, 0.0, 0.0))
```

(continues on next page)
\texttt{sage: p}
Graphics object consisting of 1 graphics primitive

\texttt{\texttt{xmax}(\texttt{xmax}=\texttt{None})}

EXEMPLARY:

\begin{verbatim}
\texttt{sage: g = line([\((-1,1), (3,2)\)])}
\texttt{sage: g.xmax()}
3.0
\texttt{sage: g.xmax(10)}
\texttt{sage: g.xmax()}
10.0
\end{verbatim}

\texttt{\texttt{xmin}(\texttt{xmin}=\texttt{None})}

EXEMPLARY:

\begin{verbatim}
\texttt{sage: g = line([\((-1,1), (3,2)\)])}
\texttt{sage: g.xmin()} 
-1.0
\texttt{sage: g.xmin(-3)}
\texttt{sage: g.xmin()}
-3.0
\end{verbatim}

\texttt{\texttt{ymax}(\texttt{ymax}=\texttt{None})}

EXEMPLARY:

\begin{verbatim}
\texttt{sage: g = line([\((-1,1), (3,2)\)])}
\texttt{sage: g.ymax()}
2.0
\texttt{sage: g.ymax(10)}
\texttt{sage: g.ymax()}
10.0
\end{verbatim}

\texttt{\texttt{ymin}(\texttt{ymin}=\texttt{None})}

EXEMPLARY:

\begin{verbatim}
\texttt{sage: g = line([\((-1,1), (3,2)\)])}
\texttt{sage: g.ymin()}
1.0
\texttt{sage: g.ymin(-3)}
\texttt{sage: g.ymin()}
-3.0
\end{verbatim}

\texttt{sage.plot.graphics.GraphicsArray(*args, **kwargs)}

This is deprecated (see github issue #28675). Use 	exttt{sage.plot.multigraphics.GraphicsArray} instead.

\texttt{sage.plot.graphics.is_Graphics(x)}

Return True if \texttt{x} is a Graphics object.

EXEMPLARY:
```python
sage: from sage.plot.graphics import is_Graphics
sage: is_Graphics(1)
False
sage: is_Graphics(disk((0.0, 0.0), 1, (0, pi/2)))
True
```

## 5.2 Graphics arrays and insets

This module defines the classes `MultiGraphics` and `GraphicsArray`. The class `MultiGraphics` is the base class for 2-dimensional graphical objects that are composed of various `Graphics` objects, arranged in a given canvas. The subclass `GraphicsArray` is for `Graphics` objects arranged in a regular array.

**AUTHORS:**

- Eric Gourgoulhon (2019-05-24): initial version, refactoring the class `GraphicsArray` that was defined in the module `graphics`.

```python
class sage.plot.multigraphics.GraphicsArray(array)
```

**INPUT:**

- `array` – either a list of lists of `Graphics` elements (generic case) or a single list of `Graphics` elements (case of a single-row array)

**EXAMPLES:**

An array made of four graphics objects:

```python
sage: g1 = plot(sin(x^2), (x, 0, 6), axes_labels=['$x$', '$y$'],
......:        axes=False, frame=True, gridlines='minor')
sage: y = var('y')
sage: g2 = streamline_plot((sin(x), cos(y)), (x,-3,3), (y,-3,3),
......:        aspect_ratio=1)
sage: g3 = graphs.DodecahedralGraph().plot()
sage: g4 = polar_plot(sin(5*x)^2, (x, 0, 2*pi), color='green',
......:        fontsize=8) \
......: + circle((0,0), 0.5, rgbcolor='red', fill=True, alpha=0.1,
......:        legend_label='pink')
sage: g4.set_legend_options(loc='upper right')
sage: G = graphics_array([[g1, g2], [g3, g4]])
sage: G
Graphics Array of size 2 x 2
```

If one constructs the graphics array from a single list of graphics objects, one obtains a single-row array:

```python
sage: G = graphics_array([g1, g2, g3, g4])
sage: G
Graphics Array of size 1 x 4
```
Chapter 5. Infrastructure and Low-Level Functions
5.2. Graphics arrays and insets
We note that the overall aspect ratio of the figure is 4/3 (the default), which makes $g_1$ elongated, while the aspect ratio of $g_2$, which has been specified with the parameter aspect_ratio=1 is preserved. To get a better aspect ratio for the whole figure, one can use the option figsize in the method show():

```
sage: G.show(figsize=[8, 3])
```

We can access individual elements of the graphics array with the square-bracket operator:

```
sage: G = graphics_array([[$g_1$, $g_2$], [$g_3$, $g_4$]])  # back to the 2x2 array
sage: print(G)
Graphics Array of size 2 x 2
sage: G[0] is $g_1$
True
sage: G[1] is $g_2$
True
sage: G[2] is $g_3$
True
sage: G[3] is $g_4$
True
```

Note that with respect to the square-bracket operator, $G$ is considered as a flattened list of graphics objects, not as an array. For instance, $G[0, 1]$ throws an error:

```
sage: G[0, 1]
Traceback (most recent call last):
...
TypeError: list indices must be integers or slices, not tuple
```

$G[:]$ returns the full (flattened) list of graphics objects composing $G$:

```
sage: G[:]
[Graphics object consisting of 1 graphics primitive,
Graphics object consisting of 1 graphics primitive,
Graphics object consisting of 51 graphics primitives,
Graphics object consisting of 2 graphics primitives]
```

The total number of Graphics objects composing the array is returned by the function len:
The square-bracket operator can be used to replace elements in the array:

\[
\text{sage: } G[0] = g4
\]

\[
\text{sage: } G
\]

Graphics Array of size 2 x 2

append(g)

Append a graphics to the array.

Currently not implemented.

ncols()

Number of columns of the graphics array.

EXAMPLES:

\[
\text{sage: } R = \text{rainbow}(6)
\]

\[
\text{sage: } L = [\text{plot}(x^n, (x,0,1), \text{color}=R[n]) \text{ for } n \text{ in range}(6)]
\]

\[
\text{sage: } G = \text{graphics_array}(L, 2, 3)
\]

\[
\text{sage: } G.ncols()
\]

3

(continues on next page)
sage: graphics_array(L).ncols()
6

nrows()  
Number of rows of the graphics array.

EXAMPLES:

sage: R = rainbow(6)  
sage: L = [plot(x^n, (x,0,1), color=R[n]) for n in range(6)]  
sage: G = graphics_array(L, 2, 3)  
sage: G.nrows()
2  
sage: graphics_array(L).nrows()
1

position(index)  
Return the position and relative size of an element of self on the canvas.

INPUT:
• index – integer specifying which element of self

OUTPUT:
• a 4-tuple (left, bottom, width, height) giving the location and relative size of the element on the canvas, all quantities being expressed in fractions of the canvas width and height

EXAMPLES:

sage: g1 = plot(sin(x), (x, -pi, pi))  
sage: g2 = circle((0,1), 1.)  
sage: G = graphics_array([g1, g2])  
sage: G.position(0)  # tol 5.0e-3  
(0.025045451349937315, 0.03415488992713045, 0.4489880779745068, 0.9345951100728696)  
sage: G.position(1)  # tol 5.0e-3  
(0.5170637412999687, 0.20212705964722733, 0.4489880779745068, 0.5986507706326758)

class sage.plot.multigraphics.MultiGraphics(graphics_list)
Bases: WithEqualityById, SageObject

Base class for objects composed of Graphics objects.

Both the display and the output to a file of MultiGraphics objects are governed by the method save(), which is called by the rich output display manager, via graphics_from_save().

The user interface is through the functions multi_graphics() (generic multi-graphics) and graphics_array() (subclass GraphicsArray).

INPUT:
• **graphics_list** – a list of graphics along with their positions on the common canvas; each element of **graphics_list** is either
  
  - a pair (graphics, position), where graphics is a *Graphics* object and position is the 4-tuple (left, bottom, width, height) specifying the location and size of the graphics on the canvas, all quantities being in fractions of the canvas width and height

  - or a single *Graphics* object; its position is then assumed to occupy the whole canvas, except for some padding; this corresponds to the default position (left, bottom, width, height) = (0.125, 0.11, 0.775, 0.77)

**EXAMPLES:**

A multi-graphics made from two graphics objects:

```sage
sage: g1 = plot(sin(x^3), (x, -pi, pi))
sage: g2 = circle((0,0), 1, color='red')
sage: G = multi_graphics([g1, (g2, (0.2, 0.55, 0.3, 0.3))])
sage: G
Multigraphics with 2 elements
```

Since no position was given for g1, it occupies the whole canvas. Moreover, we note that g2 has been drawn over g1 with a white background. To have a transparent background instead, one has to construct g2 with the keyword **transparent** set to True:
We can add a new graphics object to \( G \) via the method `append()`: 

```
    sage: g3 = complex_plot(zeta, (-20, 10), (-20, 20),
    ....:     axes_labels=['$x$', '$y$'], frame=True)
    sage: G.append(g3, pos=(0.63, 0.12, 0.3, 0.3))
    sage: G
    Multigraphics with 3 elements
```

We can access the individual elements composing \( G \) with the square-bracket operator:

```
    sage: print(G[0])
    Graphics object consisting of 1 graphics primitive
g7: G[0] is g1
    True
    sage: G[1] is g2
    True
    sage: G[2] is g3
    True
```
5.2. Graphics arrays and insets
G[::] returns the full list of graphics objects composing G:

```sage
G[::]
[Graphics object consisting of 1 graphics primitive,
 Graphics object consisting of 1 graphics primitive,
 Graphics object consisting of 1 graphics primitive]
```

```sage
len(G)
3
```

**append**(graphics, pos=None)

Append a graphics object to self.

**INPUT:**

- graphics – the graphics object (instance of Graphics) to be added to self
- pos – (default: None) 4-tuple \((\text{left}, \text{bottom}, \text{width}, \text{height})\) specifying the location and size of graphics on the canvas, all quantities being in fractions of the canvas width and height; if None, graphics is assumed to occupy the whole canvas, except for some padding; this corresponds to the default position \((\text{left}, \text{bottom}, \text{width}, \text{height}) = (0.125, 0.11, 0.775, 0.77)\)

**EXAMPLES:**

Let us consider a multigraphics with 2 elements:

```sage
g1 = plot(chebyshev_T(4, x), (x, -1, 1), title='n=4')
g2 = plot(chebyshev_T(8, x), (x, -1, 1), title='n=8',
 ....:   color='red')
G = multi_graphics([(g1, (0.125, 0.2, 0.4, 0.4)),
 ....:   (g2, (0.55, 0.4, 0.4, 0.4))])
sage: G
Multigraphics with 2 elements
```

We append a third plot to it:

```sage
g3 = plot(chebyshev_T(16, x), (x, -1, 1), title='n=16',
 ....:   color='brown')
G.append(g3, pos=(0.55, 0.11, 0.4, 0.15))
sage: G
Multigraphics with 3 elements
```

We may use append to add a title:

```sage
title = text("Chebyshev polynomials", (0, 0), fontsize=16,
 ....:   axes=False)
G.append(title, pos=(0.18, 0.8, 0.7, 0.1))
sage: G
Multigraphics with 4 elements
```

**See also:**

- `inset()`

**inset**(graphics, pos=None, fontsize=None)

Add a graphics object as an inset.

**INPUT:**

- graphics – the graphics object (instance of Graphics) to be added as an inset
5.2. Graphics arrays and insets
Chebyshev polynomials

n=4

n=8

n=16

5.2. Graphics arrays and insets
• pos – (default: None) 4-tuple (left, bottom, width, height) specifying the location and relative size of the inset on the canvas, all quantities being expressed in fractions of the canvas width and height; if None, the value (0.7, 0.7, 0.2, 0.2) is used

• fontsize – (default: None) integer, font size (in points) for the inset; if None, the value of 6 points is used, unless fontsize has been explicitly set in the construction of graphics (in this case, it is not overwritten here)

OUTPUT:

• instance of MultiGraphics

EXAMPLES:

Let us consider a graphics array of 2 elements:

\[
\begin{align*}
sage: G &= \text{graphics_array}([\text{plot} (\sin, (0, 2\pi)), \\
& \quad \quad \quad \quad \text{plot} (\cos, (0, 2\pi))]) \\
\end{align*}
\]

\[
\begin{align*}
sage: G \\
\text{Graphics Array of size 1 x 2}
\end{align*}
\]

and add some inset at the default position:

\[
\begin{align*}
sage: c &= \text{circle} ((0, 0), 1, \text{color}= 'red', \text{thickness}= 2, \text{frame}= \text{True}) \\
sage: G.\text{inset} (c) \\
\text{Multigraphics with 3 elements}
\end{align*}
\]
5.2. Graphics arrays and insets
We may customize the position and font size of the inset:

```python
sage: G.inset(c, pos=(0.3, 0.7, 0.2, 0.2), fontsize=8)
```

Multigraphics with 3 elements

---

```python
matplotlib(figure=None, figsize=None, **kwds)
```

Construct or modify a Matplotlib figure by drawing `self` on it.

**INPUT:**

- `figure` – (default: `None`) Matplotlib figure (class `matplotlib.figure.Figure`) on which `self` is to be displayed; if `None`, the figure will be created from the parameter `figsize`
- `figsize` – (default: `None`) width or `[width, height]` in inches of the Matplotlib figure in case `figure` is `None`; if `figsize` is `None`, Matplotlib's default (6.4 x 4.8 inches) is used
- `kwds` – options passed to the `matplotlib()` method of each graphics object constituting `self`

**OUTPUT:**

- a `matplotlib.figure.Figure` object; if the argument `figure` is provided, this is the same object as `figure`.

**EXAMPLES:**

Let us consider a `GraphicsArray` object with 3 elements:
sage: G = graphics_array([plot(sin(x^k), (x, 0, 3))
.....: for k in range(1, 4)])

If matplotlib() is invoked without any argument, a Matplotlib figure is created and contains the 3 graphics element of the array as 3 Matplotlib Axes:

sage: fig = G.matplotlib()
sage: fig
<Figure size 640x480 with 3 Axes>
sage: type(fig)
<class 'matplotlib.figure.Figure'>

Specifying the figure size (in inches):

sage: G.matplotlib(figsize=(8., 5.))
<Figure size 800x500 with 3 Axes>

If a single number is provided for figsize, it is considered to be the width; the height is then computed according to Matplotlib’s default aspect ratio (4/3):

sage: G.matplotlib(figsize=8.)
<Figure size 800x600 with 3 Axes>

An example of use with a preexisting created figure, created by pyplot:

sage: import matplotlib.pyplot as plt
sage: fig1 = plt.figure(1)
sage: fig1
<Figure size 640x480 with 0 Axes>
sage: fig_out = G.matplotlib(figure=fig1)
sage: fig_out
<Figure size 640x480 with 3 Axes>

Note that the output figure is the same object as the input one:

sage: fig_out is fig1
True

It has however been modified by G.matplotlib(figure=fig1), which has added 3 new Axes to it.

Another example, with a figure created from scratch, via Matplotlib’s Figure:

sage: from matplotlib.figure import Figure
sage: fig2 = Figure()
sage: fig2
<Figure size 640x480 with 0 Axes>
sage: G.matplotlib(figure=fig2)
<Figure size 640x480 with 3 Axes>
sage: fig2
<Figure size 640x480 with 3 Axes>

plot()
Return self since self is already a graphics object.

EXAMPLES:
sage: g1 = plot(cos, 0, 1)
sage: g2 = circle((0,0), 1)
sage: G = multi_graphics([g1, g2])
sage: G.plot() is G
True

position(index)
Return the position and relative size of an element of self on the canvas.

INPUT:

  • index – integer specifying which element of self

OUTPUT:

  • a 4-tuple (left, bottom, width, height) giving the location and relative size of the element on the canvas, all quantities being expressed in fractions of the canvas width and height

EXAMPLES:

sage: g1 = plot(sin(x^2), (x, 0, 4))
sage: g2 = circle((0,0), 1, rgbcolor='red', fill=True, axes=False)
sage: G = multi_graphics([g1, (g2, (0.15, 0.2, 0.1, 0.15))])
sage: G.position(0)  # tol 1.0e-13
(0.125, 0.11, 0.775, 0.77)
sage: G.position(1)  # tol 1.0e-13
(0.15, 0.2, 0.1, 0.15)

save(filename, figsize=None, **kwds)
Save self to a file, in various formats.

INPUT:

  • filename – (string) the file name; the image format is given by the extension, which can be one of the following:
    - .eps,
    - .pdf,
    - .png,
    - .ps,
    - .sobj (for a Sage object you can load later),
    - .svg,
    - empty extension will be treated as .sobj.
  • figsize – (default: None) width or [width, height] in inches of the Matplotlib figure; if none is provided, Matplotlib’s default (6.4 x 4.8 inches) is used
  • kwds – keyword arguments, like dpi=..., passed to the plotter, see show()

EXAMPLES:

sage: F = tmp_filename(ext='.png')
sage: L = [plot(sin(k*x), (x,-pi,pi)) for k in [1..3]]
sage: G = graphics_array(L)
sage: G.save(F, dpi=500, axes=False)
**save_image**(filename=None, *args, **kwds)

Save an image representation of self. The image type is determined by the extension of the filename. For example, this could be .png, .jpg, .gif, .pdf, .svg. Currently this is implemented by calling the `save()` method of self, passing along all arguments and keywords.

**Note:** Not all image types are necessarily implemented for all graphics types. See `save()` for more details.

**EXAMPLES:**
```
sage: plots = [[plot(m*cos(x + n*pi/4), (x, 0, 2*pi))
....: for n in range(3)] for m in range(1,3)]
sage: G = graphics_array(plots)
sage: G.save_image(tmp_filename(ext='.png'))
```

**show**(**kwds)

Show self immediately.

This method attempts to display the graphics immediately, without waiting for the currently running code (if any) to return to the command line. Be careful, calling it from within a loop will potentially launch a large number of external viewer programs.

**OPTIONAL INPUT:**
- **dpi** – dots per inch
- **figsize** – width or [width, height] of the figure, in inches; the default is 6.4 x 4.8 inches
- **axes** – boolean; if True, all individual graphics are endowed with axes; if False, all axes are removed (this overrides the axes option set in each graphics)
- **frame** – boolean; if True, all individual graphics are drawn with a frame around them; if False, all frames are removed (this overrides the frame option set in each graphics)
- **fontsize** – positive integer, the size of fonts for the axes labels (this overrides the fontsize option set in each graphics)

**OUTPUT:**
This method does not return anything. Use `save()` if you want to save the figure as an image.

**EXAMPLES:**
This draws a graphics array with four trig plots and no axes in any of the plots and a figure width of 4 inches:
```
sage: G = graphics_array([[plot(sin), plot(cos)],
....: [plot(tan), plot(sec)]])
sage: G.show(axes=False, figsize=4)
```

Same thing with a frame around each individual graphics:
```
sage: G.show(axes=False, frame=True, figsize=4)
```

Actually, many options are possible; for instance, we may set fontsize and gridlines:
```
sage: G.show(axes=False, frame=True, figsize=4, fontsize=8,
....: gridlines='major')
```
5.3 Plotting primitives

```python
class sage.plot.primitive.GraphicPrimitive(options)
    Bases: WithEqualityById, SageObject

    Base class for graphics primitives, e.g., things that knows how to draw themselves in 2D.

    EXAMPLES:

    We create an object that derives from GraphicPrimitive:

    sage: P = line([(-1,-2), (3,5)])
sage: P[0]
Line defined by 2 points
sage: type(P[0])
<class 'sage.plot.line.Line'>
```

```python
options()

    Return the dictionary of options for this graphics primitive.

    By default this function verifies that the options are all valid; if any aren’t, then a verbose message is printed
    with level 0.

    EXAMPLES:

    sage: from sage.plot.primitive import GraphicPrimitive
    sage: GraphicPrimitive({}).options()
{}
```

```python
plot3d(**kwds)

    Plots 3D version of 2D graphics object. Not implemented for base class.

    EXAMPLES:

    sage: from sage.plot.primitive import GraphicPrimitive
    sage: G=GraphicPrimitive({})
```
```python
sage: G.plot3d()
Traceback (most recent call last):
...
NotImplementedError: 3D plotting not implemented for Graphics primitive
```

**set_options** *(new_options)*

Change the options to *new_options*.

**EXAMPLES:**

```python
sage: from sage.plot.circle import Circle
sage: c = Circle(0,0,1,{}

sage: c.set_options({'thickness': 0.6})

sage: c.options()
{
    'thickness': 0.6...
```

**set_zorder** *(zorder)*

Set the layer in which to draw the object.

**EXAMPLES:**

```python
sage: P = line([-2,-3), (3,4)], thickness=4)

sage: p=P[0]

sage: p.set_zorder(2)

sage: p.options()['zorder']
2

sage: Q = line([-2,-4), (3,5)], thickness=4,zorder=1,hue=.5)

sage: P=Q # blue line on top

Graphics object consisting of 2 graphics primitives

sage: q=Q[0]

sage: q.set_zorder(3)

sage: P=Q # teal line on top

Graphics object consisting of 2 graphics primitives

sage: q.options()['zorder']
3
```

**class** *sage.plot.primitive.GraphicPrimitive_xydata*(options)

Bases: *GraphicPrimitive*

**get_minmax_data** *

Returns a dictionary with the bounding box data.

**EXAMPLES:**

```python
sage: d = polygon([[1,2], [5,6], [5,0]], rgbcolor=(1,0,1))[0].get_minmax_data()

sage: d['ymin']
0.0

sage: d['xmin']
1.0

sage: d = point((3, 3), rgbcolor=hue(0.75))[0].get_minmax_data()

sage: d['xmin']
3.0
```

(continues on next page)
sage: d['ymin']
3.0

sage: l = line(((100, 100), (120, 120)))[0]
sage: d = l.get_minmax_data()
sage: d['xmin']
100.0
sage: d['xmax']
120.0

5.4 Plotting utilities

class sage.plot.misc.Fast_callable_plotWrapper(ff, imag_tol)
Bases: Fast_callable_floatWrapper

A fast-callable wrapper for plotting that returns nan instead of raising an error whenever
the imaginary tolerance is exceeded.  

A detailed rationale for this can be found in the superclass documentation.

EXAMPLES:
The float incarnation of “not a number” is returned instead of an error being thrown if the answer is complex:

sage: from sage.plot.misc import Fast_callable_plotWrapper
sage: f = sqrt(x)
sage: ff = fast_callable(f, vars=[x], domain=CDF)
sage: fff = Fast_callable_plotWrapper(ff, imag_tol=1e-8)
sage: fff(1)
1.0
sage: fff(-1)
nan

sage.plot.misc.get_matplotlib_linestyle(linestyle, return_type)

Function which translates between matplotlib linestyle in short notation (i.e., '-', '--', ':', '-.'). and long notation
(i.e., 'solid', 'dashed', 'dotted', 'dashdot').

If linestyle is none of these allowed options, the function raises a ValueError.

INPUT:

• linestyle - The style of the line, which is one of
  - "-" or "solid"
  - "--" or "dashed"
  - ".-" or "dash dot"
  - ":" or "dotted"
  - "None" or " " or "" (nothing)

The linestyle can also be prefixed with a drawing style (e.g., "steps--")
  - "default" (connect the points with straight lines)
- "steps" or "steps-pre" (step function; horizontal line is to the left of point)
- "steps-mid" (step function; points are in the middle of horizontal lines)
- "steps-post" (step function; horizontal line is to the right of point)

If linestyle is None (of type NoneType), then we return it back unmodified.

- **return_type** - The type of linestyle that should be output. This argument takes only two values - "long" or "short".

**EXAMPLES:**

Here is an example how to call this function:

```python
sage: from sage.plot.misc import get_matplotlib_linestyle
sage: get_matplotlib_linestyle(':', return_type='short')
'short'

sage: get_matplotlib_linestyle(':', return_type='long')
'dotted'
```

**sage.plot.misc.setup_for_eval_on_grid**(*funcs*, *ranges*, *plot_points=None*, *return_vars=False*, *imaginary_tolerance=1e-08*)

Calculate the necessary parameters to construct a list of points, and make the functions fast_callable.

**INPUT:**

- **funcs** – a function, or a list, tuple, or vector of functions
- **ranges** – a list of ranges. A range can be a 2-tuple of numbers specifying the minimum and maximum, or a 3-tuple giving the variable explicitly.
- **plot_points** – a tuple of integers specifying the number of plot points for each range. If a single number is specified, it will be the value for all ranges. This defaults to 2.
- **return_vars** – (default False) If True, return the variables, in order.
- **imaginary_tolerance** – (default: 1e-8); if an imaginary number arises (due, for example, to numerical issues), this tolerance specifies how large it has to be in magnitude before we raise an error. In other words, imaginary parts smaller than this are ignored in your plot points.

**OUTPUT:**

- **fast_funcs** - if only one function passed, then a fast callable function. If funcs is a list or tuple, then a tuple of fast callable functions is returned.
- **range_specs** - a list of range_specs: for each range, a tuple is returned of the form (range_min, range_max, range_step) such that srange(range_min, range_max, range_step, include_endpoint=True) gives the correct points for evaluation.

**EXAMPLES:**

```python
sage: x,y,z=var('x,y,z')
sage: f(x,y)=x+y-z
sage: g(x,y)=x+y
sage: h(y)=-y
sage: sage.plot.misc.setup_for_eval_on_grid(f, [(0, 2),(1,3),(-4,1)], plot_points=5)
((<sage...>, [(0.0, 2.0, 0.5), (1.0, 3.0, 0.5), (-4.0, 1.0, 1.25)]))
sage: sage.plot.misc.setup_for_eval_on_grid([g,h], [(0, 2),(-1,1)], plot_points=5)
((<sage...>, <sage...>), [(0.0, 2.0, 0.5), (-1.0, 1.0, 0.5)])
```

(continues on next page)
sage: sage.plot.misc.setup_for_eval_on_grid([sin, cos], [(-1, 1)], plot_points=9)
((<sage...>, <sage...>), [(-1.0, 1.0, 0.25)])
sage: sage.plot.misc.setup_for_eval_on_grid([lambda x: x^2, cos], [(-1, 1)], plot_
  →points=9)
((<function <lambda> ...>, <sage...>), [(-1.0, 1.0, 0.25)])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,-1,1),(y,-2,2)]
((<sage...>,), [(-1.0, 1.0, 2.0), (-2.0, 2.0, 4.0)])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,-1,1),(y,-1,1)], plot_
  →points=[4,9])
((<function <lambda> ...>, <sage...>), [(-1.0, 1.0, 0.6666666666666666), (-1.0, 1.0, 0.25)])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,-1,1),(y,-1,1)], plot_
  →points=[4,9,10])
Traceback (most recent call last):
  ...)
ValueError: plot_points must be either an integer or a list of integers, one for␣  →each range
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(1,-1),(y,-1,1)], plot_points=[4,
  →9,10])
Traceback (most recent call last):
  ...
ValueError: Some variable ranges specify variables while others do not

Beware typos: a comma which should be a period, for instance:

sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x, 1, 2), (y, 0,1, 0.2)], plot_
  →points=[4,9,10])
Traceback (most recent call last):
  ...)
ValueError: At least one variable range has more than 3 entries: each should either␣  →have 2 or 3 entries, with one of the forms (xmin, xmax) or (x, xmin, xmax)
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(y,1,-1),(x,-1,1)], plot_points=5)
(<sage...>, [(-1.0, 1.0, 0.5), (-1.0, 1.0, 0.5)])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,-1),(x,-1,1)], plot_points=5)
Traceback (most recent call last):
  ...
ValueError: range variables should be distinct, but there are duplicates
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,1),(y,-1,1)])
Traceback (most recent call last):
  ...
ValueError: plot start point and end point must be different
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(x,1,-1),(y,-1,1)], return_
  →vars=True)
(<sage...>, [(-1.0, 1.0, 2.0), (-1.0, 1.0, 2.0)], [x, y])
sage: sage.plot.misc.setup_for_eval_on_grid(x+y, [(y,1,-1),(x,-1,1)], return_
  →vars=True)
(<sage...>, [(-1.0, 1.0, 2.0), (-1.0, 1.0, 2.0)], [y, x])
sage.plot.misc.unify_arguments(funcs)
Return a tuple of variables of the functions, as well as the number of “free” variables (i.e., variables that defined in a callable function).

INPUT:
• `funcs` – a list of functions; these can be symbolic expressions, polynomials, etc

**OUTPUT:** functions, expected arguments

• A tuple of variables in the functions

• A tuple of variables that were “free” in the functions

**EXAMPLES:**

```
sage: x,y,z=var('x,y,z')
sage: f(x,y)=x+y-z
sage: g(x,y)=x+y
sage: h(y)=-y
sage: sage.plot.misc.unify_arguments((f,g,h))
((x, y, z), (z,))
sage: sage.plot.misc.unify_arguments((g,h))
((x, y), ())
sage: sage.plot.misc.unify_arguments((f,z))
((x, y, z), (z,))
sage: sage.plot.misc.unify_arguments((h,z))
((y, z), (z,))
sage: sage.plot.misc.unify_arguments((x+y,x-y))
((x, y), (x, y))
```
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