Standard Semirings

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The Sage Development Team

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CHAPTER ONE

NON NEGATIVE INTEGER SEMIRING

sage.rings.semirings.non_negative_integer_semiring.NN = Non negative integer semiring
class sage.rings.semirings.non_negative_integer_semiring.NonNegativeIntegerSemiring
    Bases: sage.sets.non_negative_integers.NonNegativeIntegers

A class for the semiring of the non negative integers
This parent inherits from the infinite enumerated set of non negative integers and endows it with its natural semiring structure.

EXAMPLES:

```
sage: NonNegativeIntegerSemiring()
Non negative integer semiring

For convenience, NN is a shortcut for NonNegativeIntegerSemiring():

```
sage: NN == NonNegativeIntegerSemiring()
True
sage: NN.category()
Category of facade infinite enumerated commutative semirings
```

Here is a piece of the Cayley graph for the multiplicative structure:

```
sage: G = NN.cayley_graph(elements=range(9), generators=[0,1,2,3,5,7])
sage: G
Looped multi-digraph on 9 vertices
sage: G.plot()
Graphics object consisting of 48 graphics primitives
```

This is the Hasse diagram of the divisibility order on NN.

```
sage: Poset(NN.cayley_graph(elements=[1..12], generators=[2,3,5,7,11])).show()
```

Note: as for NonNegativeIntegers, NN is currently just a “facade” parent; namely its elements are plain Sage Integers with Integer Ring as parent:

```
sage: x = NN(15); type(x)
<class 'sage.rings.integer.Integer'>
sage: x.parent()
Integer Ring
sage: x+3
18
```
additive_semigroup_generators()

Returns the additive semigroup generators of self.

EXAMPLES:

```
sage: NN.additive_semigroup_generators()
Family (0, 1)
```
TROPICAL SEMIRINGS

AUTHORS:

- Travis Scrimshaw (2013-04-28) - Initial version

```python
class sage.rings.semirings.tropical_semiring.TropicalSemiring(base, use_min=True):
    Bases: sage.structure.parent.Parent, sage.structure.unique_representation.UniqueRepresentation
    The tropical semiring.
    Given an ordered additive semigroup $R$, we define the tropical semiring $T = R \cup \{+\infty\}$ by defining tropical addition and multiplication as follows:
    
    $a \oplus b = \min(a, b), \quad a \odot b = a + b.$
    
    In particular, note that there are no (tropical) additive inverses (except for $\infty$), and every element in $R$ has a (tropical) multiplicative inverse.
    
    There is an alternative definition where we define $T = R \cup \{-\infty\}$ and alter tropical addition to be defined by
    
    $a \oplus b = \max(a, b).$
    
    To use the max definition, set the argument `use_min = False`.
```

**Warning:** `zero()` and `one()` refer to the tropical additive and multiplicative identities respectively. These are not the same as calling $T(0)$ and $T(1)$ respectively as these are not the tropical additive and multiplicative identities respectively.

Specifically do not use `sum(...)` as this converts 0 to 0 as a tropical element, which is not the same as `zero()`. Instead use the `sum` method of the tropical semiring:

```python
sage: T = TropicalSemiring(QQ)
sage: sum([T(1), T(2)]) # This is wrong
0
sage: T.sum([T(1), T(2)]) # This is correct
1
```

Be careful about using code that has not been checked for tropical safety.

**INPUT:**

- `base` – the base ordered additive semigroup $R$
• use_min – (default: True) if True, then the semiring uses \( a \oplus b = \min(a, b) \); otherwise uses \( a \oplus b = \max(a, b) \)

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2); elt
2
```

Recall that tropical addition is the minimum of two elements:

```
sage: T(3) + T(5)
3
```

Tropical multiplication is the addition of two elements:

```
sage: T(2) * T(3)
5
sage: T(0) * T(-2)
-2
```

We can also do tropical division and arbitrary tropical exponentiation:

```
sage: T(2) / T(1)
1
sage: T(2) ^ (-3/7)
-6/7
```

Note that “zero” and “one” are the additive and multiplicative identities of the tropical semiring. In general, they are not the elements 0 and 1 of \( R \), respectively, even if such elements exist (e.g., for \( R = \mathbb{Z} \)), but instead the (tropical) additive and multiplicative identities \(+\infty\) and 0 respectively:

```
sage: T.zero() + T(3) == T(3)
True
sage: T.one() * T(3) == T(3)
True
sage: T.zero() == T(0)
False
sage: T.one() == T(1)
False
```

Element

alias of TropicalSemiringElement

additive_identity()

Return the (tropical) additive identity element \(+\infty\).

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

gens()

Return the generators of self.

EXAMPLES:
```python
sage: T = TropicalSemiring(QQ)
sage: T.gens()
(1, +infinity)
```

\section*{Infinity}

Return the (tropical) additive identity element $+\infty$.

\section*{Examples:}
```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

\section*{Multiplicative Identity}

Return the (tropical) multiplicative identity element 0.

\section*{Examples:}
```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

\section*{One}

Return the (tropical) multiplicative identity element 0.

\section*{Examples:}
```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

\section*{Zero}

Return the (tropical) additive identity element $+\infty$.

\section*{Examples:}
```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

\section*{Class sage.rings.semirings.tropical_semiring.TropicalSemiringElement}

\section*{Bases: sage.structure.element.Element}

An element in the tropical semiring over an ordered additive semigroup $R$. Either in $R$ or $\infty$. The operators $+, \cdot$ are defined as the tropical operators $\oplus, \odot$ respectively.

\section*{Lift}

Return the value of \texttt{self} lifted to the base.

\section*{Examples:}
```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2)
sage: elt.lift()
2
sage: elt.lift().parent() is QQ
True
```

(continues on next page)
sage: T.additive_identity().lift().parent()
The Infinity Ring

multiplicative_order()

Return the multiplicative order of self.

EXAMPLES:

sage: T = TropicalSemiring(QQ)
sage: T.multiplicative_identity().multiplicative_order()
1
sage: T.additive_identity().multiplicative_order()
+Infinity

class sage.rings.semirings.tropical_semiring.TropicalToTropical

Map from the tropical semiring to itself (possibly with different bases). Used in coercion.
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