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sage.rings.semirings.non_negative_integer_semiring.NN = Non negative integer semiring

class sage.rings.semirings.non_negative_integer_semiring.NonNegativeIntegerSemiring
    Bases: sage.sets.non_negative_integers.NonNegativeIntegers

A class for the semiring of the non negative integers

This parent inherits from the infinite enumerated set of non negative integers and endows it with its natural
semiring structure.

EXAMPLES:

sage: NonNegativeIntegerSemiring()
Non negative integer semiring

For convenience, NN is a shortcut for NonNegativeIntegerSemiring():

sage: NN == NonNegativeIntegerSemiring()
True

sage: NN.category()
Category of facade infinite enumerated commutative semirings

Here is a piece of the Cayley graph for the multiplicative structure:

sage: G = NN.cayley_graph(elements=range(9), generators=[0,1,2,3,5,7])
sage: G
Looped multi-digraph on 9 vertices
sage: G.plot()
Graphics object consisting of 48 graphics primitives

This is the Hasse diagram of the divisibility order on NN.

    sage: Poset(NN.cayley_graph(elements=[1..12], generators=[2,3,5,7,11])).show()

Note: as for NonNegativeIntegers, NN is currently just a “facade” parent; namely its elements are plain
Sage Integers with Integer Ring as parent:

sage: x = NN(15); type(x)
<type 'sage.rings.integer.Integer'>
sage: x.parent()
Integer Ring
sage: x+3
18
additive_semigroup_generators()  
Returns the additive semigroup generators of self.

EXAMPLES:

```
sage: NN.additive_semigroup_generators()
Family (0, 1)
```
TROPICAL SEMIRINGS

AUTHORS:

- Travis Scrimshaw (2013-04-28) - Initial version

```python
class sage.rings.semirings.tropical_semiring.TropicalSemiring(base, use_min=True):
    Bases: sage.structure.parent.Parent, sage.structure.unique_representation.UniqueRepresentation

The tropical semiring.

Given an ordered additive semigroup \( R \), we define the tropical semiring \( T = R \cup \{+\infty\} \) by defining tropical addition and multiplication as follows:

\[
a \oplus b = \min(a, b), \quad a \odot b = a + b.
\]

In particular, note that there are no (tropical) additive inverses (except for \( \infty \)), and every element in \( R \) has a (tropical) multiplicative inverse.

There is an alternative definition where we define \( T = R \cup \{-\infty\} \) and alter tropical addition to be defined by

\[
a \oplus b = \max(a, b).
\]

To use the \( \max \) definition, set the argument `use_min = False`.

**Warning:** \( \texttt{zero()} \) and \( \texttt{one()} \) refer to the tropical additive and multiplicative identities respectively. These are not the same as calling \( \texttt{T(0)} \) and \( \texttt{T(1)} \) respectively as these are not the tropical additive and multiplicative identities respectively.

Specifically do not use \( \texttt{sum(...)} \) as this converts \( 0 \) to \( 0 \) as a tropical element, which is not the same as \( \texttt{zero()} \). Instead use the \( \texttt{sum} \) method of the tropical semiring:

```python
sage: T = TropicalSemiring(QQ)
sage: sum([T(1), T(2)]) # This is wrong
0
sage: T.sum([T(1), T(2)]) # This is correct
1
```

Be careful about using code that has not been checked for tropical safety.

**INPUT:**

- `base` – the base ordered additive semigroup \( R \)
• use_min – (default: True) if True, then the semiring uses \( a \oplus b = \min(a, b) \); otherwise uses \( a \oplus b = \max(a, b) \)

EXAMPLES:

```python
sage: T = TropicalSemiring(QQ)
sage: elt = T(2); elt
2
Recall that tropical addition is the minimum of two elements:

```python
sage: T(3) + T(5)
3
```

Tropical multiplication is the addition of two elements:

```python
sage: T(2) * T(3)
5
sage: T(0) * T(-2)
-2
```

We can also do tropical division and arbitrary tropical exponentiation:

```python
sage: T(2) / T(1)
1
sage: T(2)^(-3/7)
-6/7
```

Note that “zero” and “one” are the additive and multiplicative identities of the tropical semiring. In general, they are **not** the elements 0 and 1 of \( R \), respectively, even if such elements exist (e.g., for \( R = \mathbb{Z} \)), but instead the (tropical) additive and multiplicative identities \(+\infty\) and 0 respectively:

```python
sage: T.zero() + T(3) == T(3)
True
sage: T.one() * T(3) == T(3)
True
sage: T.zero() == T(0)
False
sage: T.one() == T(1)
False
```

**Element**

alias of `TropicalSemiringElement`

**additive_identity()**

Return the (tropical) additive identity element \(+\infty\).

EXAMPLES:

```python
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

**gens()**

Return the generators of `self`.

EXAMPLES:
```python
sage: T = TropicalSemiring(QQ)
sage: T.gens()
(1, +infinity)
```

### infinity()

Return the (tropical) additive identity element \(+\infty\).

**EXAMPLES:**

```python
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

### multiplicative_identity()

Return the (tropical) multiplicative identity element 0.

**EXAMPLES:**

```python
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

### one()

Return the (tropical) multiplicative identity element 0.

**EXAMPLES:**

```python
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

### zero()

Return the (tropical) additive identity element \(+\infty\).

**EXAMPLES:**

```python
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

### class sage.rings.semirings.tropical_semiring.TropicalSemiringElement

An element in the tropical semiring over an ordered additive semigroup \(R\). Either in \(R\) or \(+\infty\). The operators \(+,\cdot\) are defined as the tropical operators \(\oplus,\odot\) respectively.

### lift()

Return the value of \(self\) lifted to the base.

**EXAMPLES:**

```python
sage: T = TropicalSemiring(QQ)
sage: elt = T(2)
sage: elt.lift()
2
sage: elt.lift().parent() is QQ
True
sage: T.additive_identity().lift().parent()
The Infinity Ring
```
\texttt{multiplicative\_order()}

Return the multiplicative order of \texttt{self}.

\textbf{EXAMPLES:}

\begin{verbatim}
\texttt{sage: T = TropicalSemiring(QQ)}
\texttt{sage: T.multiplicative_identity().multiplicative_order()}
\texttt{1}
\texttt{sage: T.additive_identity().multiplicative_order()}
\texttt{+Infinity}
\end{verbatim}

class \texttt{sage.rings.semirings.tropical_semiring.TropicalToTropical}

\textbf{Bases:} \texttt{sage.categories.map.Map}

Map from the tropical semiring to itself (possibly with different bases). Used in coercion.
CHAPTER THREE

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