Standard Semirings

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The Sage Development Team

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CHAPTER ONE

NON NEGATIVE INTEGER SEMIRING

class sage.rings.semirings.non_negative_integer_semiring.NonNegativeIntegerSemiring
    Bases: NonNegativeIntegers
    
    A class for the semiring of the non negative integers
    
    This parent inherits from the infinite enumerated set of non negative integers and endows it with its natural
    semiring structure.
    
    EXAMPLES:

    sage: NonNegativeIntegerSemiring()
    Non negative integer semiring
    
    For convenience, NN is a shortcut for NonNegativeIntegerSemiring():

    sage: NN == NonNegativeIntegerSemiring()
    True
    sage: NN.category()
    Category of facade infinite enumerated commutative semirings
    
    Here is a piece of the Cayley graph for the multiplicative structure:

    sage: G = NN.cayley_graph(elements=range(9), generators=[0,1,2,3,5,7])
    # needs sage.graphs
    sage: G
    # needs sage.graphs
    Looped multi-digraph on 9 vertices
    sage: G.plot()
    # needs sage.graphs sage.plot
    Graphics object consisting of 48 graphics primitives
    
    This is the Hasse diagram of the divisibility order on NN.
    
    sage: P = Poset(NN.cayley_graph(elements=[1..12], generators=[2,3,5,7,11])).show() # needs
    sage.combinat sage.graphs sage.plot
    
    Note: as for NonNegativeIntegers, NN is currently just a “facade” parent; namely its elements are plain Sage
    Integers with Integer Ring as parent:

    sage: x = NN(15); type(x)
    <class 'sage.rings.integer.Integer'>
    sage: x.parent()
    
    (continues on next page)
additive_semigroup_generators()

Returns the additive semigroup generators of self.

EXAMPLES:

```
sage: NN.additive_semigroup_generators()
Family (0, 1)
```
CHAPTER

TWO

TROPICAL SEMIRINGS

AUTHORS:

• Travis Scrimshaw (2013-04-28) - Initial version

class sage.rings.semirings.tropical_semiring.TropicalSemiring(base, use_min=True)

    Bases: Parent, UniqueRepresentation

    The tropical semiring.

    Given an ordered additive semigroup $R$, we define the tropical semiring $T = R \cup \{+\infty\}$ by defining tropical addition and multiplication as follows:

    $a \oplus b = \min(a, b), \quad a \odot b = a + b.$

    In particular, note that there are no (tropical) additive inverses (except for $\infty$), and every element in $R$ has a (tropical) multiplicative inverse.

    There is an alternative definition where we define $T = R \cup \{-\infty\}$ and alter tropical addition to be defined by

    $a \oplus b = \max(a, b).$

    To use the $\max$ definition, set the argument $use\_min = False$.

    Warning: $zero()$ and $one()$ refer to the tropical additive and multiplicative identities respectively. These are not the same as calling $T(0)$ and $T(1)$ respectively as these are not the tropical additive and multiplicative identities respectively.

    Specifically do not use $\text{sum}(\ldots)$ as this converts 0 to 0 as a tropical element, which is not the same as $zero()$. Instead use the $\text{sum}$ method of the tropical semiring:

    sage: T = TropicalSemiring(QQ)
    sage: sum([T(1), T(2)]) # This is wrong
    0
    sage: T.sum([T(1), T(2)]) # This is correct
    1

    Be careful about using code that has not been checked for tropical safety.

INPUT:

• base – the base ordered additive semigroup $R$

• use\_min – (default: True) if True, then the semiring uses $a \oplus b = \min(a, b)$; otherwise uses $a \oplus b = \max(a, b)$
EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2); elt
2
```

Recall that tropical addition is the minimum of two elements:

```
sage: T(3) + T(5)
3
```

Tropical multiplication is the addition of two elements:

```
sage: T(2) * T(3)
5
sage: T(0) * T(-2)
-2
```

We can also do tropical division and arbitrary tropical exponentiation:

```
sage: T(2) / T(1)
1
sage: T(2)^(-3/7)
-6/7
```

Note that “zero” and “one” are the additive and multiplicative identities of the tropical semiring. In general, they are not the elements 0 and 1 of \( R \), respectively, even if such elements exist (e.g., for \( R = \mathbb{Z} \)), but instead the (tropical) additive and multiplicative identities \(+\infty\) and 0 respectively:

```
sage: T.zero() + T(3) == T(3)
True
sage: T.one() * T(3) == T(3)
True
sage: T.zero() == T(0)
False
sage: T.one() == T(1)
False
```

Element

alias of `TropicalSemiringElement`

```
additive_identity()
```

Return the (tropical) additive identity element \(+\infty\).

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

gens()

Return the generators of `self`.

EXAMPLES:
sage: T = TropicalSemiring(QQ)
sage: T.gens()
(1, +infinity)

infinity()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity

multiplicative_identity()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: T.one()
0

one()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: T.one()
0

zero()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity

class sage.rings.semirings.tropical_semiring.TropicalSemiringElement

Bases: Element

An element in the tropical semiring over an ordered additive semigroup $R$. Either in $R$ or $\infty$. The operators $+$, $\cdot$ are defined as the tropical operators $\oplus$, $\odot$ respectively.

lift()

Return the value of self lifted to the base.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: elt = T(2)
sage: elt.lift()
2
sage: elt.lift().parent() is QQ
(continues on next page)
True

```
sage: T.additive_identity().lift().parent()
The Infinity Ring
```

### multiplicative_order()

Return the multiplicative order of `self`.

**EXAMPLES:**

```
sage: T = TropicalSemiring(QQ)
sage: T.multiplicative_identity().multiplicative_order()
1
sage: T.additive_identity().multiplicative_order()
+Infinity
```

### class sage.rings.semirings.tropical_semiring.TropicalToTropical

**Bases:** Map

Map from the tropical semiring to itself (possibly with different bases). Used in coercion.
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